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ASSESSMENT OF NATURAL FREQUENCY FROM MICROTREMOR MEASUREMENT USING PHASE SPECTRUM

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ABSTRACT

A technique to estimate the natural frequency of the ground by using the phase spectrum from right-side autocorrelogram of a horizontal component of microtremors is proposed. It is known that autocorrelation function has a null phase spectrum because its symmetry. But, a particular phase spectral curve appears when only right-side autocorrelogram is taken into account. The intersection of the phase spectral curve and the abscissa is showing good correspondence to the natural frequency of the ground at measured sites. Some results by using this method are shown and good correspondence between the minimum zero-crossing frequency of phase spectrum and the first predominant frequency of transfer function for horizontal motion of surface layers was obtained. Also, certain correspondence was obtained for the results of Nakamura's method. Even in the case that some strong artificial noise (human activity noise) is included in the analyzed time frame, the first predominant frequency could be clearly extracted. It can be concluded that the proposed method is available as an additional tool to predict much better the natural frequency of the measured sites.

KEYWORDS

Natural frequency, right-side autocorrelogram, phase spectrum, Fourier transform, autocorrelation, zero-crossing of phase spectrum.

INTRODUCTION

The technique of microtremor measurements has been widely used all over the world for seismic microzoning purposes since the pioneering works of Prof. Kiyoshi Kanai (Kanai and Tanaka, 1961). Random data of microtremors are usually considered as a stationary random process. At the beginning, autocorrelogram was used as an auxiliary tool to evaluate approximately the predominant period of the ground, but fell into disuse after the Cooley and Tukey algorism (Cooley and Tukey, 1965) for Fourier transform (FFT) was developed. It is sometimes difficult, however, to find the value of the first vibration mode of the ground from amplitude spectrum of a horizontal component of microtremors. After a certain lapse of time, Nakamura proposed the practical method by using H/V (horizontal/vertical) spectral ratio as a tool to find it (Nakamura, 1989), known as Nakamura's method which has been used frequently thereafter. But again, sometimes estimation of the first predominant frequency is difficult because several peaks appear in H/V spectral ratio diagram.

Autocorrelation function finds out clearly deterministic noise data which might be masked in a random noise (Bendat and Piersol, 1971). It is well known that an autocorrelogram presents a null phase spectrum ($\phi = 0$) because it is always a real-valued even function (symmetrical function). However, a particular phase spectral curve appears when only right-side autocorrelogram is considered. In this paper, a new method as an auxiliary tool for estimating the natural frequency of the ground based on phase spectrum of right-side autocorrelogram applied to one horizontal component of microtremors is proposed.

METHODOLOGY

Data processing of the proposed method is shown schematically in Fig. 1. Outline of the process is as follows. At first, a corrected horizontal microtremor record which has zero amplitude average ($\mu_x=0$) is used to avoid a certain error, especially in lower frequency range. After the autocorrelogram is obtained, only right-side autocorrelogram is taken into account. Then, its phase spectrum can be got by applying fast Fourier transform analysis. Smoothing of phase spectral curve by using equivalent band width filter is better to find the intersections with abscissa. These intersections are considered as the predominant frequencies in Fourier amplitude spectrum of the original microtremor record. The minimum zero-crossing frequency of phase spectral curve can be used to estimate the first natural frequency of the ground.

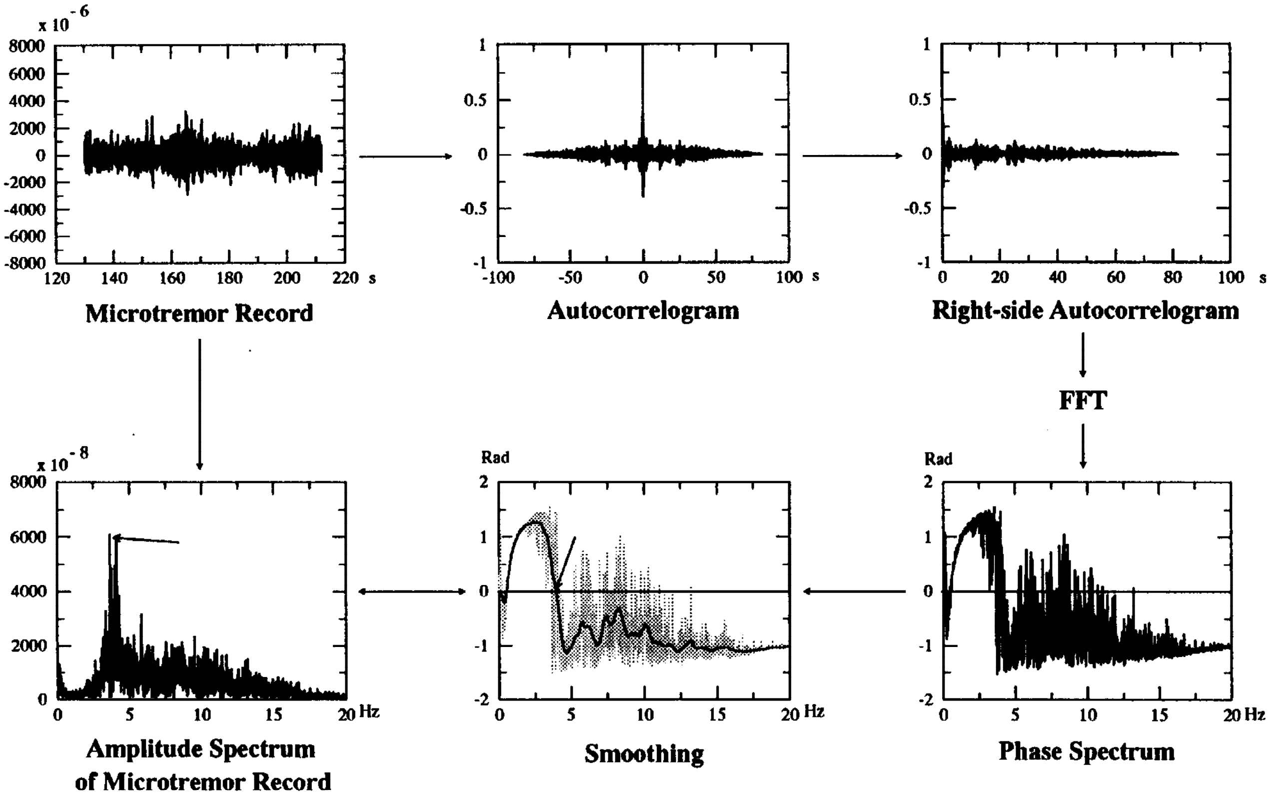


Fig.1. Data processing of the proposed method for microtremor record.

Mathematical expression of the content mentioned above is shown below. If a time function such as microtremors $f(t)$ has the Fourier transform $F(\omega)$, then its autocorrelogram $\int_{-\infty}^{\infty} f^*(u)f(u+t)du$ has the Fourier amplitude $|F(\omega)|^2$, and consequently the right-side autocorrelogram $\int_{0}^{\infty} f^*(u)f(u+t)du$ has the Fourier amplitude $|F(\omega)|^2/2$. So, the predominant frequencies of the Fourier amplitude spectrum from original record $f(t)$ is corresponding to the predominant frequencies of the Fourier amplitude spectrum of its right-side autocorrelogram. It can be possible to discuss both of amplitude spectrum and phase spectrum, because the right-side autocorrelogram has phase characteristics.

Now let consider the case that the time function $f(t)$ is stationary, the right-side autocorrelogram can be expressed as the following equation.

$$y(t) = \begin{cases} 0 \; ; \; t < 0 \\ e^{-\kappa t} \cos(\omega_0 t) \; ; \; t \geq 0 \wedge \kappa > 0 \end{cases} \tag{1}$$

in which, κ is the attenuation factor and ω_0 is the initial natural frequency. Shape of Eq.(1) is shown in Fig. 2 for various values of κ. The greater κ is, the greater attenuation is.

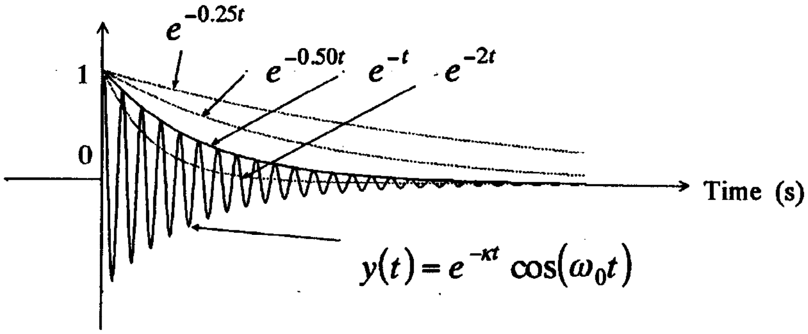


Fig. 2. Right-side autocorrelogram for the case of stationary time function.

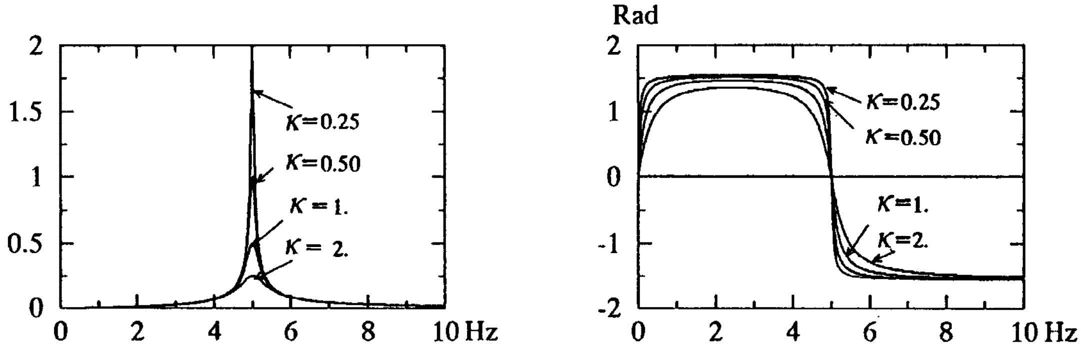
After Fourier transform is applied, Fourier spectrum of Eq.(1) is given by

$$|F[y(t)]| = \frac{\sqrt{\left[\kappa\left(\kappa^2 + \omega_0^2 + \omega^2\right)\right]^2 + \left[\omega\left(\kappa^2 - \omega_0^2 + \omega^2\right)\right]^2}}{\left(\kappa^2 + \omega_0^2 - \omega^2\right)^2 + 4\kappa^2\omega^2} \tag{2}$$

and the phase spectrum is given by

$$\phi = \arctan\left[\frac{-\omega\left(\kappa^2 - \omega_0^2 + \omega^2\right)}{\kappa\left(\kappa^2 + \omega_0^2 + \omega^2\right)}\right] \tag{3}$$

Examples of the spectra of the cases for $\omega_0 = 5$ Hz and $\kappa = 0.25, 0.5, 1, 2$ are shown in Fig.3. It can be seen that there is an evident relation between both, the predominant frequency of Fourier spectra and the zero-crossing frequency of phase spectra, i.e., the frequency at which a sharp peak appears in Fourier amplitude spectrum is corresponding to the zero-crossing frequency in phase spectrum. As κ is increase, peak value of Fourier amplitude spectrum becomes small like a low hill and slope at zero-crossing point in phase spectrum becomes gentle. Phase angle varies of the range of $\pm\pi/2$. Strictly speaking, microtremor is not a stationary process, but has a close resemblance. So that, the methodology mentioned above is applied to the case of microtremor hereafter.



Amplitude Spectrum **Phase Spectrum**

Fig. 3. Amplitude spectra and phase spectra for right-side autocorrelogram.

APPLICATION AND RESULTS

Microtremors observed in Kushiro City were used as analytical examples of the proposed method. Locations of observation sites are shown in Fig. 4 and the results of data processing are shown in Fig. 5. There can be seen good correspondence between the zero-crossing frequencies of phase spectrum and the predominant frequencies of Fourier amplitude spectrum in each case.

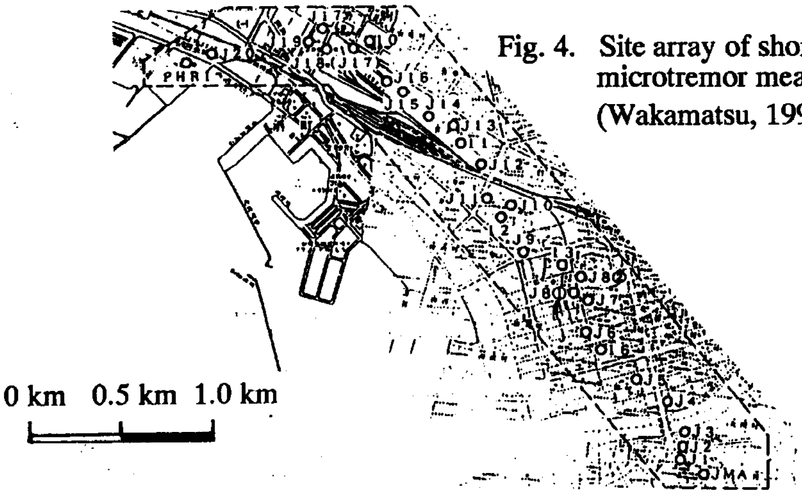


Fig. 4. Site array of short period sensor microtremor measurement. (Wakamatsu, 1994).

Figure 6 shows schematically the predominant periods (white circles) at each observation site with soil profiles evaluated by Wakamatsu (1994) by applying Nakamura's method. Wakamatsu (1994) pointed out that the longest period at each site may be corresponding to the first mode from the bottom of the alluvial deposits and the second or higher period is produced mainly from the upper surface layers, so that the bottom of the alluvial deposits in the area of which the boundary estimated as dotted line might be deeper. Black triangles shows the predominant periods determined by the proposed method. Again, good correspondence can be seen between both methods, i.e., the results of the proposed method supports Wakamatsu's suggestion.

The results of both Nakamura's method and the proposed method were compared in detail for the case of the microtremor observation point in Kushiro Meteorological Observatory (JMA) as shown in Fig. 7. Table 1 shows the two borehole data at the points A and B. Transfer functions between the ground surface and the bedrock (A: GL-20m, B: GL-21m) were computed by using these data as shown in Fig. 8. The first natural frequencies are 3.81 and 4.0 Hz for the points A and B, respectively.

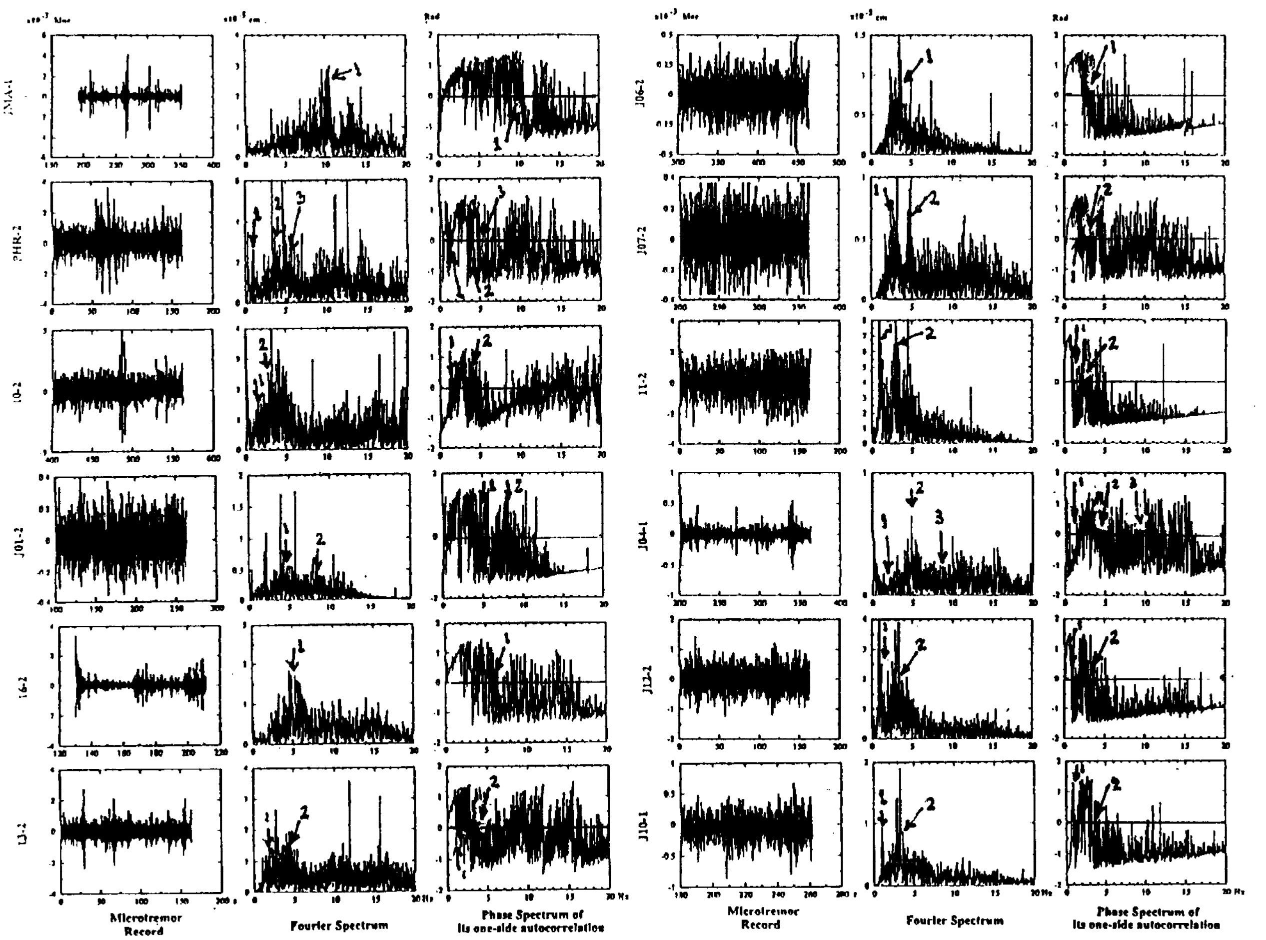
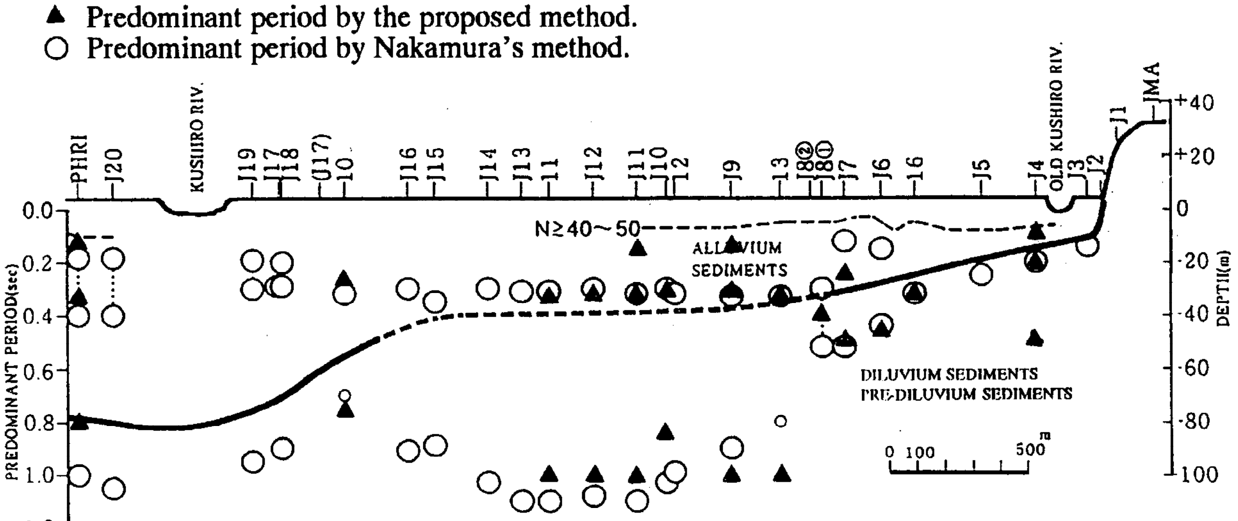


Fig. 5. N-S component microtremors, Fourier amplitude spectrum and phase spectrum of its right-side autocorrelogram at the observation sites in Kushiro City.



(After Wakamatsu, 1994)

Fig. 6. A comparison between predominant period of microtremors by using the proposed method and Nakamura's method over cross section for JMA-PHRI observation sites. The section of alluvium sediments is also plotted. The thick line shows the bottom of alluvium sediments and fine broken line shows the boundary indicating N-value over 40~50.

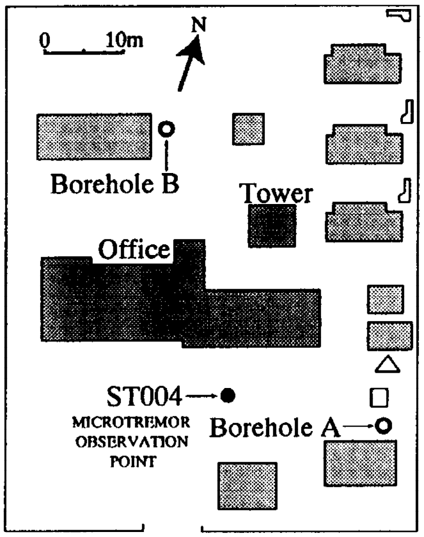


Fig. 7. Location of the two boreholes and ST04 at Kushiro Local Meteorological Observatory.

Borehole A

Depth GL - (m)	Soil classification	P wave velocity (m/s)	S wave velocity (m/s)	Poisson ratio	Density (g/cm^3)
0 ~ 1.00	Quarry sand	230	110	0.352	1.56
~ 1.90	Silty volc. ash, sand		140	0.206	
~ 6.85	Silty volcanic ash				1.63
~ 8.15	Volcanic ash	1,130	260	0.472	1.67
~ 14.00	Fine sand with volc. ash		310	0.459	1.78
~ 17.15	Fine sand with volc. ash		350	0.447	
~ 20.00	Sandstone	2,850	650	0.473	1.89

Borehole B

Depth GL - (m)	Soil classification	P wave velocity (m/s)	S wave velocity (m/s)	Poisson ratio	Density (g/cm^3)
0 ~ 1.00	Silty volcanic ash	230	120	0.313	1.59
~ 5.50	Silty volcanic ash		140	0.206	
~ 6.80	Volcanic ash	610	220	0.425	1.65
~ 12.10	Fine sand with volc. ash		340	0.275	1.77
~ 13.00	Fine sand		310	0.326	
~ 15.30	Sand	1,600		0.480	
~ 18.40	Coarse sand		340	0.476	
~ 21.00	Mudstone	2,600	510	0.480	1.82

Table 1. Borehole data for A and B at Kushiro Local Meteorological Observatory.

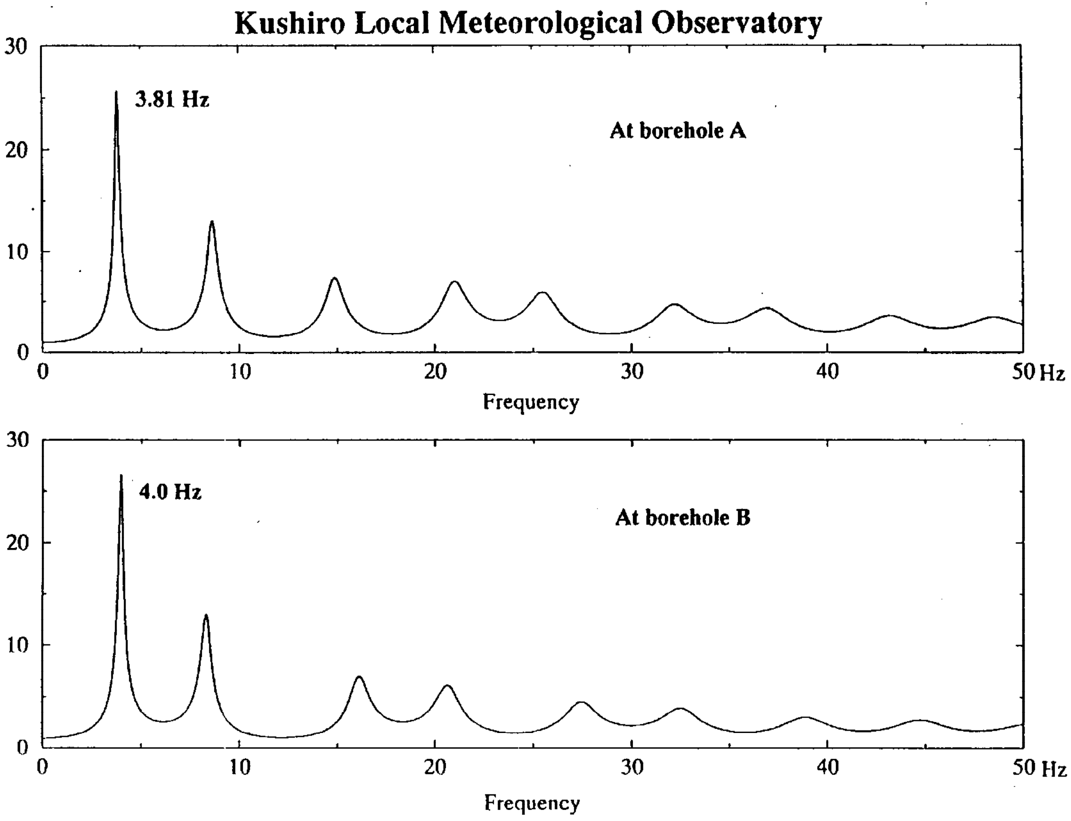


Fig. 8. SH wave transfer function for two boreholes near ST04 at Kushiro Local Meteorological Observatory.

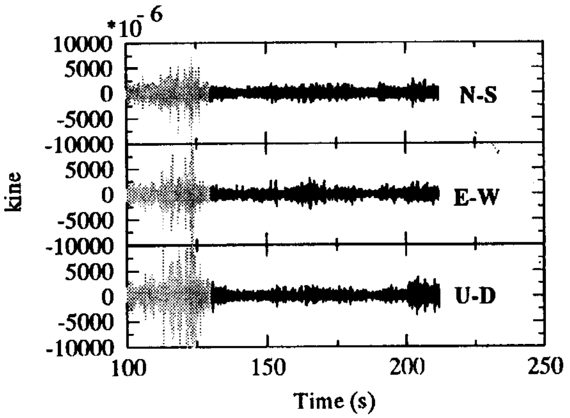


Fig. 9. Stationary type microtremors at Kushiro Local Meteorological Observatory.

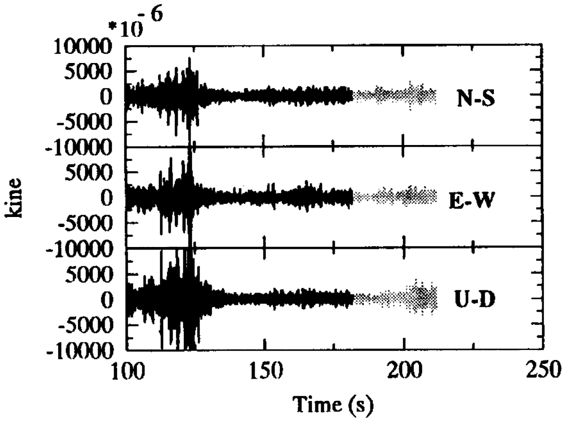


Fig. 10. Microtremors with some strong disturbances at Kushiro Local Meteorological Observatory.

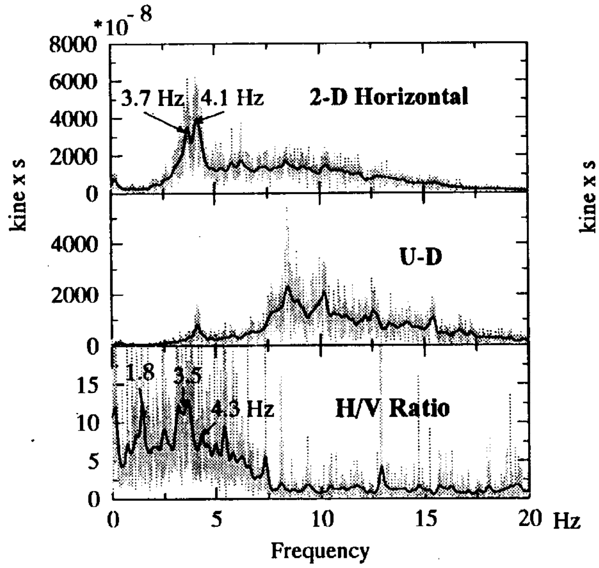


Fig. 11. Fourier spectra for stationary type microtremors.

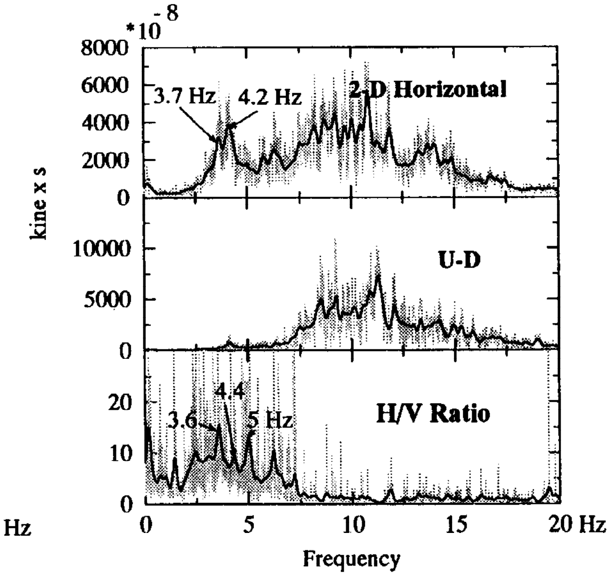


Fig. 12. Fourier spectra for microtremors with some strong disturbances .

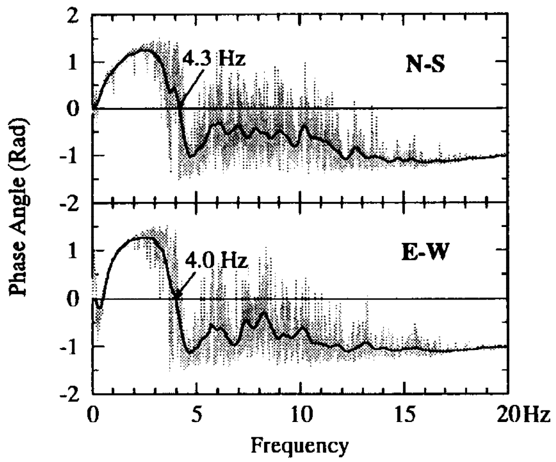


Fig. 13. Phase spectra of the right-side autocorrelogram for stationary type microtremors.

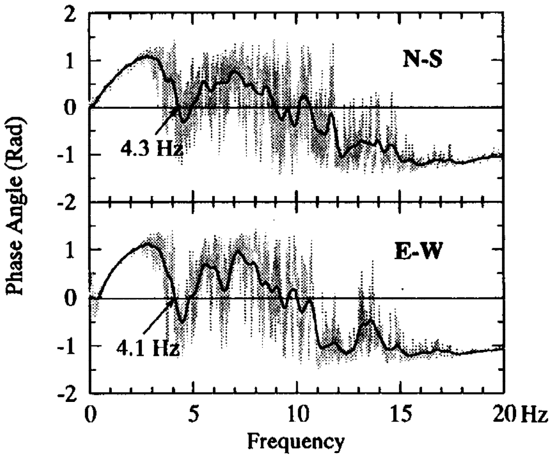


Fig. 14. Phase spectra of the right-side autocorrelation for some strong disturbances.

Two parts of microtremor record were selected as analytical objects, i.e., stationary type microtremors and microtremors with some strong disturbance, three components of which are shown in Fig. 9 and Fig. 10. 2-D horizontal Fourier spectra, Fourier spectra of component (U-D) and H/V spectral ratio by Nakamura's method are shown in Figs. 11 and Fig. 12, respectively. Phase spectra based on the proposed method for both cases are shown in Fig. 13 and Fig. 14. As for the case of stationary type microtremors in Fig. 11, predominant frequencies are 3.7 and 4.1 Hz in 2-D horizontal Fourier spectrum, whereas 1.8, 3.5 and 4.3 Hz in H/V spectral ratio diagram. The frequency 1.8 Hz should be rejected because the spectral amplitude at this frequency is small in either 2-D horizontal Fourier spectrum or U-D spectrum. In relation to the proposed method, the values of 4.3 and 4.0 Hz were obtained from Fig. 13 for N-S and E-W components respectively for the case of stationary type microtremors.

In the case of microtremors with some strong disturbance, two of the smallest predominant frequencies 3.7 and 4.2 Hz were obtained from 2-D horizontal Fourier spectrum and three predominant frequencies 3.6, 4.4 and 5.0 Hz were obtained from Nakamura's H/V spectrum in Fig. 12. While the values of 4.3 and 4.1 Hz for N-S and E-W components were obtained from the phase spectra in Fig.14 as the results of the proposed method. It can be said, therefore, that there is good correspondence between the results of the proposed phase spectral method and the natural frequency of the ground at analyzed site. Likewise, the results of Nakamura's method also shows good correspondence but there appear frequently many peaks which make complication of finding the natural frequency of the ground. The proposed method is easier and simpler than Nakamura's method, because only one horizontal component of microtremor is sufficient to estimate the natural frequency of the ground.

CONCLUSIONS

A relation between the zero-crossings of the phase spectrum from right-side autocorrelogram and the predominant frequencies from Fourier amplitude spectrum of microtremor has been investigated and a new method based on zero-crossing phase spectrum was proposed to estimate the natural frequency of the ground. It can be concluded that there is good correspondence between the estimated first predominant frequency by the proposed method and the natural frequency of the ground as well as Nakamura's method, for the case of stationary type microtremors and even though in the case of some strong artificial noise are included. However, the proposed method is easier and simpler than Nakamura's method because only one horizontal component of microtremor is sufficient to estimate the natural frequency of the ground.

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