Paper No. 301. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

NONSTATIONARY EARTHQUAKE GROUND MOTION MODEL

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ABSTRACT

A versatile, nonstationary stochastic ground motion model accounting for the time-variation of both the intensity and frequency content typical of real earthquake ground motions is formulated and validated. An extension of the Thomson's spectrum estimation method based on the prolate spheroidal sequences and prolate spheroidal wave functions is used to adaptively estimate the evolutionary power spectral density function of the target ground acceleration record. The parameters of this continuous-time, analytical, stochastic earthquake model are determined by fitting the analytical evolutionary power spectral density (psd) function of the model to the target evolutionary psd function estimated from the target record through an adaptive nonlinear least-squares algorithm. The proposed model is calibrated against actual earthquake records and validated by comparing the second-order statistics of traditional ground motion parameters and the probabilistic linear elastic response spectra simulated using the earthquake model with their deterministic counterparts obtained directly from the target record.

KEYWORDS

Nonstationary ground motion model; sigma-oscillatory process; Thomson's spectrum estimation; evolutionary power spectral density function.

INTRODUCTION

To account for the random nature of earthquake ground motion time histories, various stochastic ground motion models, stationary or nonstationary, have been developed and applied over the years. Several comprehensive review papers (Liu 1969; Ahmadi 1979; Shinozuka 1988; Shinozuka and Deodatis 1988; Kozin 1988) examine and compare the stochastic earthquake ground motion models available and provide earthquake engineers with a more solid basis for selecting the appropriate model for a given situation.

Actual earthquake records exhibit clearly a temporal variation of intensity and frequency content. The

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frequency nonstationarity is due to the different arrival times of the P (Primary or "Push"), S (Secondary or Shear), and surface (Rayleigh and Love) waves which propagate at different velocities through the earth crust. Few past studies have shown that the nonstationarity in frequency content of earthquake ground motions can have a significant effect on the response of both linear and nonlinear structures (Saragoni & Hart 1974; Yeh & Wen 1990; Papadimitriou 1990). Saragoni and Hart (1974) developed a fully nonstationary model by juxtaposing time segments of Gamma function modulated filtered Gaussian white noise with the filter properties varying from segment to segment. Thus, the frequency content of this model varies as a stepwise function of time. Kubo and Penzien (1979) proposed a nonstationary earthquake simulation model as the product of a constant intensity process having time-varying frequency content and a deterministic intensity or envelope function. They estimated the time-varying frequency characteristics of the target earthquake record using short time (or moving window) Fourier transforms. Lin and Yong (1987) formulated evolutionary Kanai-Tajimi earthquake models as convolutions of a nonstationary shot noise process and deterministic Green's functions borrowed from one-dimensional wave propagation in linear elastic and visco-elastic media. Other researchers used simultaneously time and frequency modulating functions to construct a fully nonstationary earthquake model (Grigoriu *et al.* 1988; Yeh and Wen 1990). Der Kiureghian and Crempien (1989) defined an evolutionary earthquake model composed of individually modulated component stationary (band-limited white noise) processes. Fan and Ahmadi (1990) extended the original, site-dependent, stationary Kanai-Tajimi earthquake model to account for amplitude and spectral nonstationarities. Papadimitriou and Beck (1990) produced a parsimonious nonstationary earthquake model by applying a second-order filter with slowly-varying parameters to a time modulated white noise. Conte *et al.* (1992) developed a time-varying ARMA model estimated from actual earthquake accelerograms using an iterative Kalman filtering procedure. Recently, several authors have developed fully nonstationary earthquake models using principles of geophysics and stochastic wave propagation (Deodatis *et al.* 1990; Zhang *et al.* 1991). In this paper, a new, versatile, fully nonstationary, stochastic earthquake model from the family of sigma-oscillatory processes is proposed and validated.

FORMULATION OF GROUND MOTION MODEL

Oscillatory processes and evolutionary spectral analysis were introduced by Priestley (1965, 1967). Although this approach has been proven widely applicable, it suffers from some limitations. For example, the class of oscillatory processes is not closed with respect to the sum of independent elements, and the coherency of a bivariate oscillatory process turns out to be independent of time (Battaglia 1979). An attempt to get free of these limitations is presented by Battaglia who introduces the concept of sigma-oscillatory processes and defines an evolutionary spectral analysis (time-frequency distribution analysis with a physical frequency parameter) for this kind of processes. A sigma-oscillatory process, Y(t), is defined as the sum of a finite number of mutually (statistically) independent oscillatory processes, i.e.,

$$Y(t) = \sum_{k=1}^{p} X_k(t) \qquad (1)$$

in which the component processes $\{X_k(t),\ k = 1, 2, \ldots, p\}$ are oscillatory processes admitting the spectral representation

$$X_k(t) = \int_{-\infty}^{\infty} A_k(t, \omega) e^{\mathbf{j}\omega t} dZ_k(\omega) \qquad (2)$$

In the above equation, the bold **j** denotes $\sqrt{-1}$, $A_k(t, \omega)$ is a frequency-time (deterministic) modulating function, and the quantities $\{dZ_k(\omega)\}$ denote zero-mean, mutually independent, orthogonal increment processes having the properties

$$E[dZ_k(\omega)] = 0, \qquad k = 1, 2, \ldots, p \qquad (3)$$

$$E[dZ_j^*(\omega_1) dZ_k(\omega_2)] = \delta(j-k)\delta(\omega_1 - \omega_2)\Phi_{Z_k Z_k}(\omega_1) d\omega_1 d\omega_2 \qquad (4)$$

where $j, k = 1, 2, \ldots, p$, $E[\ ]$ represents the ensemble-average or expectation operator, $\delta(\ )$ is the Dirac delta function, and the superposed * denotes the complex conjugate. The spectral representation in Eq. (2) can be physically interpreted as the limit of a "sum" of sine waves with increasing frequencies and time-varying random amplitudes $\{A_k(t, \omega)dZ_k(\omega)\}$. Each component process $X_k(t)$ of the sigma-oscillatory process $Y(t)$ has the following evolutionary spectrum

$$\Phi_{X_kX_k}(t, \omega) = |A_k(t, \omega)|^2 \Phi_{Z_kZ_k}(\omega) \tag{5}$$

with respect to the oscillatory family of functions, $\mathcal{F}_k = \{A_k(t, \omega)e^{j\omega t}\}$, which should be viewed as functions of ω indexed by t. For simplicity, it is assumed that each spectrum is absolutely continuous with respect to ω. According to Priestley's definition of oscillatory processes, the modulating function $A_k(t, \omega)$ (viewed as a function of t for each ω) must be such that the modulus of its Fourier transform $H_k(\theta, \omega)$ has an absolute maximum at the origin (i.e., $\theta = 0$) and

$$A_k(t, \omega) = \int_{-\infty}^{\infty} e^{j\theta t} H_k(\theta, \omega) d\theta \tag{6}$$

The mean-square function of the sigma-oscillatory process $Y(t)$ can be expressed as

$$E[|Y(t)|^2] = \sum_{k=1}^{p} E[|X_k(t)|^2] = \int_{-\infty}^{\infty} \sum_{k=1}^{p} [|A_k(t, \omega)|^2 \Phi_{Z_kZ_k}(\omega)] d\omega \tag{7}$$

which gives a decomposition over frequency of the "total energy" or variance of $Y(t)$ at time t. Therefore, the evolutionary (time-varying) power spectrum of $Y(t)$ can be meaningfully defined with respect to the oscillatory family of functions $\mathcal{F}_Y = \bigcup_{k=1}^{p} \mathcal{F}_k$ by

$$\Phi_{YY}(t, \omega) = \sum_{k=1}^{p} |A_k(t, \omega)|^2 \Phi_{Z_kZ_k}(\omega) \tag{8}$$

Notice that the sum of two independent sigma-oscillatory processes remains a sigma-oscillatory process whose evolutionary spectrum is the sum of the evolutionary spectra of the two individual processes. Moreover, the characteristic width of the family $\mathcal{F}_Y$, and the characteristic width of the process $Y(t)$ are defined as

$$B_{\mathcal{F}_Y} = \min_{1 \le k \le p} B_{\mathcal{F}_K}; \quad B_Y = \min_{1 \le k \le p} B_{X_k} \tag{9}$$

where $B_{\mathcal{F}_k} = \left[\sup_{\omega} \int_{-\infty}^{\infty} |\theta| |H_k(\theta, \omega)| d\theta / \int_{-\infty}^{\infty} |H_k(\theta, \omega)| d\theta\right]^{-1}$ is the characteristic width of $\mathcal{F}_k$ and B_{X_k} is the characteristic width of the component process $X_k(t)$ defined as $B_{X_k} = \sup_{\mathcal{F}_k \in \mathcal{G}_k} B_{\mathcal{F}_k}$ in which $\mathcal{G}_k$ is the class of families $\mathcal{F}_k$ with respect to which $X_k(t)$ admits the spectral representation in Eq. (2). If the process $X_k(t)$ is stationary, B_{X_k} is infinite. If B_{X_k} is finite, the nonstationary process $X_k(t)$ is termed semi-stationary. The characteristic width is a measure of the nonstationarity of a process; roughly speaking, $2\pi B_{X_k}$ or $2\pi B_Y$ may be interpreted as the maximum time interval over which $X_k(t)$ or $Y(t)$ can be treated as approximately stationary.

Here, the fully nonstationary, stochastic earthquake ground acceleration model, $\ddot{U}_g(t)$, is defined as a sum of zero-mean, independent, uniformly modulated Gaussian processes. Each uniformly modulated process consists of the product of a deterministic time modulating function, $A_k(t)$, and a stationary Gaussian process, $S_k(t)$. Therefore, the proposed stochastic earthquake model is a particular sigma-oscillatory Gaussian process defined as:

$$\ddot{U}_g(t) = \sum_{k=1}^{p} X_k(t) = \sum_{k=1}^{p} A_k(t) S_k(t) \tag{10}$$

Furthermore, the modified gamma function is used as time modulating function, i.e.,

$$A_k(t) = \alpha_k (t - \zeta_k)^{\beta_k} e^{-\gamma_k (t - \zeta_k)} H(t - \zeta_k) \tag{11}$$

where α_k and γ_k are positive constants, β_k is a positive integer, and ζ_k represents the "arrival time" of the k-th sub-process, $X_k(t)$; $H(t)$ denotes the unit step function. The k-th zero-mean stationary

Gaussian process, $S_k(t)$, is characterized by its autocorrelation function

$$R_{S_kS_k}(\tau) = e^{-\nu_k\tau}\cos(\eta_k\tau) \tag{12}$$

and its power spectral density function

$$\Phi_{S_kS_k}(\omega) = \frac{\nu_k}{2\pi}\left[\frac{1}{\nu_k^2 + (\omega+\eta_k)^2} + \frac{1}{\nu_k^2 + (\omega-\eta_k)^2}\right] \tag{13}$$

in which ν_k and η_k are two free parameters representing the frequency bandwidth and predominant (or central) frequency of the process $S_k(t)$, respectively. The stationary processes, $\{S_k(t),\ k = 1, 2, \ldots, p\}$, are normalized to a unit variance. According to Eq. (7), the mean square ground acceleration is given by

$$E[|\ddot{U}_g(t)|^2] = \int_{-\infty}^{\infty}\sum_{k=1}^{p}|A_k(t)|^2\Phi_{S_kS_k}(\omega)d\omega = \sum_{k=1}^{p}|A_k(t)|^2 \tag{14}$$

where

$$\int_{-\infty}^{\infty}\Phi_{S_kS_k}(\omega)d\omega = R_{S_kS_k}(\tau)|_{\tau=0} = E[|S_k(t)|^2] = 1 \tag{15}$$

From Eq. (8), the evolutionary power spectral density function of $\ddot{U}_g(t)$ is

$$\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega) = \sum_{k=1}^{p}|A_k(t)|^2\Phi_{S_kS_k}(\omega) \tag{16}$$

It is worth noting that the ground acceleration process, $\ddot{U}_g(t)$, is not separable, although its component processes are individually separable (i.e., uniformly modulated). Each uniformly modulated component process, $X_k(t)$, is characterized by a uni-modal power-spectral density function in the frequency domain and a "uni-modal" mean square function in the time domain. Therefore, each component process captures the complex time-frequency distribution of the earthquake ground acceleration in a local time-frequency region.

ESTIMATION OF MODEL PARAMETERS

The parameters of the earthquake ground acceleration model defined above are estimated such that the analytical evolutionary power spectral density function in Eq. (16) best fits, in the least square sense, the evolutionary PSD function of the target earthquake accelerogram estimated using the short-time Thomson's multiple-window method. Thomson's spectral estimate enjoys attractive statistical properties: it is consistent, has high resolution and estimation capacity, and it is not hampered by the usual trade-off between bias (leakage) and variance (Thomson 1982, Drosopoulous and Haykin 1992). The details of the parameter estimation procedure are given in (Conte and Peng 1996).

APPLICATION EXAMPLES AND MODEL VALIDATION

The proposed stochastic earthquake model has been applied to several real earthquake records having different nonstationarity characteristics. One of the applications presented here is the CHAN1: 90 Deg component of the Loma Prieta earthquake of October 17, 1989, recorded at Capitola site and show in Fig. 3(a). Fig. 1 represents the estimated time-varying power spectral density function, $\hat{\Phi}_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, based on the short-time Thomson's multiple-window spectrum estimation method. It shows how the frequency content of the target earthquake ground acceleration evolves in time. Fig. 2 portrays the analytical time-varying power spectral density function, $\Phi_{\ddot{U}_g\ddot{U}_g}(t, \omega)$, of the identified nonstationary stochastic
model.

The first level of model validation is performed by simulating a sample of 100 artificial accelerograms from the identified earthquake model and computing the second-order statistics (i.e., mean and standard-deviation) of ten ground motion parameters traditionally used to characterize earthquake intensity. These ground motion parameters include peak ground acceleration (PGA), velocity (PGV), and displacement (PGD); ratios of peak ground motions, PGV/PGA and PGD/PGA; root-mean-square acceleration (RMSA), velocity (RMSV), and displacement (RMSD); Arias intensity (AI); and Housner spectral intensity (SI_ξ) of damping ratio ξ. In simulating the analytical ground motion model, the component processes are generated independently using the spectral representation method (Shinozuka and Jan 1972) and combined to form one realization of the ground acceleration process. The artificial ground motions simulated are baseline-corrected in the frequency domain by using a simple rectangular high-pass filter with a cut-off frequency of 0.10 Hz and applying a least-square straight line fitting to both the integrated ground velocity and displacement records. Table 1 presents the values of the above ground motion parameters for the target earthquake record and the corresponding second-order statistics generated from the identified earthquake model. In each case, the statistical interval defined by "mean ± one standard deviation" contains the target parameter except for the RMSD which is slightly overpredicted by the identified earthquake model. A typical artificial ground acceleration time history simulated using the identified earthquake model is given in Fig. 3(b). Notice the strong similarities between the artificial and target accelerograms.

The second level of model validation consists of comparing target linear elastic response spectra with their probabilistic counterparts generated from the identified earthquake model. Fig. 4 shows the probabilistic linear elastic true relative displacement response spectrum, (S_D), for a probability of exceedence of 95%, 70%, 50%, 30%, and 5%. It is observed that the deterministic target response spectrum falls almost entirely within the (5-95%) statistical range of the probabilistic response spectrum in the period interval of practical interest from 0.1 to 10 sec. In fact, the target response spectrum falls between the sample maximum and minimum for the whole period range from 0.01 to 100 sec, except for a very short period segment around 1.08 sec.

CONCLUSIONS

A versatile, fully nonstationary, analytical stochastic earthquake ground motion model based on the theory of sigma-oscillatory processes is formulated and validated in this paper. First, the time-varying power spectral density function of the target real earthquake ground acceleration record is estimated using the short-time Thomson's multiple-window spectrum estimation method which is consistent, has high resolution, and is not hampered by the usual trade-off between bias (leakage) and variance. Then, the stochastic earthquake model corresponding to the target ground motion is built by identifying the "order" of the model (= number of independent component processes) and estimating the model parameters through an adaptive nonlinear least-squares algorithm. The parameter estimation procedure consists of minimizing the L_2-norm of the error between the analytical time-varying power spectral density (PSD) function of the earthquake model and the estimated time-varying PSD function, subjected to simple inequality constraints. Based on the application examples considered, it is found that the proposed earthquake model is able to capture very well the temporal variation of both the intensity and frequency content of real earthquake ground motions. Due to its analytical formulation, the proposed nonstationary earthquake model can be used for analytical linear (Conte and Peng, 1995) and nonlinear random vibration studies. This realistic stochastic earthquake model is currently used to gain better insight into the effects of the temporal variation of the frequency content of earthquake ground motions on linear and nonlinear structural response.

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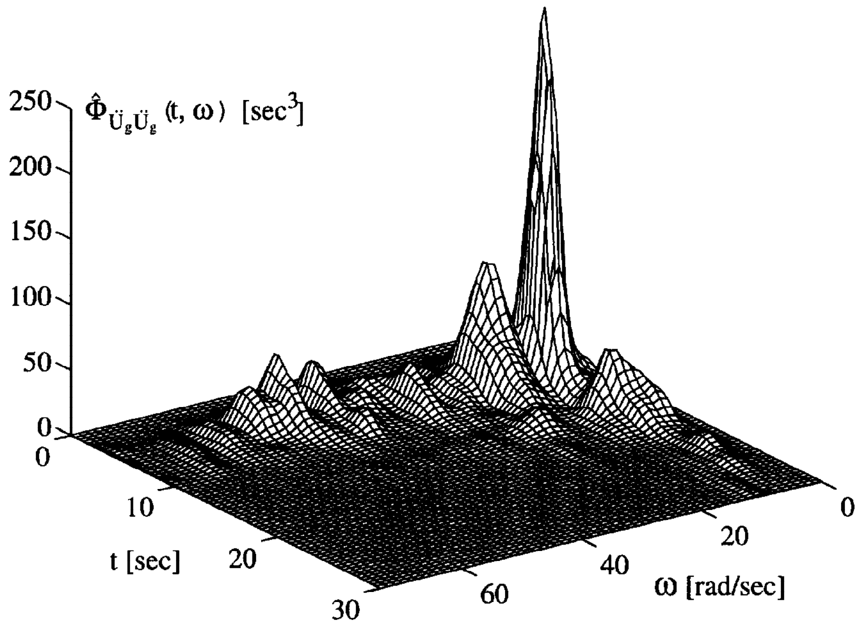

Fig. 1. Estimated Time-Varying Power Spectral Density Function for Capitola 1989 Earthquake Ground Acceleration

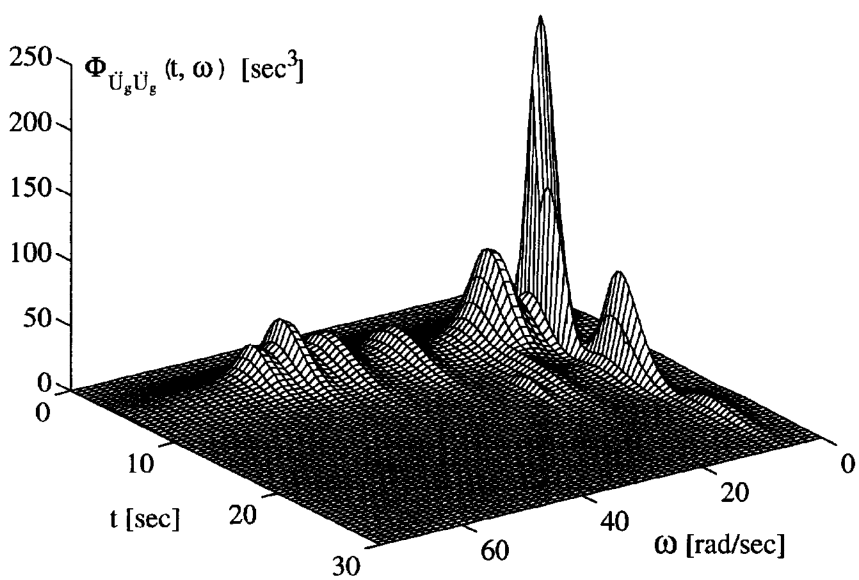

Fig. 2. Analytical Time-Varying Power Spectral Density Function of Sigma-Oscillatory Process Model for Capitola 1989 Earthquake Ground Acceleration

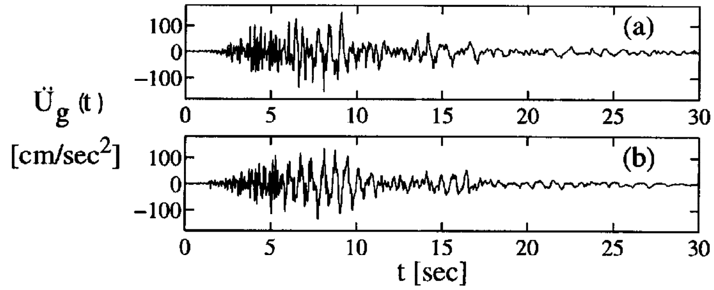

Fig. 3. Actual and Artificial Ground Acceleration Time Histories of Loma Prieta 1989 Earthquake Recorded at Capitola Site

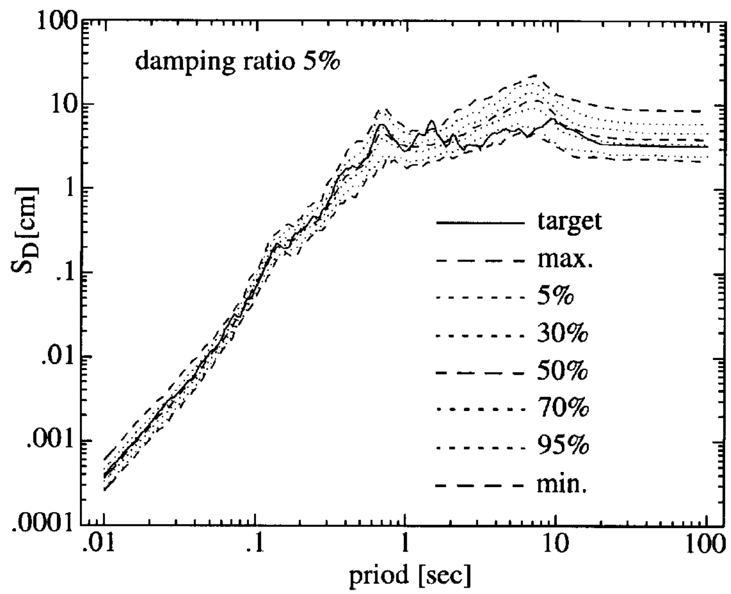

Fig. 4. Probabilistic Linear Elastic True Relative Displacement Response Spectrum for Capitola 1989 Earthquake

Table 1. Ground Motion Parameters for Capitola 1989 Earthquake and Statistics from the Identified Ground Motion Model

Parameter	Target	Mean	Std	C.O.V.	Max	Min
PGA [cm/sec^2]	153.92	145.93	24.38	0.17	235.82	102.04
PGV [cm/sec]	12.11	12.94	2.98	0.23	24.78	7.63
PGD [cm]	3.14	4.12	1.20	0.29	8.63	2.22
PGV/PGA [sec]	0.08	0.09	0.02	0.19	0.14	0.05
PGD/PGA [sec^2]	0.02	0.03	0.01	0.32	0.06	0.01
RMSA [cm/sec^2]	27.50	26.65	2.62	0.10	35.10	22.21
RMSV [cm/sec]	2.67	2.85	0.33	0.12	3.93	2.20
RMSD [cm]	0.94	1.44	0.36	0.25	2.48	0.79
AI [cm/sec]	92.24	87.55	17.60	0.20	150.37	60.22
$SI_{0.05}$ [cm]	46.96	43.17	6.46	0.15	61.14	31.31