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EFFECTS OF SCATTERED WAVE INDUCED AT THE INTERFACE OF SOIL LAYERS ON SEISMIC RESPONSE OF SUBSURFACE GROUND

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ABSTRACT

A modified Reflection / Transmission Matrices method is proposed to consider scattered wave induced on the boundary between soil layers. The proposed method can calculate the Reflection / Transmission Matrices by less CPU time, which utilizes the approximate function of the matrices. The case studies using this method show that the scattered wave has two effects on the characteristics of the ground response. One is to give other peaks on frequency response function which can not be represented by one dimensional analysis and the other is the same effect with the frequency dependent damping.

KEYWORDS

Scattering wave; Reflection / Transmission matrix method; Irregular boundary; Soil layer.

INTRODUCTION

When an important structure is designed such as nuclear power plants, high rise buildings and long spanned bridges, it is very important to predict strong ground motion on the site. In the process of prediction, the source, path and site characteristics must be assumed. In many analyses of site effects, one dimensional soil profile have been assumed. One dimensional analyses, however, does not explain the records obtained by vertical array observation system of seismic ground motion. The wave forms obtained by one dimensional analysis are always simple and the amplitude of high frequency range are smaller than those of actual records. Many researchers pointed out that scattering is a reason of difference between calculation and observation.

Two types of wave scattering must be considered when the local site effect is analyzed. One is the scattering caused by heterogeneity of soil properties and the other is the scattering induced on irregular interface between soil layers. In this paper, the latter type of scattering is examined. Analyzing this type of scattering is to analyze the seismic response of two or three dimensional soil profile such as basin structure, etc. We developed a new method to calculate the seismic response of irregularly layered ground by small computation time and to estimate the effects of wave scattering. The proposed method is based on Reflection / Transmission matrix method(Kennett 1983 and 1984, Kohketsu 1987).

REFLECTION / TRANSMISSION MATRIX METHOD

Reflection / Transmission Factor of One-dimensional Structure

In this paper, only SH wave is considered. In case of one dimensional soil profile, an incident wave with horizontal wave number k_q generates reflected and transmitted (refracted) waves having k_q on the soil boundaries. In other words, Snell's law must be satisfied. As shown in Fig.1, the upward wave in layer 1 on the reference level 1, Φ_1^-, and the downward wave in layer 2 on the reference level 2, Φ_2^+, can be represented by Eqs.(1) and (2).

$$\Phi_2^+(k_q) = r_U^{(1D)}(k_q) \cdot \Phi_2^-(k_q) \tag{1}$$

$$\Phi_1^-(k_q) = t_U^{(1D)}(k_q) \cdot \Phi_2^-(k_q) \tag{2}$$

where, Φ_2^- is the upward incident wave in layer 2, $r_U^{(1D)}$ and $t_U^{(1D)}$ are the reflection and transmission factors of upward incident wave as follows;

$$t_U^{(1D)}(k_q) = \frac{\exp[-j\nu_2 d_2]}{\exp[j\nu_1 d_1]} \cdot \frac{2\mu_2 \nu_2}{\mu_1 \nu_1 + \mu_2 \nu_2} \tag{3}$$

$$r_U^{(1D)}(k_q) = \frac{\exp[-j\nu_2 d_2]}{\exp[j\nu_2 d_2]} \cdot \frac{\mu_2 \nu_2 - \mu_1 \nu_1}{\mu_1 \nu_1 + \mu_2 \nu_2} \tag{4}$$

$$\nu_1 = \sqrt{\frac{\rho_1 \omega^2}{\mu_1^2} - k_q^2}, \quad \nu_2 = \sqrt{\frac{\rho_2 \omega^2}{\mu_2^2} - k_q^2} \tag{5a,5b}$$

where, ω is angular frequency, μ shear modulus, ρ density, d distance from the boundary to reference section and ν vertical wavenumber for each layer which is specified by the subscript.

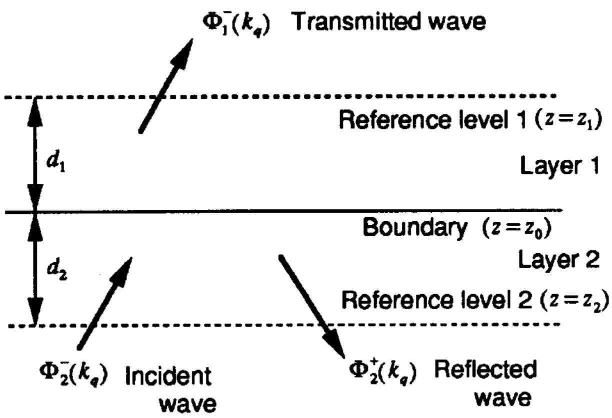


Fig.1 Reflection and transmission in one dimensional model.

Reflection / Transmission Matrix of Two-dimensional Soil Profile

On the other hand, in case of two dimensional analysis, incident wave with horizontal wavenumber k_q generates not only reflected and transmitted waves with k_q but also waves with other horizontal wave number. This is called "Wavenumber coupling"(Kennett 1986). The discrete reflection / transmission matrix is consisted of a column concerned with incident wavenumber and a raw with reflection / transmission wavenumber, as shown in the following equations;

$$\{\Phi_2^+\} = \sum_{p=-N}^{N} r_U^{(2D)}(k_p; k_q) \cdot \Phi_2^-(k_q) \Delta k_p = [R_U]^{(2D)} \{\Phi_2^-\} \tag{6}$$

$$\{\Phi_1^-\} = \sum_{p=-N}^{N} t_U^{(2D)}(k_p;k_q)\cdot\Phi_2^-(k_q)\Delta k_p = [T_U]^{(2D)}\{\Phi_2^-\} \tag{7}$$

Takenaka(1990) deduced the R/T matrices for two dimensional structures, $[R_U]^{(2D)}$, $[R_D]^{(2D)}$, $[T_U]^{(2D)}$, and $[T_D]^{(2D)}$, under Rayleigh's assumption as follows;

$$\begin{aligned}
[T_D]^{(2D)} &= [E_2^-][M_2^+]^{-1}[S_2^+ - S_1^-]^{-1}[S_1^+ - S_1^-][M_1^+][E_1^+] \\
[R_D]^{(2D)} &= [E_1^+][M_1^-]^{-1}[S_2^+ - S_1^-]^{-1}[S_1^+ - S_2^+][M_1^+][E_1^+] \\
[T_U]^{(2D)} &= [E_1^+][M_1^-]^{-1}[S_2^+ - S_1^-]^{-1}[S_2^+ - S_2^-][M_2^-][E_2^-] \\
[R_U]^{(2D)} &= [E_2^-][M_2^+]^{-1}[S_2^+ - S_1^-]^{-1}[S_1^- - S_2^-][M_2^-][E_2^-]
\end{aligned} \tag{8}$$

where,

$$[S_i^\pm] = [N_i^\pm]^{-1}[M_i^\pm]\,. \tag{9}$$

In Eq.(8), $[E_i]$ is the phase matrix which represents the phase difference between the reference level and the boundary, $[M_i]$ the conversion matrix from $\{\Phi\}$ to the displacement on the boundary and $[N_i]$ the conversion matrix from $\{\Phi\}$ to the stress on the boundary. Those matrices are functions of boundary shape function $z=h(x)$. The superscripts "+" and "-" of those matrices represent the downward wave and the upward wave. To calculate the reflection and transmission matrix by Eq.(8), five inverse matrices must be computed for each frequency.

Ground Response using Reflection / Transmission Matrix

Ground seismic response can be calculated by the following process. Fig.2 shows the two layers model which consists of surface layer 1 connected to a half-space seismic base layer, layer 2. In this model, as the transmitted wave from layer 2 to layer 1 is reflected on the ground surface and it becomes the incident wave at the boundary between layer 1 and layer 2, the reflection wave on the boundary becomes upward wave in layer 1. Consequently, infinite number of reflected and transmitted wave must be considered, as shown in Fig. 2. As those R/T waves make Neumann's series, the ground displacement $u(k,\omega)$ which is two times of upward wave in layer 1 is given by Eq.(10).

$$\begin{aligned}
u(k,\omega) &= 2\left\{T_U^{23} + (R_D^{23})T_U^{23} + (R_D^{23})^2 T_U^{23} + \cdots\right\}\Phi_{IN}^- \\
&= 2\left(I - R_D^{23}\right)^{-1} T_U^{23}\Phi_{IN}^-
\end{aligned} \tag{10}$$

where, I is a unit matrix, ()-1 represents the inverse matrix.

The seismic response of multi-layered ground can be computed because the effects on two boundaries are integrated to a set of R/T matrices using the similar method as Eq.(10).

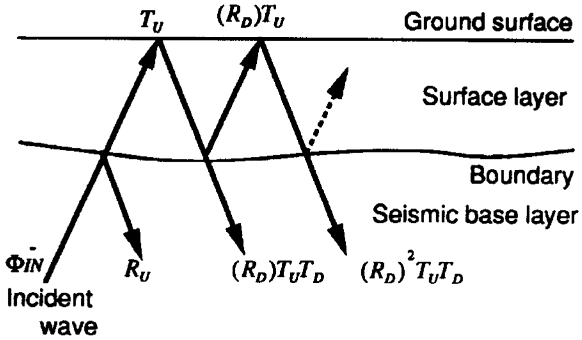


Fig.2 Multiple reflection and transmission in the two layers model.

APPROXIMATION OF R/T MATRIX

Function Shape of R / T Matrix

When R/T matrices are computed using Eq.(8), it needs large CPU time because of calculation of many inverse matrices. Then we propose a new approximate method to calculate R/T matrix.

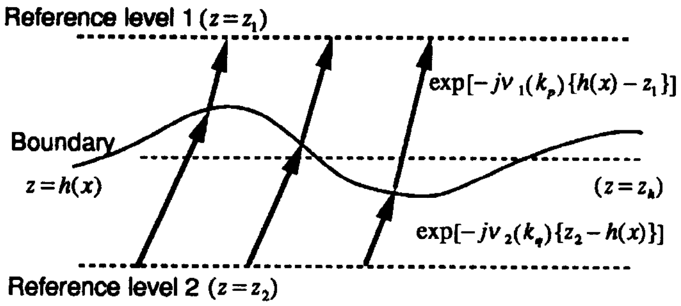


Fig.3 Phase shift induced by irregular boundary.

Fig.3 shows the model which has two half-space layers with contact boundary at $z=h(x)$. Reference levels 1 and 2 are assumed at $z=z_1$ and $z=z_2$. z_h is the average of $h(x)$. The incident plane wave ϕ_0 with horizontal wave number k_q is described by Eq.(11), if amplitude is a unit and phase angle is zero at x=0 on the reference level 2.

$$\phi_0 = \exp[jk_q x] \tag{11}$$

The phase difference between the reference level 2 and the boundary is given by the following equation;

$$\phi_{b2} = \exp[-j\nu_2(k_q)\{z_2 - h(x)\}] \tag{12}$$

where, ν_i is vertical wavenumber in i-th layer The phase difference of the transmitted wave between the boundary and the reference level 1, ϕ_{1b}, is given in Eq.(13).

$$\phi_{1b} = \exp[-j\nu_1(k_p)\{h(x) - z_1\}] \tag{13}$$

Then the phase difference between the reference level 1 and 2 is determined from Eqs.(11) to (13). If z_1=z_2=z_h are assumed, the total phase difference is given by Eq.(14).

$$\begin{aligned} g_{t_U}(k_p;k_q) &= \phi_{1b}\cdot\phi_{b2}\cdot\phi_0 \\ &= \exp[-j\{\nu_1(k_p) - \nu_2(k_q)\}h(x) + jk_q x] \end{aligned} \tag{14}$$

In the next, Eq.(14) is converted from space domain to wavenumber domain by Fourier transform which yields

$$G_{t_U}(k_p;k_q) = \frac{1}{L}\int_0^L g_{t_U}(k_p;k_q)\cdot\exp[-jk_p x]dx. \tag{15}$$

Then R/T matrices are expected to have the functional shape similar to Eq.(15). By examining the difference between Eq.(15) and Eq.(8) by numerical experiments, approximate equation is deduced as follows;

$$\bar{t}_U(k_p - k_q;k_p) = (-1)^{p-q} t_U^{(1D)} G_{t_U}(k_p;k_q) \tag{16}$$

Approximate Equation of R / T Matrix

In order to determine each element of R/T matrix using Eqs.(14)-(16), it is necessary to calculate Fourier transform and it requires similar CPU time to the case of Eq.(8). Then a new approximation is proposed to reduce the CPU time. It can be done by replacing k_p by k_q in Eq.(14) as shown by;

$$\tilde{g}_{t_U}(k_q) = \exp\left[-j\{v_1(k_q) - v_2(k_q)\}h(x) + jk_q x\right] \tag{17}$$

The new approximated $\tilde{t}_U$ is defined by letting Eq.(17) into Eq.(15) and (16).

$$\tilde{t}_U(k_p - k_q; k_q) = (-1)^{p-q} t_U^{(1D)} \frac{1}{L}\int_0^L \tilde{g}_{t_U}(k_q) \cdot \exp[-jk_p x]dx \tag{18}$$

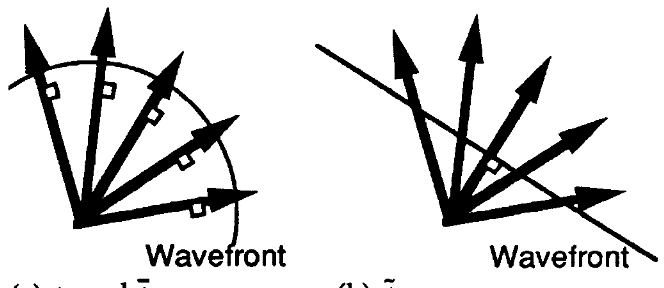
In this approximation, wave front of scattered wave is assumed to be straight even in the case of cylindrical transmission, as is shown in Fig. 4. This assumption may induce large phase error in the region in which k_p is significantly different from k_q Fortunately, the amplitude in those region is so small that the effect of phase error is not significant. As Eqs.(17) and (18) can determine a raw of R/T matrix by calculating once Fourier transform, CPU time is greatly decreased from the case of Eq.(8). Other R/T matrices can be defined by the same method, as follows;

$$\begin{aligned} \tilde{r}_U(k_p - k_q; k_q) &= (-1)^{p-q} r_U^{(1D)} \frac{1}{L}\int_0^L \tilde{g}_{r_U}(k_q) \cdot \exp[-jk_p x]dx \\ \tilde{g}_{r_U}(k_q) &= \exp\left[j\{v_2(k_q) + v_2(k_q)\}h(x) + jk_q x\right] \end{aligned} \tag{19}$$

$$\begin{aligned} \tilde{t}_D(k_p - k_q; k_q) &= (-1)^{p-q} t_D^{(1D)} \frac{1}{L}\int_0^L \tilde{g}_{t_D}(k_q) \cdot \exp[-jk_p x]dx \\ \tilde{g}_{t_D}(k_q) &= \exp\left[-j\{v_1(k_q) - v_2(k_q)\}h(x) + jk_q x\right] \end{aligned} \tag{20}$$

$$\begin{aligned} \tilde{r}_D(k_p - k_q; k_q) &= (-1)^{p-q} r_D^{(1D)} \frac{1}{L}\int_0^L \tilde{g}_{r_D}(k_q) \cdot \exp[-jk_p x]dx \\ \tilde{g}_{r_D}(k_q) &= \exp\left[-j\{v_1(k_q) + v_1(k_q)\}h(x) + jk_q x\right] \end{aligned} \tag{21}$$

It was examined by Sawada et.al(1994) that these equations can be applied to general type of boundaries.



(a) t_U and $\tilde{t}_U$ (b) $\tilde{t}_U$

Fig.4 Schematic representation of approximation.

EFFECT OF WAVE SCATTERING ON GROUND SEISMIC RESPONSE

Model Analyzed

Seismic ground response are analyzed considering the effect of scattered wave generated on the irregular boundary between soil layers.

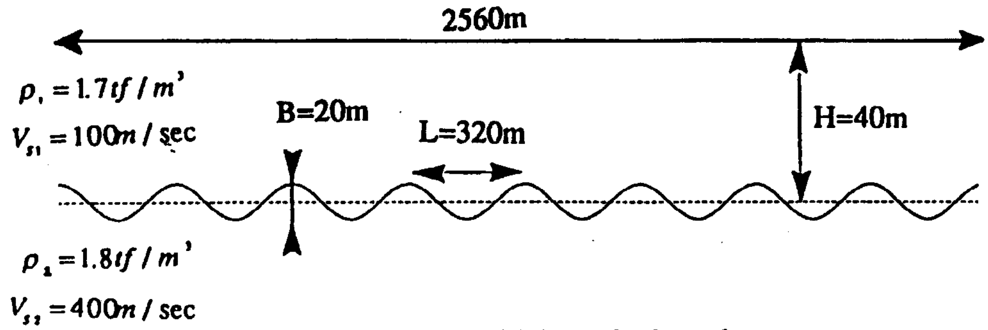


Fig.5 Basic model with irregular boundary.

Fig.5 shows the model analyzed. The boundary is assumed to be sinusoidal in horizontal direction. This boundary shape is not "irregular" but it is adopted to make the problem simple. We set the basic model case of which average thickness of surface layer H is 40 m, wavelength of the boundary L is 320 m and amplitude of fluctuation of boundary B is 20 m. Case studies are conducted for different sets of values of configuration of model. Only vertical incident SH wave is considered as the incident wave. The number of grid for discrete horizontal wavenumber is 255, and numerical computation are performed at every 0.05 Hz in the frequency domain from 0.05 to 10.0 Hz. Internal damping is not considered.

Effect of Receiver Position

Fig.6 shows the response functions for three different receiver points; receiver A is located at ground

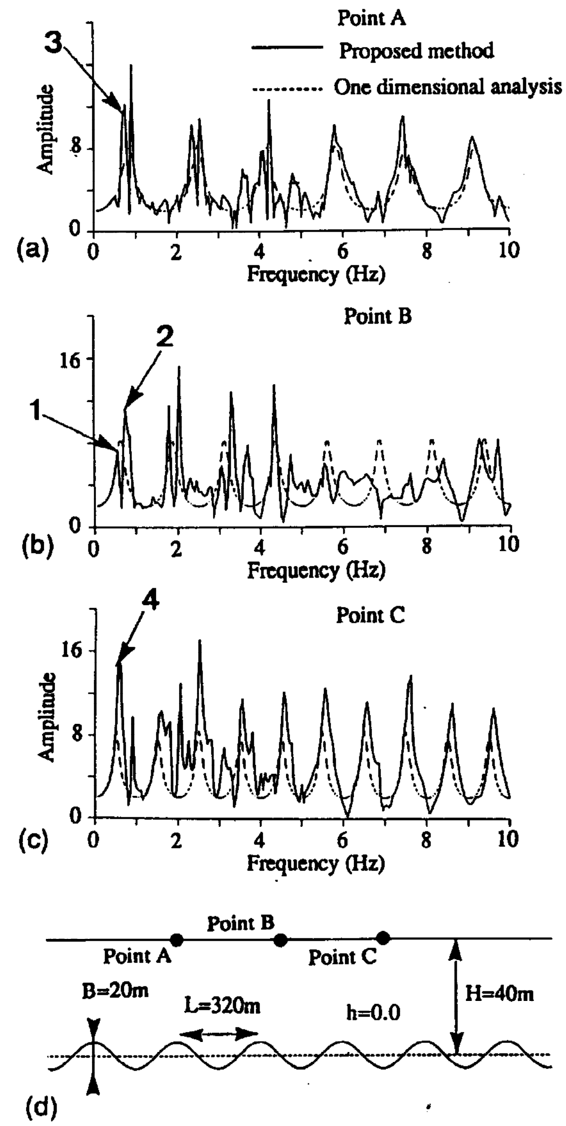


Fig.6 Effect of position of receiver

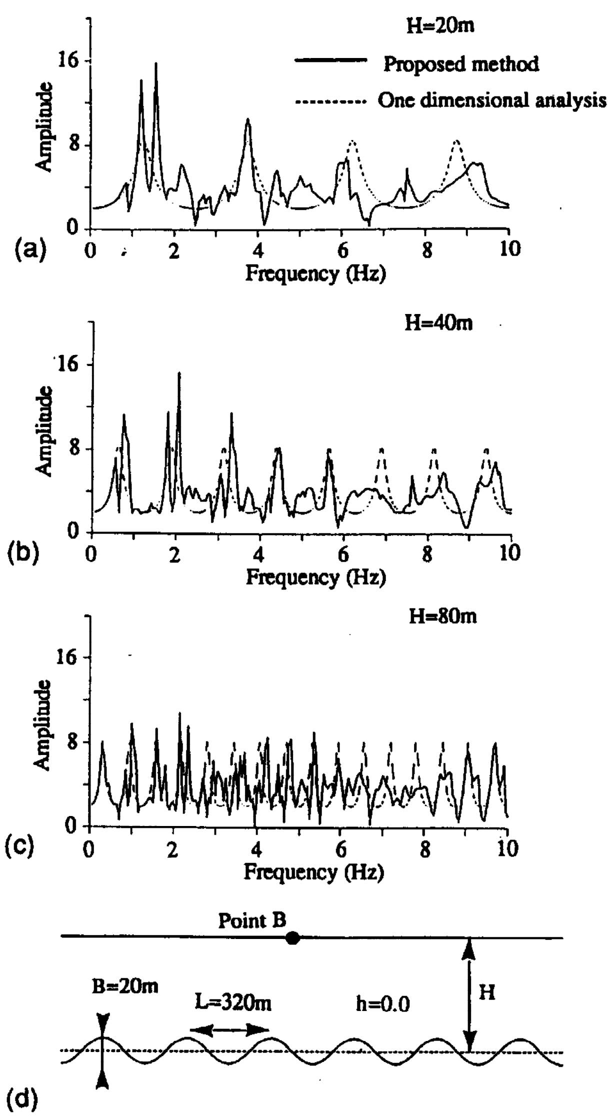


Fig.7 Effects of thickness of surface layer.

surface just over the peak of the boundary, B is over the mid point and C is over the crest. The dashed lines in Fig.6 indicate the response functions of one dimensional soil profiles which have the same depth H with the receiver point. The solid lines are the response functions which consider scattering effects and they are different from the dash lines; it has different peaks from dash lines in the low frequency range. For example, the first peak, indicated by the arrow number 1 in Fig.6, appears around 0.5 Hz, and the second peak (arrow 2) is around 0.8 Hz. These frequencies correspond to the natural frequency of the soil profile just under the receiver points A and C. On the other hand, as the natural frequency of the structure under the receiver point B is 0.6 Hz, the frequency response functions at the receiver points A and C have peaks of 0.6 Hz as shown in Fig.6(a) and (c), denoted by arrows 3 and 4. Existence of these peaks indicates the existence of waves traveling in non-vertical direction in spite of vertical incidence. In addition, transmitted wave seems to concentrate to the receiver point C because of lens effect. This is the reason why the solid line in Fig.6(c) is larger than the dashed line for entire frequency range. In Fig.6(b) no major peak appears in the frequency range from 5.5 to 8.5 Hz and the response values are smaller than that of one dimensional analysis in the range.

It can be concluded that scattering exerts two influences on the ground response function; it induces several peaks at different frequency from one dimensional analysis and it has frequency dependent damping effect. These effects are dependent on the position of receiver.

Effect of Thickness of Surface Layer

Fig.7 shows the effect of thickness of surface layer, H, on frequency response function. The parameters except H are the same with the basic case. The solid and dashed lines have same meaning as Fig.6. The peak values in low frequency range are larger, and the peak values of higher mode are smaller than that of

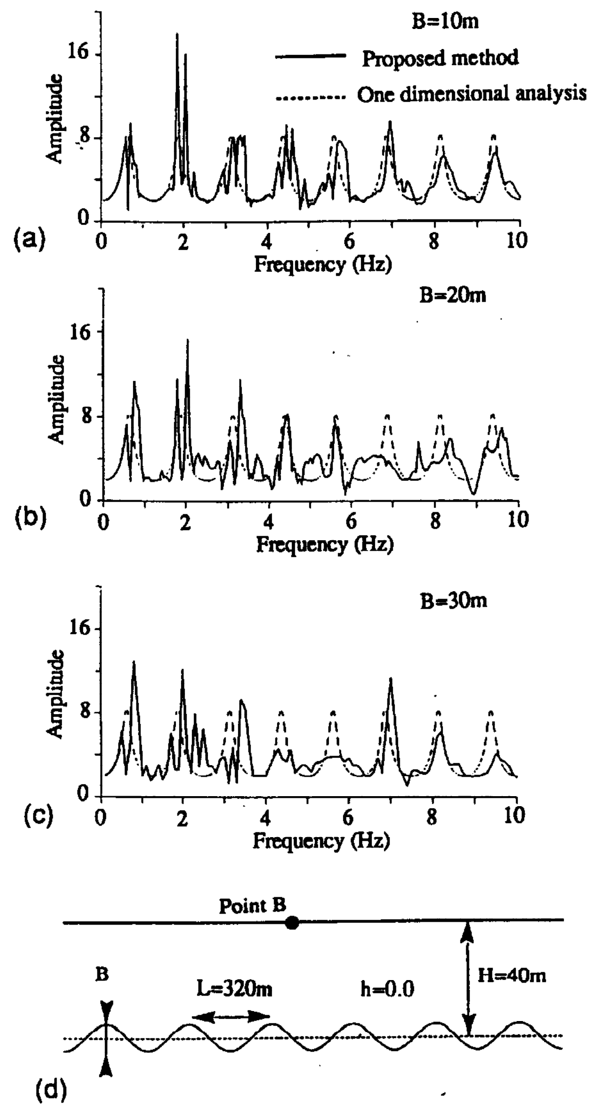


Fig.8 Effects of fluctuation amplitude of boundary.

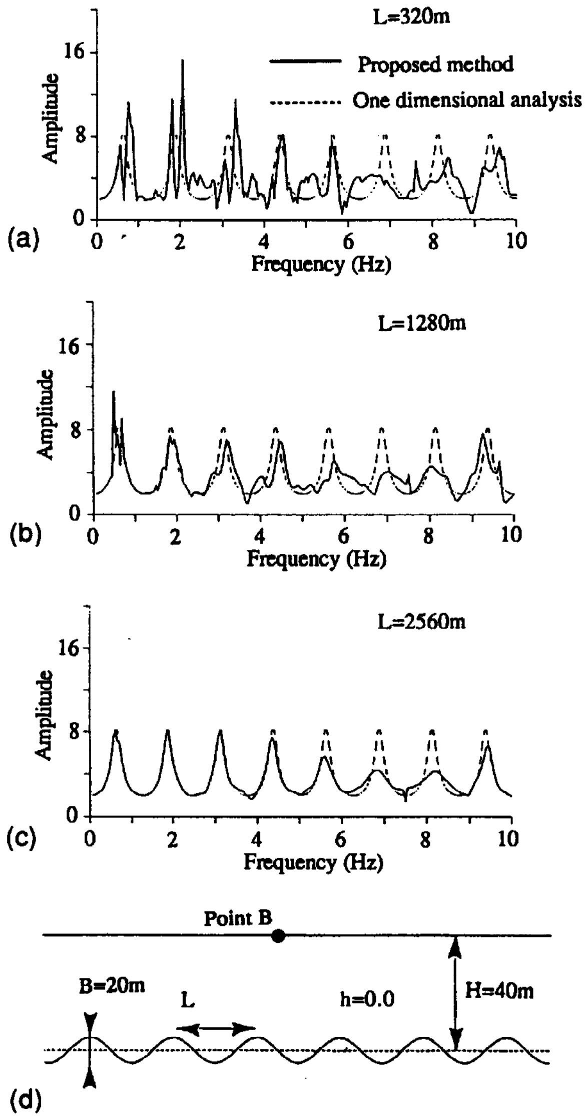


Fig.9 Effects of wavelength of irregular boundary.

one dimensional analysis as shown in Fig. 7. As the thickness increases, the amplification factor decreases and becomes closer to the one dimensional analysis. Thin surface layer or low frequency makes the wavenumber in surface layer small, and amplification becomes larger because of small phase difference of scattered waves. On the contrary, high frequency or thick surface layer makes wavenumber larger and amplification small. In other words the damping effect becomes significant because large wavenumber in surface layer causes random variation of phase among scattered waves.

It is important to note that the amplification factor is smaller than that of one dimensional analysis in the frequency range from 5.5 to 8.5 Hz in all cases shown in Fig.7. Thickness of surface layer affects to the level of amplification, but it does not exert influence on the frequency characteristic of response function.

Effects of Fluctuation Amplitude and Wavelength of Boundary

Fig.8 shows the effects of fluctuation of boundary. Small fluctuation of boundary gives the similar response function with that of one dimensional analysis. This results is acceptable because small fluctuation means more flat interface between layers. In the case of small fluctuation as shown in Fig.8(a), amplitude of frequency response function is smaller than that of one dimensional analysis in the frequency range higher than 8 Hz. As the fluctuation B is increased to 30 m, amplitude becomes smaller than that of one dimensional analysis in the frequency range higher than 4 Hz as is shown in Fig.8(c). This implies that the frequency characteristic of seismic response of soil layer is affected by fluctuation amplitude of boundary.

Fig.9 shows the effect of wavelength of boundary. Longer wavelength gives the similar response function as that of one dimensional analysis. The frequency range in which the response function is smaller than that of one dimensional analysis is 6.0 to 8.0 Hz in all cases. In the case of Fig.9(c), we can recognize the range clearly because the response function is smooth.

CONCLUSIONS

(1)The proposed method needs small CPU time to calculate Reflection / Transmission Matrices, which utilizes the approximate function of the matrices.
(2)The case studies using the proposed method show that the scattered wave have two effects on the characteristics of seismic ground response; one is to give other peaks of response function which is not represented by one dimensional analysis and the other is to give frequency dependent damping on seismic response of soil layers.
(3)The effects of scattered wave induced on the boundary between soil layers are dependent on the position of receiver.
(4)Thickness of surface layer and wavelength of boundary affect on the level of amplification factor but the frequency characteristic of response function is not affected.
(5)The frequency characteristic of response function is affected by the fluctuation amplitude of boundary.

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