11 WCEE

Copyright © 1996 Elsevier Science Ltd
Paper No. 297. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

WAVE PROPAGATION IN FLUID-SATURATED GROUND BY USE OF IMMISCIBLE MIXTURE THEORY

KAZUYA MITSUJI[1], SATOSHI KURITA[1] and MASANORI IZUMI[2]

[1] Department of Architecture, Faculty of Engineering, Tohoku University, Sendai, 980-77 JAPAN

[2] Izumi Research Institute, Shimizu Corporation, Fukoku Seimei Bldg. 2-2-2, Uchisaiwai-Cho, Chiyoda-Ku, Tokyo, 100 JAPAN

ABSTRACT

The purpose of this paper is to study the dynamic behavior of soil ground partially saturated with water. For this purpose the soil has been assumed to be a continuum mixture with two phases, one solid phase and one fluid phase, where the solid skeleton is fully saturated with fluid. Following Bowen [1] , for such soil ground the porosity of the fluid, in other words the volume fraction of the fluid phase, should be considered to be a variable during vibration, especially when one wants to calculate the seismic responses. Using the mixture theory of continuum mechanics presented by Bowen [1] , the constitutive laws, the wave attenuation and the wave propagation problems have been discussed.

KEYWORDS

Mixture theory, Volume fraction, Wave attenuation, Wave propagation, Horizontally layered ground

Introduction

Soil ground should be substantially considered to be a mixture which consists of solid and fluid in earthquake engineering. Biot's theory [2] and [3] is often applied to describe the dynamic behavior of the soft ground. According to his theory, soil is assumed to be a mixture of elastic skeleton with void which is fully saturated by pore fluid. Also the porosity, which is the ratio of solid and fluid, is defined as an important constant parameter. On the other hand, mixture theory has been developed in continuum mechanics. In this field, Bowen [1] proposed a theory in which the volume fraction has been treated as an independent variable, which is applicable to the multiphase mixture and is equivalent to the porosity when the mixture is assumed to be the two phase mixture.

In this paper, Bowen's theory has been utilized to solve earthquake engineering problems, and the simulation results have been investigated by comparison with Biot's theory. The mixture is considered to be a two phase mixture of solid particle and pore fluid, and the porosity (volume fraction) has been treated a variable which has its own constitutive equation as presented by Bowen [1]. Because of the relative motion of two different phases and the change of the volume fraction during vibration, wave propagation in the soil ground attenuates even if each phase is elastic. Such wave attenuation behaviors have been studied using the characteristic equation, by changing the coefficients related to the volume fraction. The amplitude of the surface ground containig a porous layer has been presented.

Basic Theory

The volume fraction ϕ_f is defined as:

$$\phi_a(x,t) = \frac{\rho_a(x,t)}{\gamma_a(x,t)} \tag{1}$$

where ρ_a, γ_a represent the mass of the ath constituent per unit volume of the mixture, the mass of the ath constituent per unit volume of the ath constituent respectively. From the definition, the volume fractions can be defined as many as the number of the constituents of mixture. However, in this study, a two phase mixture which is composed of the solid skeleton (soil particle) and fluid (the pore water) is focused on, hereinafter the volume fraction is referred to as the porosity. Thus according to Bowen [1], the governing equations of the wave propagation of each phase are represented in the following forms:

$$\rho_f^+ \frac{\partial^2 w_f}{\partial t^2} = \lambda_f GRAD(Divw_f) + \lambda_{sf} GRAD(Divw_s) + \Gamma_f GRAD\phi_f - \xi(\frac{\partial w_f}{\partial t} - \frac{\partial w_s}{\partial t}) \tag{2}$$

$$\rho_s^+ \frac{\partial^2 w_s}{\partial t^2} = (\lambda_s + \mu_s) GRAD(Divw_s) + +\mu_s Div(GRADw_s) + \lambda_{sf} GRAD(Divw_f)$$
$$+\Gamma_s GRAD\phi_f + \xi(\frac{\partial w_f}{\partial t} - \frac{\partial w_s}{\partial t}) \tag{3}$$

$$\frac{\partial \phi_f}{\partial t} = -\Lambda_f\{(\phi_f - \phi_f^+) + \frac{\Gamma_s}{\Phi_f} Divw_s + \frac{\Gamma_f}{\Phi_f} Divw_f\} \tag{4}$$

where w_s, w_f are displacements of solid skeleton and fluid, ρ_s, ρ_f are mass densities of solid skeleton and fluid, ϕ_f^+, ϕ_f are initial porosity and porosity, ξ is dissipation coefficient, $\xi = \phi_f^{+2}\rho_f g/k_d$, k_d is permeability coefficient, g is gravity acceleration, λ_s, μ_s are Lamé constants of the solid, λ_f is bulk modulus of fluid, λ_{sf} is bulk modulus of mixture. Λ_f has the physical dimension of the reciprocal time and determine the characteristic time of the relaxation process. Γ_s, Γ_f, Φ_f are the coefficients related to the Helmholtz free energy and the porosity. When the terms, $\Gamma_f GRAD\phi_f$ and $\Gamma_s GRAD\phi_f$, are eliminated, Biot's equations could be obtained.

Wave Attenuation

It is well known that the dispersion relation due to the interaction between solid and fluid appears in porous media. Two kinds of P wave are observed, namely P1 and P2 wave. P2 wave is slow and very highly attenuated. Substituting $w_{s,f} = C_{s,f}exp\{i(wt - kx)\}$ into eq.(2), the characteristic equations to estimate attenuation and phase velocities of P wave eq.(5) and S wave eq.(6) have been derived.

$$\{(\lambda_f - \frac{\Lambda_f\Gamma_f}{\Lambda_f + i\omega}\frac{\Lambda_f}{\Phi_f})(\lambda_s + \mu_s - \frac{\Lambda_f\Gamma_s}{\Lambda_f + i\omega}\frac{\Lambda_s}{\Phi_f}) - (\lambda_{sf} + \frac{\Lambda_f\Gamma_s}{\Lambda_f + i\omega}\frac{\Lambda_f}{\Phi_f})^2\}(\frac{k}{\omega})^4$$
$$-[\rho_f^+(\lambda_s + 2\mu_s - \frac{\Lambda_f\Gamma_s}{\Lambda_s}\Phi_f) + \rho_s^+(\lambda_f - \frac{\Lambda_f\Gamma_f}{\Lambda_f + i\omega}\frac{\Lambda_f}{\Phi_f}) - \frac{i\xi}{\omega}\{(\lambda_f - \frac{\Lambda_f\Gamma_f}{\Lambda_f + i\omega}\frac{\Lambda_f}{\Phi_f})$$
$$+(\lambda_s + 2\mu_s - \frac{\Lambda_f\Gamma_s}{\Lambda_f + i\omega}\frac{\Lambda_s}{\Phi_f}) + (2\lambda_{sf} + \frac{\Lambda_f\Gamma_s}{\Lambda_f + i\omega}\frac{\Lambda_f}{\Phi_f})^2\}](\frac{k}{\omega})^2 + \rho_s^+\rho_f^+ - \frac{i\xi}{\omega}(\rho_s^+ + \rho_f^+) = 0 \tag{5}$$

$$(-\rho_f^+\mu_s + \frac{i\xi}{\omega}\mu_s)(\frac{k}{\omega})^2 + \{\rho_s^+\rho_f^+ - \frac{i\xi}{\omega}(\rho_s^+ + \rho_f^+)\} = 0 \tag{6}$$

The nature of the porous media causes attenuation of the body waves, which may be characterized by the quality factor, Q value. The material properties of numerical examples are shown in Table 1, which are used in *Géli et al* [4]. Coefficients λ_f, λ_{sf} are expressed as $K_V\phi_f^{+2}$, $K_V\phi_f^+(1-\phi_f^+)$ respectively, and K_V is described $K_V = K_sK_f/(\phi_f^+K_s + (1-\phi_f^+)K_f)$, using bulk modulus of solid K_s, and fluid K_f.

Using the coefficients shown in Table1. , the Q value has been calculated. The simulated results are shown in Fig.1. The porosity has been assumed to be 35%, and the permeability coefficient k_d is assumed to be 1.0 and 10.0. In Fig.1, there are four figures, of which the coefficients are shown in the figures. The horizontal axis is the frequency f and the vertical axis is the Q value. The solid line is

Table 1: Material properties

	1. Surface Layer (elastic)	2. Porous Layer	3. Elastic Half Space
$V_s(m/s)$	900	900	3100
$V_p(m/s)$	1800	1800	5400
$\rho(t/m^3)$	1.72	2.07	2.8
$\rho_s(t/m^3)$		2.65	
$\rho_f(t/m^3)$		1.0	
$K_v(t/cm^2)$		51	
$k_d(cm/s)$		1.0,10.0	
$\phi_f^+(\%)$		35	

the result from Biot's theory, and the dot and dot-dashed lines are the results from what shown in the figures. It is obvious that the Q values are different, and the change of the porosity attenuates the wave.

Wave Propagation in Horizontally Layered Ground

In order to analyze the amplitude of the surface ground in frequency domain, the displacement potentials have been introduced as follows.

$$u_x^s = \frac{\partial \phi^s}{\partial x} - \frac{\partial \psi^s}{\partial z}, \quad u_z^s = \frac{\partial \phi\ s}{\partial z} + \frac{\partial \psi^s}{\partial x} \tag{7}$$

$$u_x^f = \frac{\partial \phi^f}{\partial x} - \frac{\partial \psi^f}{\partial z}, \quad u_z^f = \frac{\partial \phi\ f}{\partial z} + \frac{\partial \psi^f}{\partial x} \tag{8}$$

Substituting eq.(7) and (8) into eq.(2), the characteristic equations of P1, P2 and S wave are derived. Based on Motosaka and Ohtsuka [6] and Philippacopoulos [7] , potentials have been defined as,

$$\phi^s(x,z) = \{A_1^{(\pm)} exp(\pm\alpha_1 z) + A_2^{(\pm)} exp(\pm\alpha_2 z)\} exp(ikx) \tag{9}$$

$$\psi^s(x,z) = B^{(\pm)} exp(\pm\beta z) exp(ikx) \tag{10}$$

$$\phi^f(x,z) = \{d_1(\omega) A_1^{(\pm)} exp(\pm\alpha_1 z) + d_2(\omega) A_2^{(\pm)} exp(\pm\alpha_2 z)\} exp(ikx) \tag{11}$$

$$\psi^f(x,z) = d_3(\omega) B^{(\pm)} exp(\pm\beta z) exp(ikx) \tag{12}$$

where the time factor $e^{i\omega t}$ is omitted for simplicity and

$$\alpha_{1,2} = k^2 - \omega^2/V_{P1,P2}^2, \quad \beta = k^2 - \omega^2/V_S^2, \quad Re(\alpha_{1,2}) \geq 0, \quad Re(\beta) \geq 0$$

$$d_{1,2}(\omega) = \frac{\omega(\lambda_s + 2\mu_s - \frac{\Lambda_f \Gamma_s}{\Lambda_f + i\omega}\frac{\Gamma_f}{\Phi_f}) - V_{P1,P2}^2(-i\xi + \omega\rho_s^+)}{i\xi V_{P1,P2} - \omega(\lambda_{sf} - \frac{\Lambda_f \Gamma_s}{\Lambda_f + i\xi}\frac{\Gamma_f}{\Phi_f})}$$

$$d_3(\omega) = \frac{i\xi}{i\xi - \omega\rho_f^+}$$

As an example of the calculation of the amplitude of the surface, the layered ground shown in Fig.2 has been used as a model. By use of Haskell method [5] , the amplitude of the surface has been calculated and are shown in Fig. where the boundary conditions at each interface are assumed in the following.

At the top of the surface layer:

$$T_{xz} = 0 \quad T_{zz} = 0$$

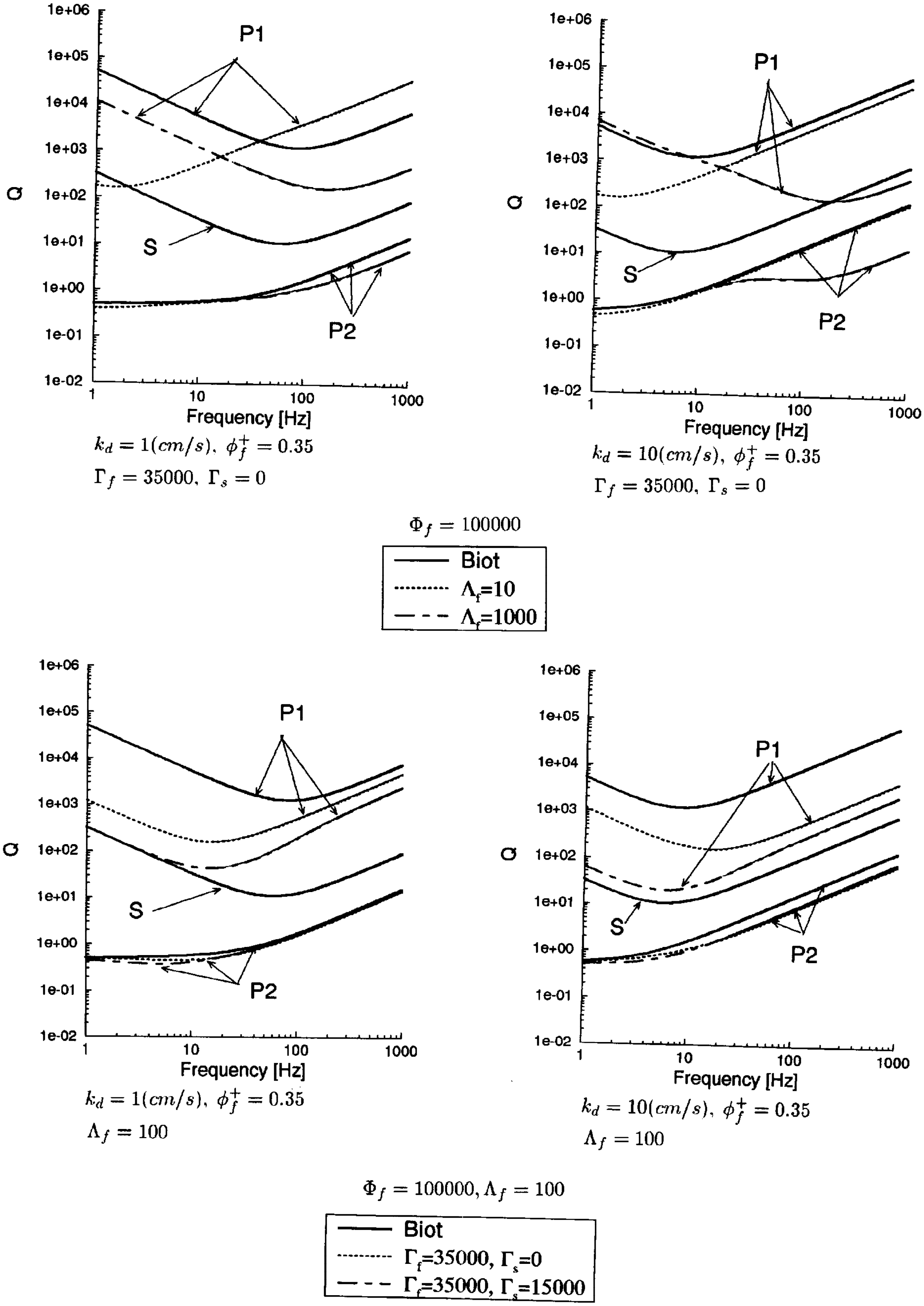

Fig. 1: The Q Value

At the top of the porous layer,

$$T^e_{xz} = T^b_{xz} \quad T^e_{zz} = T^b_{zz} \quad u^e_{x,z} = u^b_{x,z} \quad p = 0$$

At the bottom of the porous layer,

$$T^b_{xz} = T^e_{xz} \quad T^b_{zz} = T^e_{zz} \quad u^b_{x,z} = u^e_{x,z} \quad \phi^+_f(\dot{u}^b_z - \dot{u}^f_z) = 0$$

where T, u, p, mean traction, displacement, pore pressure, and supersucript b and e represent bulk and elastic media, respectively.

The simulation results are shown in Fig.3 with four figures, where the horizontal axis is the frequency f and the vertical axis is the amplitude. The coefficients of each figure are same as in Fig.1.

It is obvious that the amplitude of the surface while considering the change of the porosity is smaller compared with that is derived from Biot's theory. Fig.1 makes it clear that the change of the porosity attenuates the wave.

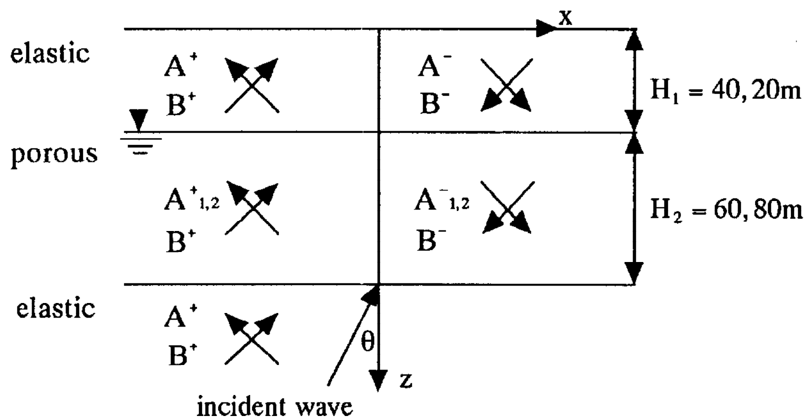

Fig. 2: Horizontally layered ground with porous layer

Conclusion

For the purpose solving dynamic problems of ground soil partially saturated with water, constitutive laws have been presented in which the porosity has been considered to be a variable with an independent constitutive law. Based on the constitutive laws, the wave attenuation problem and wave propagation problem have been solved and simulation results have been shown. It is clear that the wave attenuates if the change of the porosity is taken into account.

Acknowledgment

The authors are very grateful to Dr. M. Motosaka and Dr. Y. Ohtsuka of Kobori Research Institute, Dr. S. Xue and Dr. J. Tobita of Tohoku University, without their help this research would have been impossible.

References

[1] Bowen, R. M. (1982), "Compressible Porous Media Model by Use of the Theory of Mixtures", *Int. J. Engng Science* Vol. 20, No. 6, pp.697-735

[2] Biot, M. A. (1956), "Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid, I. Low-Frequency Range", *The J. of The Acoustic Society of America* Vol. 28, No. 2, pp.168-178

[3] Biot, M. A. (1956), "Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid, II. Higher Frequency Range", *The J. of The Acoustic Society of America* Vol. 28, No. 2, pp.179-190

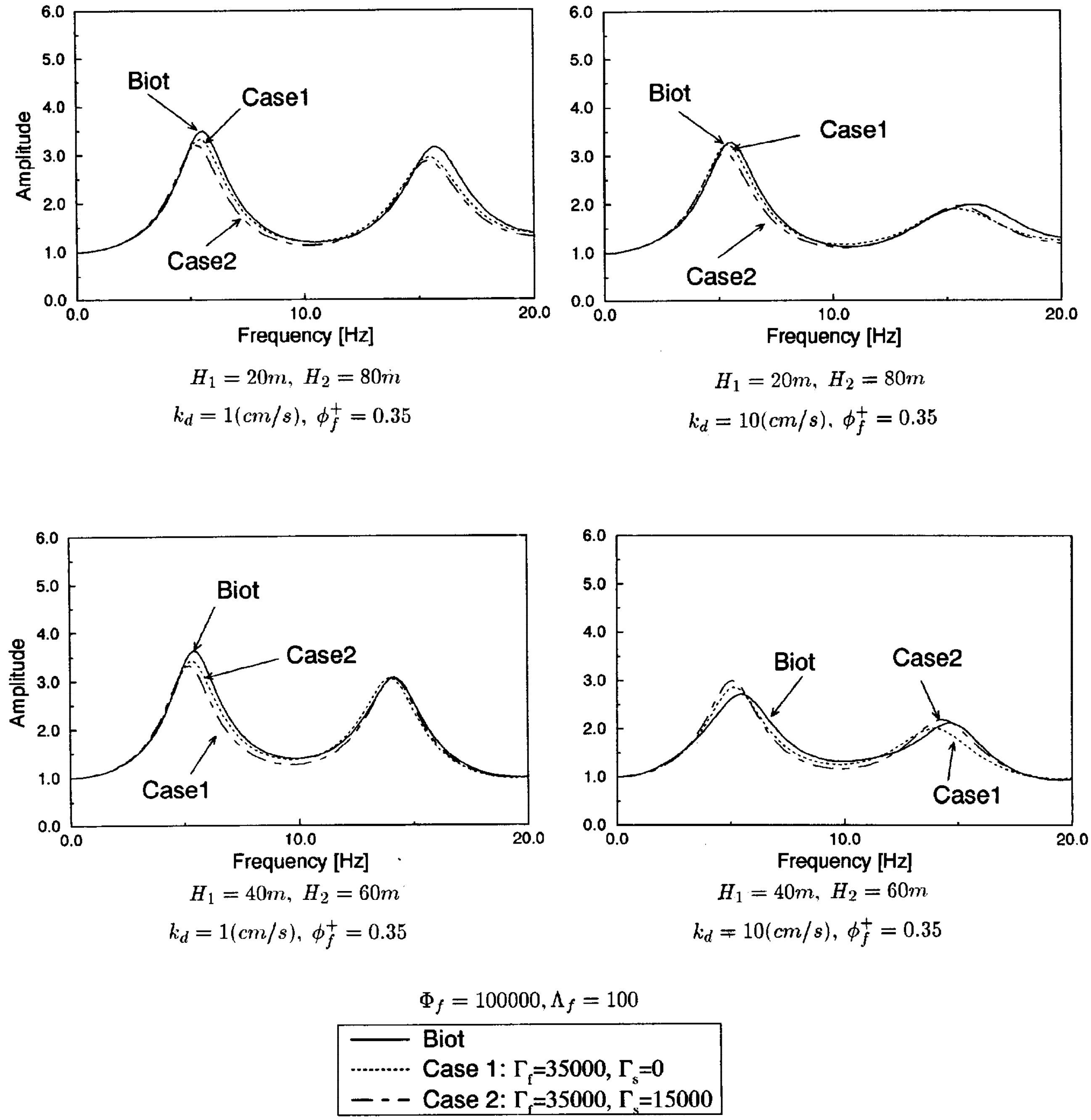

Fig. 3: Vertical motion due to P wave

[4] Géli, L. Bard P. Y. and Schmitt, D. P. (1987), "Seismic Wave Propagation in a very Permeable Water-Saturated Surface Layer", *J. of Geophysical Research* Vol. 92, No. B8, pp.7931-7944

[5] Motosaka, M. and Ohtsuka Y. (1992), "Frequency Response Characteristics for Incident P-SV Waves in Multi-Layered Irregular Strata Including Water-Saturated Layers Using Discrete Wave Number Method", *J. of Phys. Earth.* Vol. 40, pp.263-284

[6] Philippacopoulos, A. J. (1987), "Waves in a Partially Saturated Layered Half-Space: Analytic Formulation", *Bulletin of Seismological Society of America* Vol. 77, No. 5, pp.1838-1853

[7] Haskell, N. A. (1951), "The Dispersion of Surface Waves on Multilayered Media", *Bulletin of the Seismological Society of America* Vol. 41, pp.17-34