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DYNAMIC RESPONSE OF SUBSTRUCTURES IN FRICTION DAMPED BRACED FRAMES

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ABSTRACT

Efficient exact and approximate solution techniques are developed to compute the response of secondary structures, which are attached to friction damped braced frames under earthquake excitation as well as to calculate floor spectra at low computing costs. Slip deformations of the primary structure are treated as additional "internal" forcing terms acting on the linear background structure of initial stiffness and their intensity is calculated in an iterative process. Modal coupling of tuned modes is considered. A numerical example is shown for a 4-storey plane frame under earthquake excitation with a SDOF oscillator attached as the secondary system. Approximate results are compared to those derived from a fully coupled analysis, in order to evaluate their validity.

KEYWORDS

Friction braced frames; secondary structures; iterative synthesis technique; modal coupling of tuned modes.

INTRODUCTION

Since the last decade friction damped braced frames (FDBF) are used for aseismic design. This concept of dissipative bracing not only increases the stiffness against lateral deflection under moderate excitation but also dissipates a major portion of input energy due to slip of the damping devices during severe seismic excitation. The main structural components of the building remain elastic under nearly catastrophic conditions. Superior energy dissipation capabilities of FDBF are characterized by stable hysteretic non-degrading behavior and by avoiding irrecoverable damage due to inelastic action. Both analytical and experimental analyses have been performed in a series of papers in the last decade (e.g. Pall and Marsh, 1982, Vezina *et al.*, 1992). Recently, an incremental time-stepping solution procedure has been developed (Lei and Ziegler, Lei, 1994), where the nonlinear effects of friction bracing are treated as additional "internal" forcing terms acting upon the linear background structure. When compared to the classical incremental stiffness formulation, this solution procedure avoids the inefficient recalculation of the frequencies and modes of the structure every time there occurs a slip of the friction damper.

In earthquake resistant design of containment structures it is often not sufficient to use the response of the primary system as the input of the secondary structure. Such an uncoupled analysis neglects the reaction of the main structure to the dynamic response of the secondary containment. Especially, if one or more eigenfrequencies of the primary and of the secondary system become close (tuned), a coupled analysis is necessary at least in the evaluation of the secondary response. In order to avoid a modal analysis of the entire composite structure for each different containment system, it is more appealing to use a decomposition into substructure modes. While this procedure diagonalizes the equations of motion of each substructure, a successive synthesis, however, leads again to a non-diagonal system of equations (Curnier, 1983). The solution procedure can be simplified by assuming the coupling terms as an additional excitation of each substructure, and the interface constraints are solved in an iterative process. Such a procedure becomes especially useful, if the primary structure is a friction braced frame (thus nonlinear) and the response is determined with the above described solution procedure, where iterations become necessary. Recently, an elastic-plastic primary structure was considered (Adam and Fotiu, 1995).

Alternatively the response of linear substructures in particular with respect to floor spectra is obtained by considering a coupling only between the tuned frequencies, leaving all modes with well separated frequencies uncoupled (Igusa and Der Kiureghian, 1985a, 1985b). This technique was generalized to study elastic-plastic floor spectra (Fotiu and Ziegler, 1990). In this paper that method is refined to friction braced frames as the primary structure. The response is partitioned into a linear portion due to earthquake excitation of the linear background structure, which can be determined in advance, and into the portion due to internal excitation of the linear background structure resulting from nonlinear effects due to friction bracing. The second portion of the response has to be calculated iteratively in a time-stepping procedure and shifts the linear response into the actual nonlinear one. Coupling of the tuned modes can be considered in the same way formally introduced by Igusa and Der Kiureghian (1985a, 1985b) by means of this apparently linear solution technique.

The iterative synthesis method as well as the modified tuned mode technique presented in this paper provide accurate results of the response of a composite nonlinear friction braced structure for arbitrary loadings, at low computational costs. In general, an uncoupled analysis is sufficient only if primary and secondary modes are well separated.

SYSTEM ANALYSIS BY ITERATIVE SYNTHESIS

Consider a friction braced frame of n stories with friction dampers in the traditional K-type bracing, with an elastic SDOF oscillator attached to the kth floor. For simplicity, the shear-type frame model is used in the following analysis. In this model the masses are lumped at the storey levels and the beams are considered infinitely rigid compared to the more flexible columns. The kth equation of motion of the friction braced primary structure is given by where M_k, C_k are the kth floor mass and damping coefficient, respectively, W_k is the displacement of the kth floor with respect to the foundation and $\ddot{w}_g$ denotes the ground acceleration due to earthquake excitation. k and c are the stiffness and damping coefficient of the secondary structure and w is the secondary displacement with respect to the ground. Q_k is the shear restoring force of the kth storey, which can be expressed as (Lei and Ziegler, 1994)

$$Q_k = K_k \left[(W_k - W_{k-1}) - (1 - \alpha_k)(W_k - W_{k-1} - \bar{X}_k) \right] . \tag{2}$$

In (2) K_k denotes the storey stiffness before yielding: $K_k = K_k^f + K_k^b$, where K_k^f is the storey stiffness without bracing and K_k^b is the storey stiffness of the bracing, $\alpha_k = K_k^f / K_k$ is the post-yielding stiffness ratio and $\bar{X}_k$ denotes the kth displacement with respect to the $(k-1)$th floor, where slipping is initialized. The total deformation in shear Q_k can be split into an elastic part and into a yielding drift part Z_k,

$$Z_k = (1 - \alpha_k)(W_k - W_{k-1} - \bar{X}_k) . \tag{3}$$

The equations of motions of the primary structure for $i \neq k$, $i = 1, ..., n$, are given by (1), whereby the coupling terms of the SDOF oscillator are omitted. Hence, the displacement of the primary structure and of the SDOF oscillator are given by,

$$\boldsymbol{M}\ddot{\boldsymbol{W}} + \boldsymbol{C}\dot{\boldsymbol{W}} + \boldsymbol{K}\boldsymbol{W} = -\boldsymbol{M}\boldsymbol{e}\,\ddot{w}_g - \boldsymbol{G}\boldsymbol{Z} + \left[k(w - W_k) + c(\dot{w} - \dot{W}_k) \right] \boldsymbol{g}_k , \tag{4a}$$

$$m\ddot{w} + c\dot{w} + kw = -m\ddot{w}_g + k W_k + c\dot{W}_k , \tag{4b}$$

where $\boldsymbol{M}$ is the diagonal mass matrix, $\boldsymbol{K}$ and $\boldsymbol{C}$ are the initial stiffness and damping matrix of the primary structure, respectively, $e = (1, 1, ..., 1)^T$ for single point excitation, and $\boldsymbol{W}$ is the vector of the primary displacements. Furthermore, $\boldsymbol{Z}$ denotes the vector of the yielding drift, and $\boldsymbol{G}$ consists of the constraint forces at horizontally fixed storey masses due to a singular storey slip. $\boldsymbol{g}_k$ is a vector with its kth component equal to unity and all others equal to zero and m denotes the mass coefficient of the secondary structure. Performing a decomposition of $\boldsymbol{W}$ into the n mode shapes $\boldsymbol{\phi}_i$, $i = 1, ..., n$, of the primary structure,

$$\boldsymbol{W} = \boldsymbol{\Phi}\boldsymbol{Y} \ , \quad \boldsymbol{\Phi} = [\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2 \cdots \boldsymbol{\phi}_n] , \tag{5}$$

renders

$$\ddot{Y}_i + 2\zeta_i \Omega_i \dot{Y}_i + \Omega_i^2 Y_i = \frac{\mathscr{L}_i^e}{\mathcal{M}_i}\ddot{w}_g + \frac{\boldsymbol{L}_i^{*T}}{\mathcal{M}_i}\boldsymbol{Z} + \ell_i^*(\mathfrak{w} - \mathcal{W}_k) \ , \quad \ddot{w} + 2\zeta_0 \omega \dot{w} + \omega^2 w = -\ddot{w}_g + \omega \mathcal{W}_k , \tag{6}$$

with

$$\mathcal{M}_i = \boldsymbol{\phi}_i^T \boldsymbol{M} \boldsymbol{\phi}_i \ , \quad \mu_i = m / \mathcal{M}_i \ , \quad \mathscr{L}_i^e = -\boldsymbol{\phi}_i^T \boldsymbol{M}\boldsymbol{e} \ , \quad \boldsymbol{L}_i^{*T} = -\boldsymbol{\phi}_i^T \boldsymbol{G} \ , \quad \ell_i^* = \omega \mu_i \Phi_{ki} , \tag{7}$$

$$\mathfrak{w} = \omega w + 2\zeta_0 \dot{w} \ , \quad \mathcal{W}_k = \sum_{j=1}^{n} \Phi_{kj} \left[\omega Y_j + 2\zeta_0 \dot{Y}_j \right] . \tag{8}$$

Here, Ω_i, ω and ζ_i, ζ_0 are the undamped substructure eigenfrequencies and modal damping parameters of the primary structure and of the SDOF oscillator, respectively. It is now convenient to split the total modal coefficients $\boldsymbol{Y}$, w into an uncoupled elastic part $\boldsymbol{Y}^e$, w^e and a remaining part $\boldsymbol{Y}^*$, w^*, which contains contributions from inelastic deformations and coupling, $\boldsymbol{Y}=\boldsymbol{Y}^e+\boldsymbol{Y}^*$, $w=w^e+w^*$. Such a partitioning has been introduced in the analysis of elastic-plastic reinforced concrete frames under earthquake excitations (Fotiu *et al.*, 1989). Similar techniques have been applied in the form of integral equations (Fotiu *et al.*, 1994). Accordingly, (6) decomposes into

$$\ddot{Y}_i^e+2\,\zeta_i\,\Omega_i\,\dot{Y}_i^e+\Omega_i^2\,Y_i^e=\frac{\mathcal{L}_i^e}{\mathcal{M}_i}\,\ddot{w}_g\,,\quad \ddot{w}^e+2\,\zeta_0\,\omega\,\dot{w}^e+\omega^2\,w^e=-\,\ddot{w}_g\,, \tag{9}$$

$$\ddot{Y}_i^*+2\,\zeta_i\,\Omega_i\,\dot{Y}_i^*+\Omega_i^2\,Y_i^*=\frac{\boldsymbol{L}_i^{*T}}{\mathcal{M}_i}\,\boldsymbol{Z}+\ell_i^*\,(\boldsymbol{w}-\mathcal{W}_k)\,,\quad \ddot{w}^*+2\,\zeta_0\,\omega\,\dot{w}^*+\omega^2\,w^*=\omega\,\mathcal{W}_k\,. \tag{10}$$

Solutions $\boldsymbol{Y}^e$, w^e can be calculated independently from (10), while the response $\boldsymbol{Y}^*$, w^* due to both slip deformation and coupling is found by iteration. Note that since the SDOF oscillator is assumed elastic, only coupling terms appear to the right of (10). (9), (10) will be solved by the Duhamel integral. It is assumed that within each time step $\Delta t=t_b-t_a$, the right hand sides of (9), (10) vary according to a given time shape function. Then, the time integration can be carried out explicitly within the interval Δt. Assuming, for example linear shape functions, gives

$$Y_{ib}^e=\mathcal{I}_i^e(t_a)+\frac{\mathcal{L}_i^e}{\mathcal{M}_i\,\Omega_{iD}}\left[\ddot{w}_{ga}\,H_{ia}+\ddot{w}_{gb}\,H_{ib}\right]\,,\quad w_b^e=\mathcal{J}^e(t_a)-\frac{1}{\omega_D}\left[\ddot{w}_{ga}\,h_a+\ddot{w}_{gb}\,h_b\right]\,, \tag{11}$$

$$Y_{ib}^*=\mathcal{I}_i^*(t_a)+\frac{1}{\Omega_{iD}}\left\{\left[\frac{\boldsymbol{L}_i^{*T}}{\mathcal{M}_i}\,\boldsymbol{Z}_a+\ell_i^*\,(\boldsymbol{w}_a-\mathcal{W}_{ka})\right]H_{ia}+\left[\frac{\boldsymbol{L}_i^{*T}}{\mathcal{M}_i}\,\boldsymbol{Z}_b+\ell_i^*\,(\boldsymbol{w}_b-\mathcal{W}_{kb})\right]H_{ib}\right\}\,, \tag{12a}$$

$$w_b^*=\mathcal{J}^*(t_a)+\frac{\omega}{\omega_D}\left[\mathcal{W}_{ka}\,h_a+\mathcal{W}_{kb}\,h_b\right]\,. \tag{12b}$$

In (11) and (12) a subscript a (b) indicates values at time instant $t=t_a$ (t_b), and $H_{ia\,(b)}$, $h_{a\,(b)}$ are determined by evaluating the corresponding Duhamel integrals with the linear shape functions, for example,

$$H_{ia}=\int_0^{\Delta t}(1-\tau/\Delta t)\,\exp\left[-\,\zeta_i\,\Omega_i\,(\Delta t-\tau)\right]\sin\left[\Omega_{iD}\,(\Delta t-\tau)\right]d\tau\,, \tag{13}$$

and H_{ib} is given by a similar integral with $\tau/\Delta t$ substituted for $(1-\tau/\Delta t)$. A subscript $(\,.\,)_D$ indicates the damped eigenfrequency, $\Omega_{iD}=\Omega_i\sqrt{1-\zeta_i^2}$. The symbols $\mathcal{I}_i^{e,*}$, $\mathcal{J}^{e,*}$ in (11) and (12) represent the initial conditions at the beginning of the time step $t=t_a$,

$$\mathcal{I}_i^{e,*}(t_a)=\exp\left(-\,\zeta_i\,\Omega_i\,\Delta t\right)\left[\frac{1}{\Omega_{iD}}\,(\dot{Y}_{ia}^{e,*}+Y_{ia}^{e,*}\,\zeta_i\,\Omega_i)\,\sin\Omega_{iD}\Delta t+Y_{ia}^{e,*}\,\cos\Omega_{iD}\Delta t\right]\,. \tag{14}$$

Expressions for $h_{a\,(b)}$ and $\mathcal{J}^{e,*}$ are given analogously to (13) and (14), respectively, with ζ_0, ω, ω_D, $w_a^{e,*}$, $\dot{w}_a^{e,*}$ replacing ζ_i, Ω_i, Ω_{iD}, $Y_{ia}^{e,*}$, $\dot{Y}_{ia}^{e,*}$. Relations similar to (11) and (12) are established also for the velocities $\dot{Y}_{ib}^{e,*}$ and $\dot{w}_b^{e,*}$.

If the primary structure is considered purely elastic, iteration of (11), (12) is performed only in modal displacements $\boldsymbol{Y}$, w and velocities $\dot{\boldsymbol{Y}}$, $\dot{w}$. In case slip occurs, estimates of the yielding drift parts $\boldsymbol{Z}$ are obtained via the bilinear restoring force relation from previous estimates of the relative storey displacements. Compared with the incremental stiffness formulation, the solution procedure avoids the inefficient recalculation of the frequencies and modes of the structure every time there occurs a slip of the friction damper.

SYSTEM ANALYSIS BY MODAL COUPLING OF TUNED MODES

According to equations (4) the coupled system is projected into the primary substructure eigenspace. Naturally, this does not exactly diagonalize the system of equations, but if $\mu_i << 1$ is assumed, the off-diagonal coefficients in the primary modal space are small of higher order and therefore they can be neglected. In addition, only coupling between tuned modes is considered, that means these modes are closely spaced. The criterion for two modes Ω_m, ω to be tuned is $\mathcal{O}(\delta_m) = \mathcal{O}(\sqrt{\mu_m})$ (Igusa and Der Kiureghian, 1985b) , where $\mathcal{O}(\cdot)$ denotes the order of $(\cdot)$ and

$$\delta_m = (\omega^2 - \Omega_m^2)/(2\,\bar{\omega}_m^2\,\Phi_{km})\,, \quad \bar{\omega}_m^2 = (\omega^2 + \Omega_m^2)/2\,. \tag{15}$$

The modal properties of singly tuned modes are derived from equations referring to a two degree-of-freedom oscillator (Ziegler, 1995), where only coupling terms of tuned modes are considered (Igusa and Der Kiureghian, 1985a),

$$\ddot{Y}_m + 2\,\zeta_m\,\Omega_m\,\dot{Y}_m - 2\,\zeta_0\,\omega\,\mu_m\,\Phi_{km}\,\dot{w} + \Omega_m^2\,Y_m - \omega^2\,\mu_m\,\Phi_{km}\,w = \frac{\mathcal{L}_m^e}{\mathcal{M}_m}\,\ddot{w}_g + \frac{\boldsymbol{L}_m^{*T}}{\mathcal{M}_m}\,\boldsymbol{Z}\,, \tag{16a}$$

$$\ddot{w} - 2\,\zeta_0\,\omega\,\Phi_{km}\,\dot{Y}_m + 2\,\zeta_0\,\omega\,\dot{w} - \omega^2\,\Phi_{km}\,Y_m + \omega^2\,w = -\,\ddot{w}_g\,. \tag{16b}$$

Although equations (16) are non-classically damped, an equivalent classically damped system is used to obtain solutions for w and W. This appears to be justified, since in inelastic structures modal damping (classical or non-classical) contributes only a small amount to the total damping capacity of the structure. The equivalent classically damped system is found by a modal decomposition of equations (16) by means of undamped mode shapes and considering only diagonal terms in the transformed damping matrix. This eventually yields a new set of modal coefficients for the tuned modes,

$$Y_{ib}^e = \hat{\mathcal{F}}_i^e(t_a) + \frac{\hat{\mathcal{L}}_i^e}{\hat{\Omega}_{iD}}\left[\ddot{w}_{ga}\,\hat{H}_{ia} + \ddot{w}_{gb}\,\hat{H}_{ib}\right], \quad Y_{ib}^* = \hat{\mathcal{F}}_i^*(t_a) + \frac{\hat{\boldsymbol{L}}_i^{*T}}{\hat{\Omega}_{iD}}\left[\boldsymbol{Z}_a\,\hat{H}_{ia} + \boldsymbol{Z}_b\,\hat{H}_{ib}\right], \quad i = m, n+1\,, \tag{17}$$

where tuned eigenfrequencies, damping coefficients and participation factors are given by

$$\hat{\Omega}_{mD}^2 = (1 - \hat{\zeta}_m^2)\,(1-\gamma)\,\bar{\omega}^2\,, \quad \hat{\zeta}_m = \frac{1}{d}\left[\,a\,\frac{\Omega_m}{\hat{\Omega}_m}\,\zeta_m + (b - 2\,\mu_m\,\Phi_{km})\,\frac{\omega}{\hat{\Omega}_m}\,\zeta_0\,\right], \tag{18a}$$

$$\hat{\Omega}_{n+1\,D}^2 = (1 - \hat{\zeta}_{n+1}^2)\,(1+\gamma)\,\bar{\omega}^2\,, \quad \hat{\zeta}_{n+1} = \frac{1}{d}\left[\,b\,\frac{\Omega_m}{\hat{\Omega}_m}\,\zeta_m + (a + 2\,\mu_m\,\Phi_{km})\,\frac{\omega}{\hat{\Omega}_m}\,\zeta_0\,\right], \tag{18b}$$

$$\hat{\mathcal{L}}_m^e = -\frac{1}{d}\,(\frac{\mathcal{L}_m^e}{\mathcal{M}_m} + b)\,, \quad \hat{\mathcal{L}}_{n+1}^e = \frac{1}{d}\,(\frac{\mathcal{L}_m^e}{\mathcal{M}_m} - a)\,, \quad \hat{\boldsymbol{L}}_m^{*T} = \frac{\boldsymbol{L}_m^{*T}}{d\,\mathcal{M}_m}\,, \quad \hat{\boldsymbol{L}}_{n+1}^{*T} = -\,\hat{\boldsymbol{L}}_m^{*T}\,, \tag{18c}$$

with

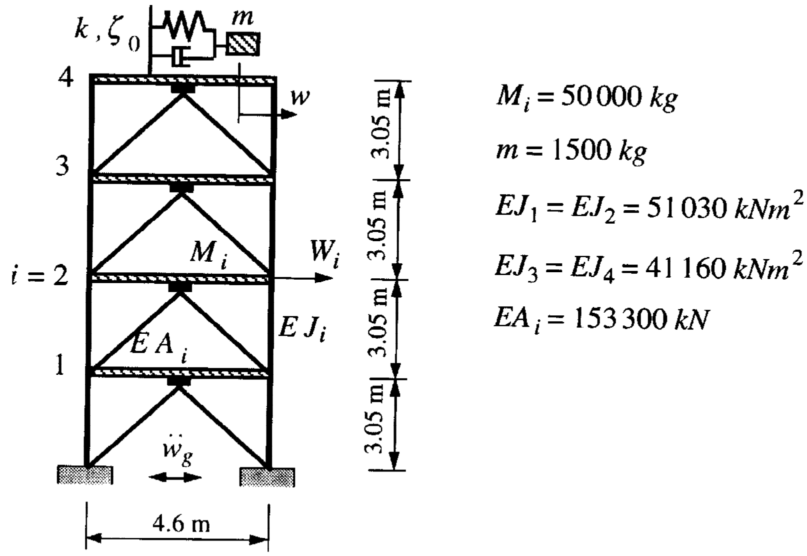

Fig.1. 4-storey friction braced frame with SDOF oscillator attached to the 4th floor

$$a = \delta_m + \sqrt{\delta_m^2 + \mu_m}\ , \quad b = \mu_m / a\ , \quad d = a + b\ , \quad \gamma = \Phi_{km}\left[\, 2\,\mu_m + \delta_m(a - b)\,\right] / d\ . \tag{19}$$

The initial conditions $\hat{\mathcal{F}}_i^{e,*}(t_a)$ and the unit impulse responses $\hat{H}_{ia}$, $\hat{H}_{ib}$ apparent in equation (16) depend on the parameters $\hat{\Omega}_i$, $\hat{\zeta}_i$. It is emphasized that equations (13), (14) provide a solution to $\hat{H}_{ia}$ and $\hat{\mathcal{F}}_i^{e,*}(t_a)$.

Detuned primary structure modes i, $i = 1, \ldots, n; n \neq m$, where $|\,\delta_i| >> \sqrt{\mu_i}$, can be evaluated with sufficient accuracy by an uncoupled analysis. Frequencies, damping ratios and participation factors of the corresponding composite system modes remain essentially unchanged, and, therefore, the solution of detuned modal coefficients are given by (11a) and (12a), where in (12a) the coupling terms to the secondary system may be omitted.

Introducing the modal coordinates into the expansion

$$\left[W_b, w_b\right]^T = \hat{\boldsymbol{\Phi}}\, Y_b\,, \quad \hat{\boldsymbol{\Phi}} = \left[\hat{\boldsymbol{\phi}}_1\ \hat{\boldsymbol{\phi}}_2 \ldots \hat{\boldsymbol{\phi}}_{n+1}\right] \tag{20}$$

gives the displacement of the coupled primary - secondary structure at the time instant t_b, where the mode shapes are derived from a perturbation analysis (Igusa and Der Kiureghian, 1985b) as follows,

$$\hat{\boldsymbol{\phi}}_m^T = \Big[\sum_{j=1, j\neq m}^{n} \frac{\mu_j\, \omega^2\, \Phi_{kj}}{\Omega_j^2 - \hat{\Omega}_m^2}\, \boldsymbol{\phi}_j^T + a\, \boldsymbol{\phi}_m^T\,, 1\,\Big]\,, \quad \hat{\boldsymbol{\phi}}_{n+1}^T = \Big[\sum_{j=1, j\neq m}^{n} \frac{\mu_j\, \omega^2\, \Phi_{kj}}{\Omega_j^2 - \hat{\Omega}_{n+1}^2}\, \boldsymbol{\phi}_j^T - b\, \boldsymbol{\phi}_{n+1}^T\,, 1\,\Big]\,, \tag{21a}$$

$$\hat{\boldsymbol{\phi}}_i^T = \Big[\,\boldsymbol{\phi}_i^T, \frac{\omega^2\, \Phi_{ki}}{\omega^2 - \Omega_i^2}\,\Big]\,, \quad i = 1, \ldots, n; n \neq m\ . \tag{21b}$$

NUMERICAL RESULTS

As an example a friction braced frame of 4 stories is considered, with an elastic SDOF oscillator attached to the 4th floor. Dimensions, stiffness and mass relations are given in Fig. 1. All modal damping parameters

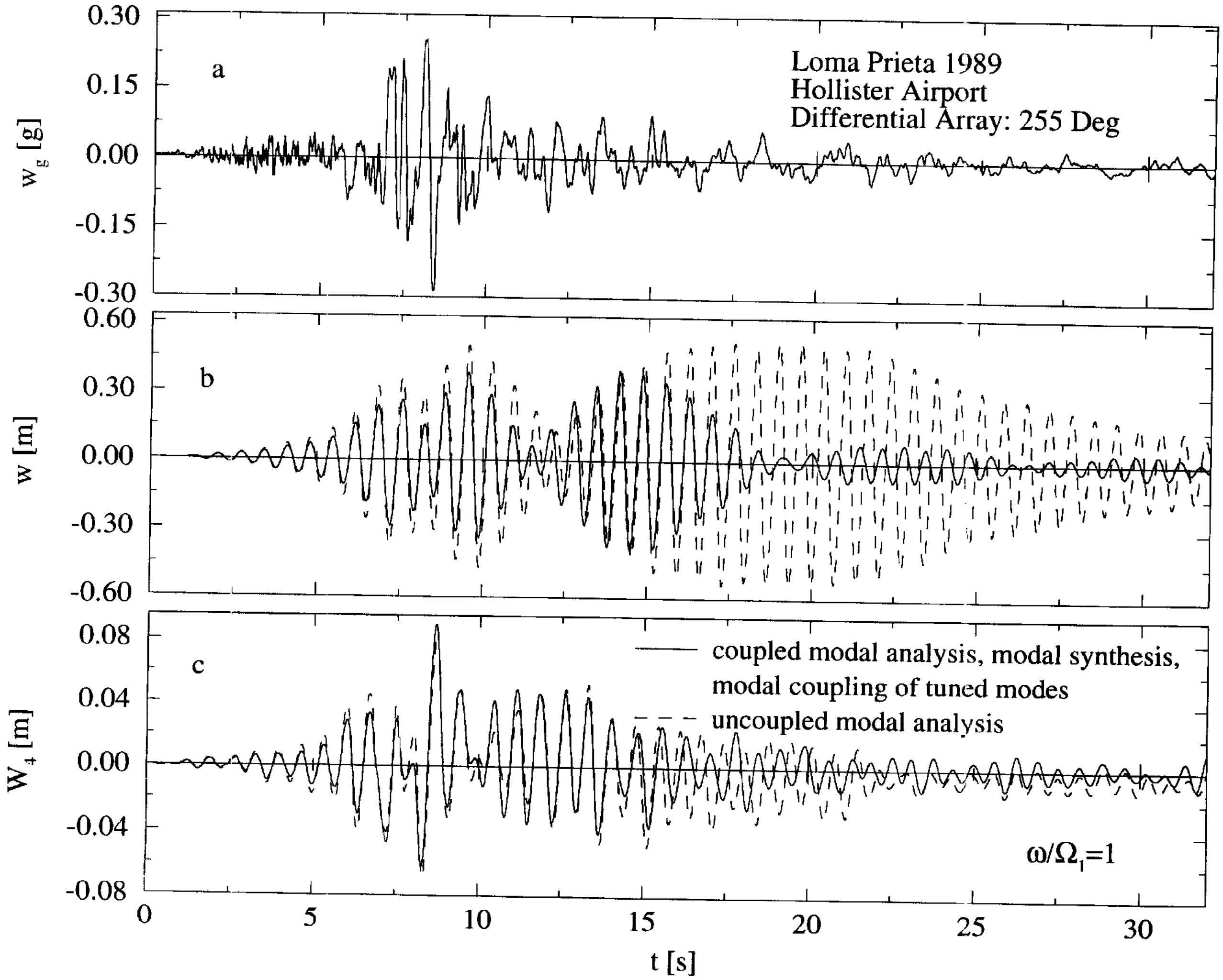

Fig. 2. Earthquake input and response. a) Ground acceleration.
b) Response of the SDOF oscillator. c) Response of the 4th floor.

are set to $\zeta_i = \zeta_0 = 0.02,\ i = 1, \ldots, 4$. The undamped natural frequencies of the primary structures are $\Omega_1 = 9.21\ rad/s$, $\Omega_2 = 36.43\ rad/s$, $\Omega_3 = 56.14\ rad/s$, $\Omega_4 = 69.89\ rad/s$. All relative storey slip deformations $\bar{X}_i$ are assumed to be $14\ mm$. In a first study the time evolution of the deflection of the SDOF oscillator under earthquake excitation is computed. It is assumed that $\omega/\Omega_1 = 1$, i.e. the SDOF is tuned to the first mode of the primary structure. A record of the Loma Prieta earthquake of October 18, 1989, recorded at Hollister Airport Differential Array, 255 Degree, is used as ground acceleration $\ddot{w}_g$. A plot of the acceleration record is given in Fig. 2a. Figs. 2b, 2c show the responses of the SDOF oscillator and of the 4th floor of the primary structure, respectively. Results of the exact (coupled) analysis, the modal synthesis and the approximate analysis by considering of modal coupling of tuned modes are given by the solid curves. There are practically no differences between the results derived by these methods. The uncoupled analysis, however, shows deviations in the range of the peak responses. In order to quantify the range of validity of an uncoupled analysis Fig. 3 shows the displacement floor response spectra calculated by four methods.

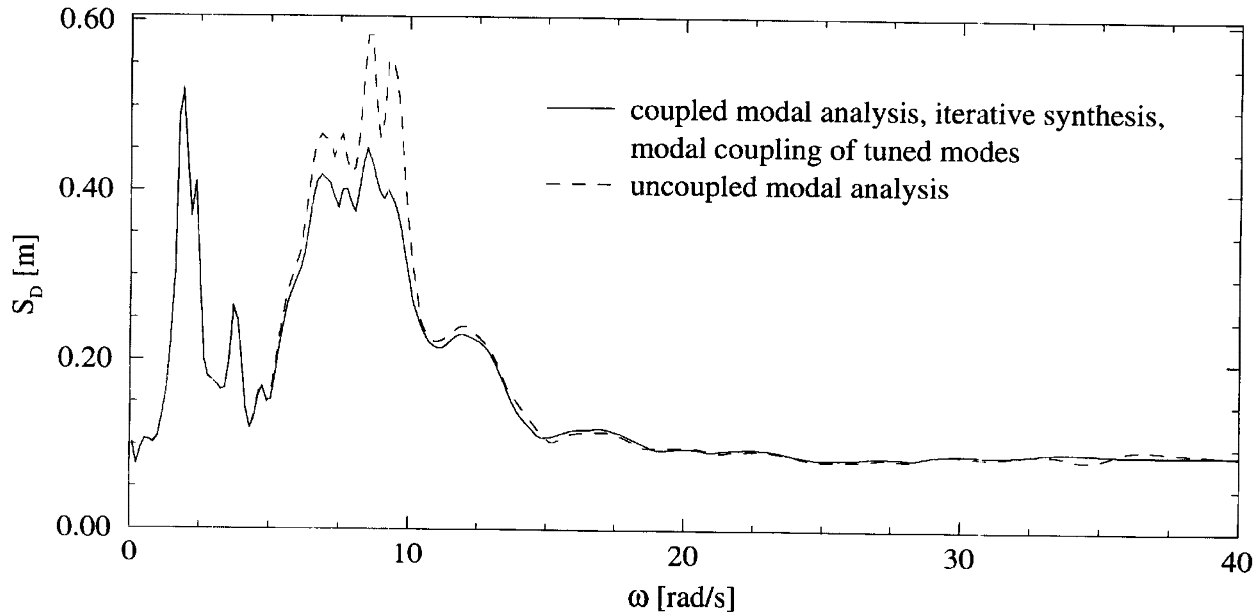

Fig. 3. Displacement floor response spectra.

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