



## AN EMPIRICAL LAW FOR PHASE DIFFERENCE SPECTRUM OF EARTHQUAKE GROUND MOTION

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### ABSTRACT

An empirical model for phase difference spectral evaluation of earthquake ground motion is established here and coefficients in the model are regressed from the earthquake acceleration records. The results indicate that the mean smoothed curve and standard variance of phase difference spectrum are both magnitude and distance dependent. The curves are nearly parallel for different magnitudes but at same distance. The shape of the curve also changes with distance. Increase of distance causes obvious difference near two ends of the curve. The standard variance of the phase difference spectrum becomes larger with the increase of magnitude and distance.

### KEYWORDS

ground motion; phase difference spectrum; empirical law; regression;

### INTRODUCTION

The relationship between Fourier spectra including the phase difference and amplitude and the non-stationary both in the amplitude and frequency content has been studied by some researchers (Ohsaki, 1979; Nigam, 1982; Zhao, 1994). Another important problem which should be studied is the dependence of the phase difference spectrum of recorded earthquake accelerograms on the magnitude and hypocentral distance. If we can make this empirical prediction, the earthquake ground motion can be easily generated for seismic

designs. The main purpose of this paper is to establish an empirical law for the phase difference spectrum of earthquake ground motions.

## THE EXPRESSION OF THE PHASE DIFFERENCE SPECTRUM

The mean smoothed curve  $\Delta\bar{\varphi}(f)$  represented a phase difference spectrum  $\Delta\varphi(f)$  can be obtained by a smoothing technique. The curve  $\Delta\bar{\varphi}(f)$  represents the general property of the phase difference spectrum varying with the frequency and  $\varepsilon(f) = \Delta\varphi(f) - \Delta\bar{\varphi}(f)$  is a stochastic process with a zero mean value. It has been found that the shape of  $\Delta\bar{\varphi}(f)$  and the variance  $\sigma_\varepsilon$  of  $\varepsilon(f)$  are dependent on the magnitude and the hypocentral distance. Because of its complexity, as the first degree approximation,  $\varepsilon(f)$  has been assumed to be a stationary stochastic process in this study, ignoring its characteristic varying with the frequency.

The regression equations of  $\Delta\bar{\varphi}(f)$  and  $\sigma_\varepsilon$  will take the forms

$$\log|\Delta\bar{\varphi}(f)| = a_1(f) + a_2(f) \cdot M + a_3(f) \cdot \log(R + 15) \quad (1)$$

$$\log\sigma_\varepsilon = d_1 + d_2 \cdot M + d_3 \log(R + 15) \quad (2)$$

Where  $M$  is the magnitude,  $R$  is the hypocentral distance in Km. Because  $\varepsilon(f)$  is not symmetry on the two sides of  $\Delta\bar{\varphi}(f)$ ,  $\varepsilon(f)$  has been assumed to be a log-normal distribution. The coefficients in eq.(1) and eq.(2) are determined by regression. By means of generation of random numbers, a phase difference spectrum can be obtained by  $\Delta\varphi(f) = \Delta\bar{\varphi}(f) + \varepsilon(f)$ .

## REGRESSION ANALYSIS OF THE PHASE DIFFERENCE SPECTRUM

Data used in this study consist of accelerograms recorded under free-field condition on the rock in the west of USA. The selected recordings represent a total of 80 horizontal time-histories with a sampling interval of 0.02 second and a total duration of 40.96 second. The magnitude is defined as a local magnitude for the magnitude less than 6.6 and a surface wave magnitude for 6.6 or above.

Through the smoothing process,  $\Delta\bar{\varphi}(f)$  and  $\sigma_\varepsilon$  can be obtained corresponding to each  $\Delta\varphi(f)$ .  $\Delta\bar{\varphi}(f)$  are calculated for the data set at the 83 selected frequencies in the range of 0.195 Hz to 25 Hz. Then regression analysis is performed on  $\Delta\bar{\varphi}(f)$  at each selected frequency. Regression results are presented for the  $\Delta\bar{\varphi}(f)$  in Table 1.

Table 1. The regressed result of  $|\Delta\bar{\varphi}(f)|$ 

Frequency(Hz)	$a_1(f)$	$a_2(f)$	$a_3(f)$	$\sigma_\varepsilon$
.195	-1.969	.211	.585	.124
.220	-1.963	.211	.584	.124
.244	-1.956	.210	.581	.123
.269	-1.948	.209	.579	.123
.293	-1.939	.208	.577	.122
.317	-1.930	.207	.574	.121
.341	-1.922	.206	.571	.121
.366	-1.912	.206	.569	.120
.391	-1.903	.205	.566	.119
.415	-1.894	.204	.563	.119
.439	-1.885	.203	.560	.118
.463	-1.876	.202	.558	.117
.488	-1.867	.201	.555	.117
.513	-1.859	.200	.552	.116
.538	-1.850	.199	.549	.115
.562	-1.842	.199	.546	.115
.585	-1.834	.198	.543	.114
.610	-1.827	.197	.540	.114
.633	-1.820	.197	.537	.113
.658	-1.813	.196	.535	.113
.685	-1.806	.195	.532	.112
.709	-1.800	.195	.529	.112
.730	-1.794	.194	.526	.112
.758	-1.788	.194	.523	.111
.781	-1.782	.193	.520	.111
.806	-1.777	.193	.517	.111
.826	-1.771	.193	.515	.110
.855	-1.766	.192	.512	.110
.877	-1.761	.192	.509	.109
.901	-1.755	.192	.506	.109
.926	-1.750	.191	.503	.109
.952	-1.745	.191	.500	.108
1.000	-1.733	.190	.493	.108
1.053	-1.722	.190	.487	.107

Table 1. The regressed result of  $|\Delta\bar{\varphi}(f)|$  (continued)

Frequency(Hz)	$a_1(f)$	$a_2(f)$	$a_3(f)$	$\sigma_\varepsilon$
1.111	-1.704	.189	.477	.105
1.176	-1.692	.188	.470	.104
1.250	-1.673	.187	.459	.102
1.333	-1.648	.186	.445	.099
1.429	-1.625	.185	.432	.096
1.538	-1.605	.184	.420	.092
1.667	-1.587	.184	.407	.087
1.818	-1.580	.184	.396	.082
2.000	-1.597	.187	.390	.077
2.083	-1.611	.189	.390	.076
2.174	-1.632	.191	.393	.075
2.273	-1.657	.193	.397	.075
2.381	-1.690	.196	.404	.076
2.500	-1.715	.197	.411	.077
2.632	-1.749	.199	.421	.080
2.778	-1.778	.201	.430	.083
2.941	-1.802	.203	.437	.086
3.125	-1.829	.205	.444	.089
3.333	-1.854	.208	.448	.091
3.571	-1.866	.210	.448	.091
3.846	-1.849	.209	.439	.089
4.167	-1.804	.205	.424	.085
4.545	-1.772	.203	.411	.083
5.000	-1.867	.213	.428	.090
5.263	-1.986	.224	.454	.096
5.556	-2.060	.232	.469	.099
5.882	-2.028	.229	.461	.095
6.250	-1.953	.221	.446	.089
6.667	-1.910	.218	.435	.086
7.143	-1.926	.220	.438	.089
7.692	-1.879	.211	.442	.098
8.333	-1.824	.199	.452	.103
9.091	-1.824	.198	.455	.098
10.000	-1.779	.193	.448	.101
10.526	-1.861	.202	.462	.112
11.111	-1.881	.206	.460	.110
11.765	-1.759	.194	.431	.098
12.500	-1.733	.194	.419	.096
13.333	-1.786	.199	.424	.100

Table 1. The regressed result of  $|\Delta\bar{\phi}(f)|$  (continued)

Frequency(Hz)	$a_1(f)$	$a_2(f)$	$a_3(f)$	$\sigma_\varepsilon$
14.286	-1.862	.206	.445	.110
15.385	-1.753	.190	.442	.101
16.667	-1.759	.191	.441	.093
18.182	-1.893	.206	.464	.113
20.000	-1.734	.194	.417	.096
20.833	-1.799	.200	.427	.102
21.739	-1.844	.204	.442	.109
22.727	-1.756	.190	.444	.101
23.810	-1.807	.196	.452	.096
25.000	-1.838	.200	.459	.109

$$\log \sigma_\varepsilon = -1.124 + 0.089M + 0.316 \log(R + 15) \quad (3)$$

The statistical standard variance of eq.(3) is 0.089.

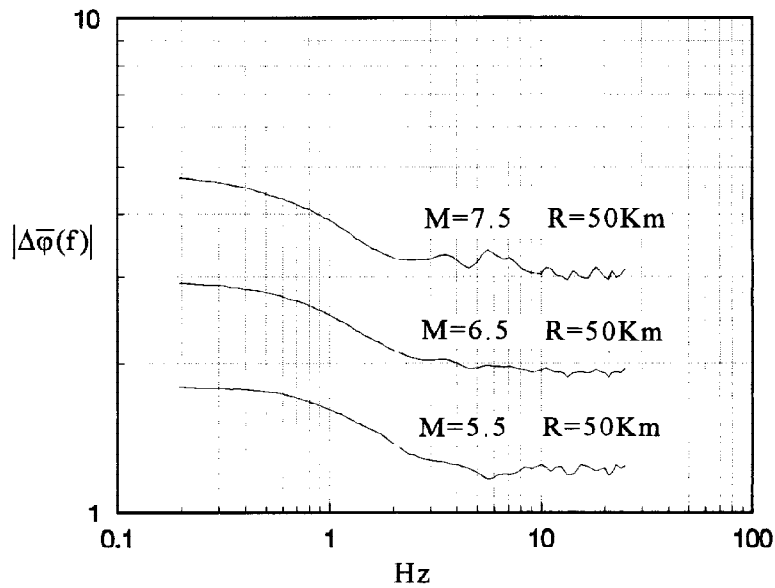


Fig. 1 The estimated mean smoothed curves of phase difference spectra with same distance but different magnitude.

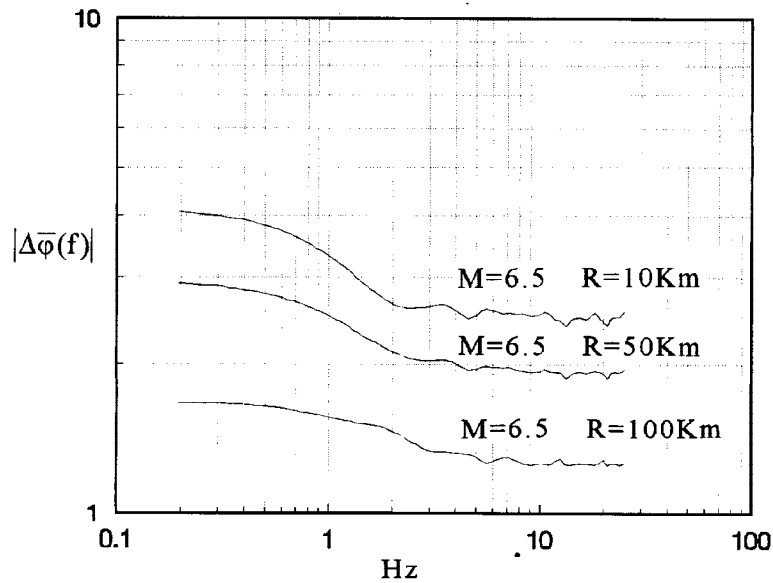


Fig. 2 The estimated mean smoothed curves of phase difference spectra with same magnitude but different distance.

Fig.1 illustrates the calculated  $\Delta\bar{\varphi}(f)$  for the hypocentral distance of 50Km and magnitudes of 5.5, 6.5 and 7.5 using the attenuation equation. This figure indicates that the  $\Delta\bar{\varphi}(f)$  curves are nearly parallel for different magnitudes with the same hypocentral distance. Fig.2 shows the calculated  $\Delta\bar{\varphi}(f)$  for the magnitude of 6.5 and hypocentral distances of 10, 50 and 100Km. It can be seen that these curves not only move up or down with different hypocentral distances, their shapes also change. With hypocentral distance increased, the difference between the two ends of the curve will become more obvious. Because the shape of  $\Delta\bar{\varphi}(f)$  is closely related to the nonstationarity of frequency content of earthquake motions, these results indicate that the hypocentral distance mainly affects the nonstationarity of accelerograms' frequency content.

The regression result of  $\sigma_e$  points out that it will become larger with the increase of the magnitude and the hypocentral distance, which means that with the increase of the magnitude and the hypocentral distance, the duration of an earthquake accelerogram will increase.

## CONCLUSION

On the basis of regression analysis of the phase difference spectrum, we conclude that it is both magnitude-dependent and distance-dependent. The mean smoothed curves of phase

difference spectra with the same hypocentral distance but different magnitudes approximate a set of parallel curves. On the other hand, the curves with the same magnitude but different hypocentral distances are not parallel, which means that the hypocentral distance is the main factor affecting the non-stationality of earthquake ground motions. The regression result of  $\sigma_{\varepsilon}$  points out that it will become larger with the increase of the magnitude and the hypocentral distance, which indicates that with the increase of the magnitude and the hypocentral distance, the duration of an earthquake accelerogram will increase.

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