PROBABILISTIC REPRESENTATION AND TRANSMISSION OF EARTHQUAKE GROUND MOTION RECORDS IN THE LOS ANGELES REGION

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ABSTRACT

This paper presents a relatively simple and straight-forward method for representing earthquake ground motion records in a compact probabilistic format which can be used as excitation input in analytical random vibration studies. The method involves two main stages of compaction. The first stage is based upon the spectral decomposition of the covariance kernel of the earthquake records by the orthogonal Karhunen-Loeve expansion. The dominant eigenvectors are subsequently least-squares-fitted to yield an analytical approximation. This compact analytical form can then be used to derive closed form solutions for the nonstationary response of structural systems. The approach is illustrated by the use of an ensemble of free-field acceleration records from the 1994 Northridge earthquake. It is shown that the proposed earthquake data-processing approach is not only useful as a data-archiving and earthquake feature-extraction tool, but it can also furnish a probabilistic measure of the average statistical characteristics of earthquake ground motion records corresponding to a spatially distributed region. Such a representation could be a valuable tool for government agencies and risk managing authorities interested in quantifying the average seismic risk over a spatially extended area, as opposed to a specific location.

KEYWORDS

Earthquake Transmission, Probabilistic Description, Northridge Earthquake, Orthogonal Decomposition, Random Vibrations

INTRODUCTION

A state-of-the-art review of current probabilistic methods and their structural and geotechnical applications indicates an active interest and growing recognition by designers and public policy authorities of the significance of probabilistic approaches in engineering applications, particularly in the areas of structural reliability and risk analysis. Among the major impediments to a more extensive use of probabilistic methods in structural dynamics applications, particularly those dealing with seismic excitations, are (1) the problems involved in the construction and representation of statistical models based on actual earthquake records, and (2) the computational difficulties encountered in subsequent analytical random vibration analyses to determine closed-form solutions for the response covariance.

This paper presents a relatively simple and straight-forward procedure for representing earthquake ground motion records, obtained from a specific region, in a compact probabilistic format that allows its convenient use later in analytical random vibration studies. The method is based on the spectral decomposition of the random (earthquake) process by the orthogonal Karhunen-Loeve expansion and subsequent use of least-squares approaches to develop an approximating analytical fit for the data-based eigenvectors of the covariance matrix of the underlying random process. The resulting compact analytical representation of the random process is then used to derive closed-form solutions for the nonstationary response of structural systems.

CONSTRUCTION OF COVARIANCE KERNELS

The first task of the procedure under discussion involved the collection of some 120 earthquake ground motion accelerograms from the California Division of Mines and Geology (CDMG) data set corresponding to the 1994 Northridge earthquake (CDMG, 1994). Each record site provided three records, i.e. one vertical, and two in perpendicular horizontal directions. These records, with a sampling rate of 0.02 seconds, were processed by CDMG by being baseline-corrected and band-pass filtered between 0.07 and 25Hz.

The available records corresponded to locations spread out around Los Angeles County. In this study, vertical and horizontal records were included in the ensemble to be processed, and the records were synchronized from a 1% g threshold in the horizontal direction at each site. Two records from Sylmar and Santa Monica were eliminated because their large magnitudes were unrepresentative of the ensemble.

A representative sample of the earthquake records is shown in Fig.1. The length of all the processed records was limited to 20 seconds which included the strong motion part of the event.

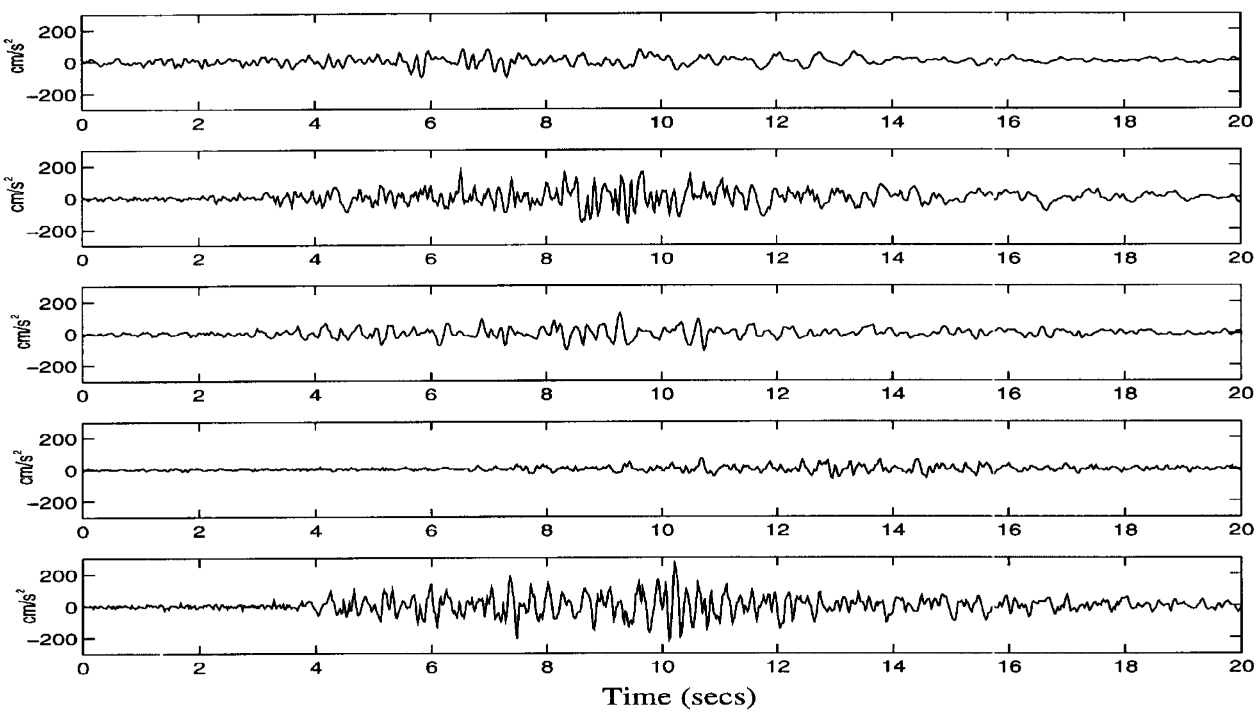

Fig. 1 Sample accelerograms used to generate the covariance matrix for the 1994 Northridge Earthquake

With a sampling interval of 0.02 seconds the covariance kernel K_{xx} as defined below, was of order (1001 x 1001) and can be seen in Fig.2,

$$K_{xx}(t_1, t_2) = E\{ [x(t_1) - \mu_x(t_1)] [x(t_2) - \mu_x(t_2)] \} \tag{1}$$

where $E\{\bullet\}$ denotes the ensemble average, and $\mu_x(t) = E[x(t)]$

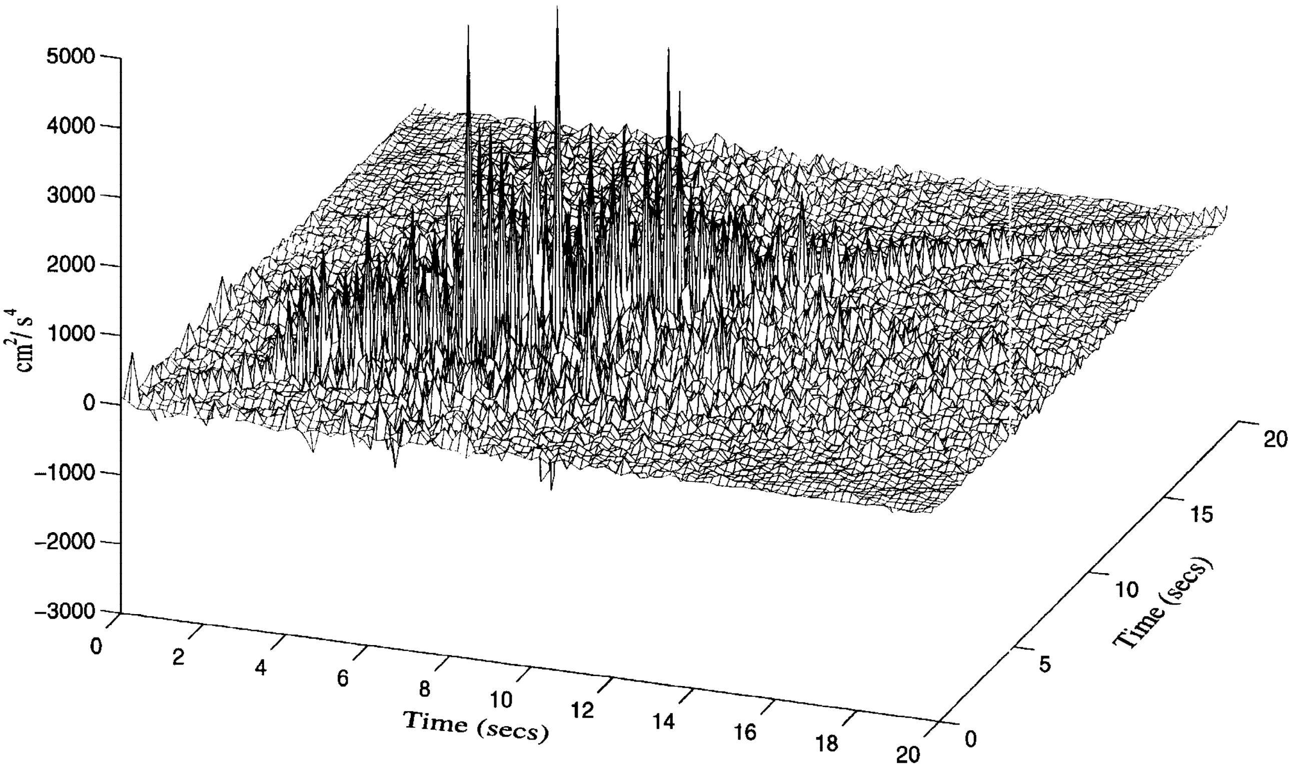

Fig.2 Exact Covariance Matrix for the first 20 s of the ensemble for the 1994 Northridge Earthquake.

EIGENVECTOR EXPANSION OF COVARIANCE MATRIX

The 120 non-vanishing eigenvalues of the covariance matrix were computed. The first (and largest) eigenvalue $\lambda_1 = 9.24\text{x}10^4$ while $\lambda_{120} = 9.42\text{x}10^{-11}$. Since the covariance matrix [C] was constructed with 120 records, all eigenvalues beyond this are zero. The rate of convergence of these eigenvalues is presented here in Fig.3

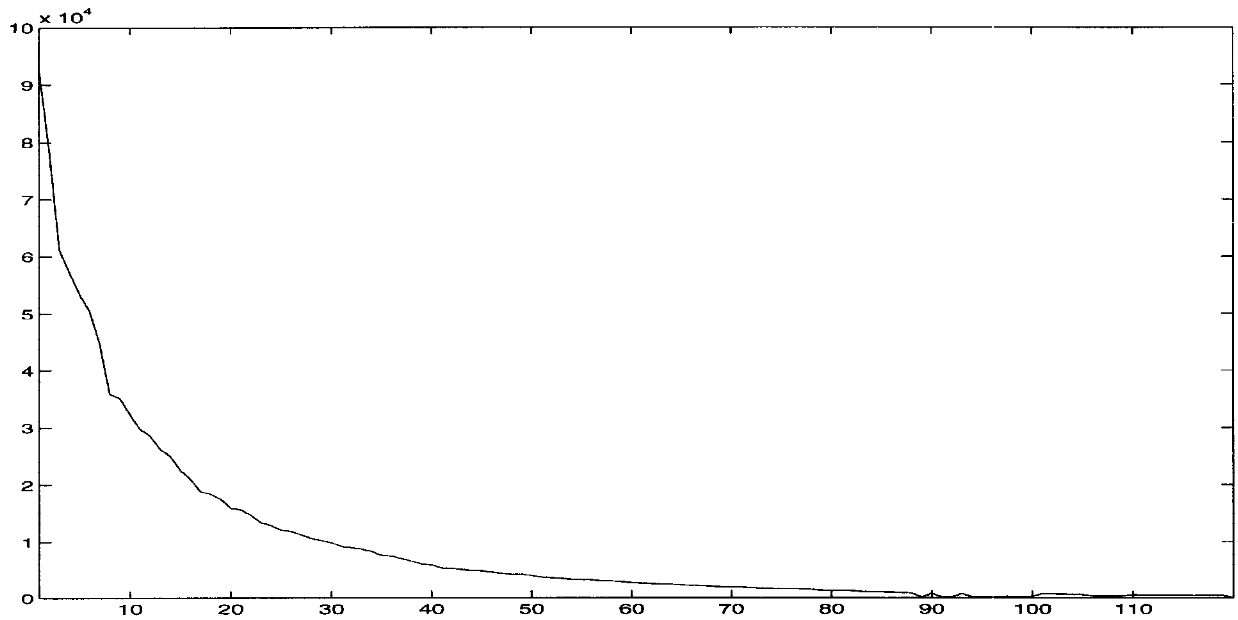

Fig. 3 Rate of convergence of the eigenvalues λ_k of the covariance matrix [C]

It is known from the Karhunen-Loeve expansion (Thomas, 1981) that

$$K_{xx}(t_1, t_2) = \sum_{k=1}^{\infty} \lambda_k \phi_k(t_1)\, \phi_k(t_2) \tag{2}$$

where the λ 's are the eigenvalues of the covariance matrix and the ϕ 's are the eigenvectors given by

$$\phi(t_1) = \frac{1}{\lambda} \int_0^{t_{max}} K_{xx}(t_1, t_2)\, \phi(t_2)\, dt_2 . \tag{3}$$

Similarly, in the discrete case, the spectral representation of a square symmetric matrix [C] is

$$[C] = \sum_{i=1}^{k} \lambda_i p_i p_i^T + [E_k] \tag{4}$$

where the p_i 's are the normalized eigenvectors of [*C*]. The Error matrix [*E*] results from the truncation of the series. To obtain a closed form representation of the random process, the eigenfunctions of the covariance matrix are approximated with a series of Chebyshev polynomials expressed as

$$p_i(t_1) \approx \hat{p}_i(t_1) = \sum_{j=0}^{m_i - 1} H_{ij} T_j(t') , \tag{5}$$

where the *H*'s can be found by the orthogonality property of the Chebyshev polynomials *T*. In the present case, the Chebyshev polynomials are function of the variable

$$t' = \frac{2t}{t_{max}} - 1 . \tag{6}$$

The estimated covariance matrix $\hat{C}$ can then be constructed with these estimated eigenvectors, and expressed in terms of the Chebyshev polynomial series form:

$$\left[\hat{C}_k(t_1, t_2)\right] = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i - 1} \sum_{l=0}^{m_i - 1} H_{ij} H_{il} T_j(t_1') \, T_l(t_2') \tag{7}$$

A comparison between the exact shape and a 200-order Chebyshev fit is shown in Fig.4 for the first three eigenvectors of the covariance matrix.

NONSTATIONARY RESPONSE OF A SDOF SYSTEM TO THE NORTHRIDGE EARTHQUAKE

To obtain the non-stationary response *y(t)* of a linear SDOF system, whose impulse function is *h(t)*, due to a non-stationary excitation random process *x(t)* whose covariance kernel is $K_{xx}(t_1, t_2)$, use can be made of the covariance of the system output, given by

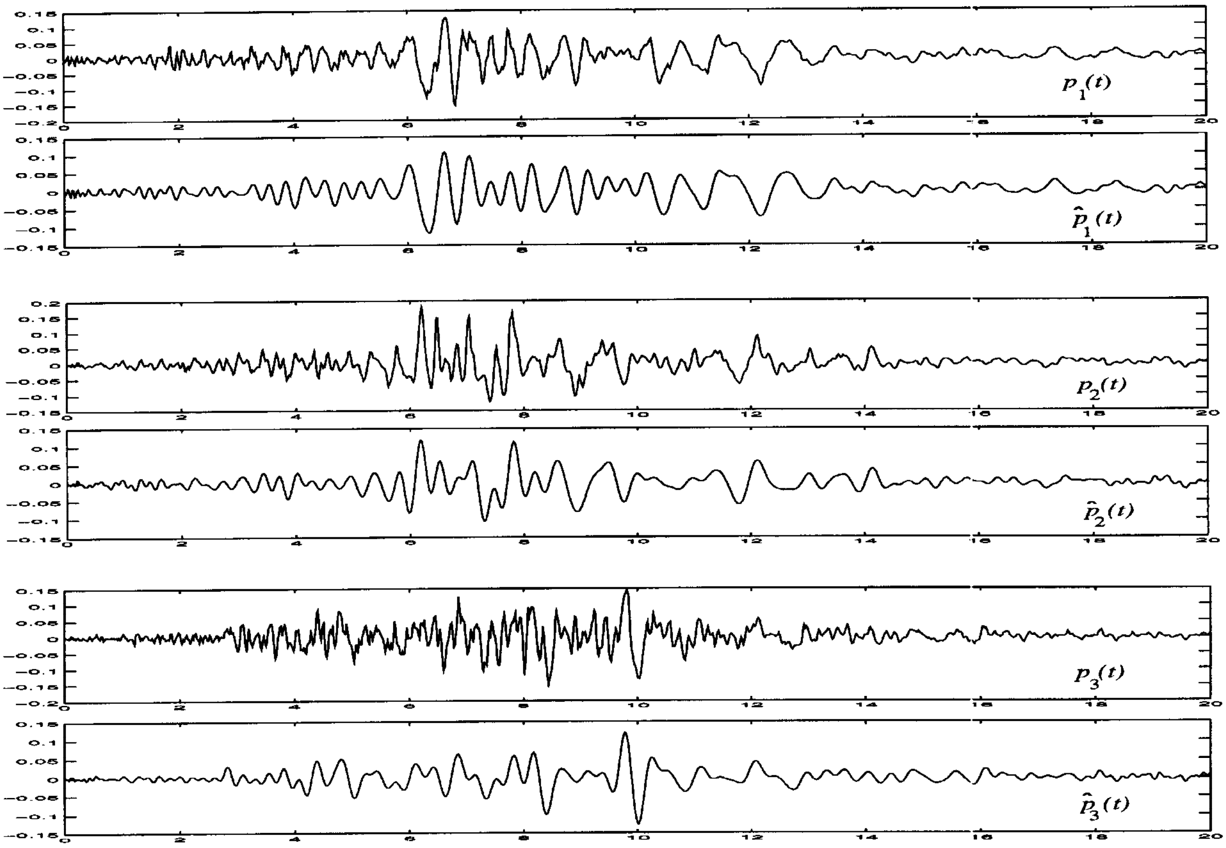

Fig. 4. Chebyshev Fit of order 200 for the first 3 eigenvectors of the Input Covariance Matrix

$$E[y(t_1)\,y(t_2)] = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)\,h(t_2-\tau_2)\,K_{xx}(\tau_1,\tau_2)\,d\tau_1 d\tau_2 \tag{8}$$

with,

$$h(t) = \frac{1}{\omega_d} e^{-\zeta\omega_n t}\sin\omega_d t$$

where ζ = ratio of critical damping, ω_n = natural frequency, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$. Inserting into this exact expression the approximated $[\hat{C}_k(t_1, t_2)]$ given above, results in

$$E[y(t_1)\,y(t_2)] \approx E^{(k)}{}_m[y(t_1)\,y(t_2)] = \sum_{i=1}^{k}\lambda_i \sum_{j=0}^{m-1}\sum_{l=0}^{m-1} H_{ij}H_{il}Y_{jl}(t_1, t_2) \tag{9}$$

where $E^{(k)}{}_m[y(t_1)\,y(t_2)]$ denotes the approximation using k eigenfunctions and m Chebychev polynomials to each eigenvector, and

$$Y_{jl}(t_1, t_2) \equiv \frac{t^2_{max}}{4\omega_d^2}\exp[\zeta\omega_n(t_{max}-t_1-t_2)]\,F_j(t_1)\,F_l(t_2) \tag{10}$$

which can be expressed as

$$Y_{jl}(t_1, t_2) = I_j(t_1) I_l(t_2) \tag{11}$$

where

$$I_i(t) = -\frac{t_{max}}{2\omega_d} e^{\xi\omega_n\left(\frac{t_{max}}{2} - t\right)} F_i(t) . \tag{12}$$

This expression, referred to as the system function, can be expressed as

$$I_i(t) = \frac{t_{max}}{2} \int_{-1}^{\left(\frac{2t}{t_{max}}\right)} h\left(t - \frac{t_{max}}{2}(\xi + 1)\right) T_i(t) \, d\xi . \tag{13}$$

Clearly the system function is only dependent upon the dynamic properties of the system and the harmonic content of the corresponding Chebyshev polynomial. Some of the time histories of various system functions are shown below in Fig. 5. Further details of this approach are available in Masri and Miller (1982) and Traina et al (1986).

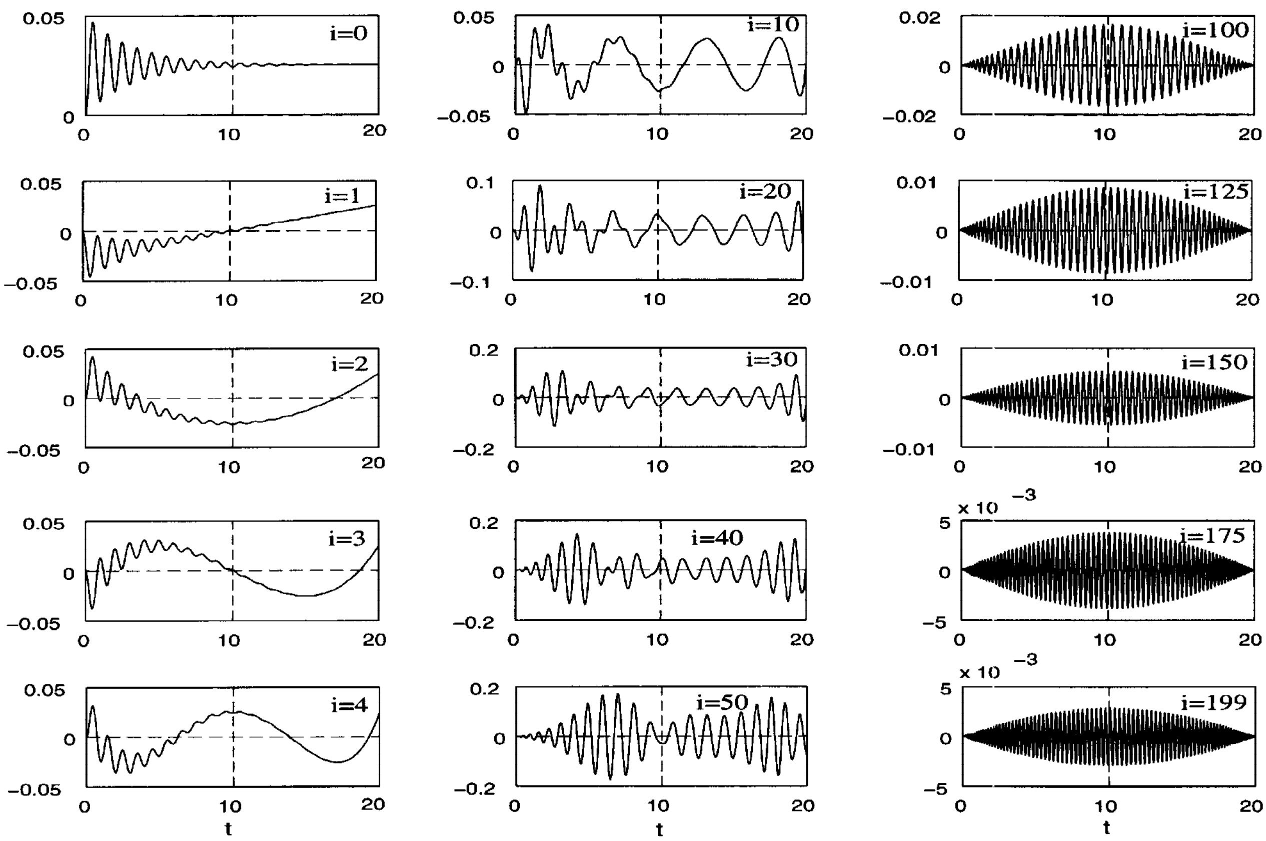

Fig.5 Time histories of System Functions, for different Chebyshev orders, ω_n =1Hz and ζ=0.05.

APPLICATIONS

After implementing this algorithm, an estimated output covariance matrix was obtained for 25 eigenvectors with a 200 order fit for each eigenvector (Fig.6). The illustrative SDOF system had a natural frequency ω_n =1Hz and a damping ratio ζ=0.05.

This least squares fit is actually of a relatively high order. This, however shows that the method works from

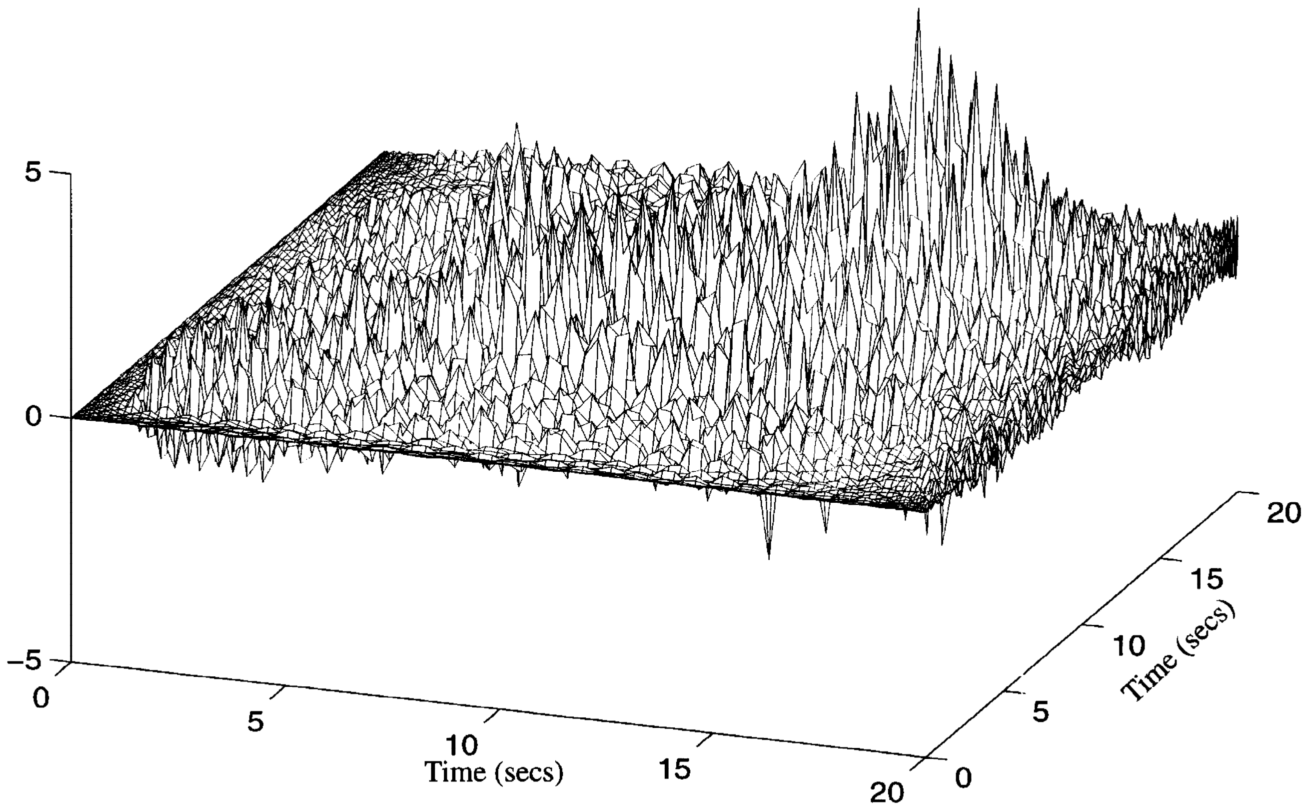

Fig. 6 Approximated Response Covariance Matrix with 25 eigenvectors and a 200 order chebyshev fit.

start to finish, and can yield excellent results. When compared with the exact solution covariance matrix diagonal in Fig. 7, one can see that the agreement is excellent, where the primary source of discrepancy was that only 25 eigenvectors were considered. As can be seen from Fig. 3, this truncation yields lower input energy than is actually present.

The application of this approach to a dataset corresponding to the 1971 San Fernando earthquake is reported in the work of Masri et al (1990).

CONCLUSIONS

A simple and compact method has been presented for obtaining analytical estimates for a linear system response to a particular non-stationary random process. Its benefits are not only in the compact form in which earthquakes can be databased, but also in the flexible form which it can be used as analytical system excitation.

By applying the approach under discussion to an ensemble of 120 records corresponding to the horizontal components of widely dispersed stations subjected to the 1994 Northridge earthquake, it is shown that the proposed earthquake data-processing approach, is not only useful as a data-archiving and earthquake feature-extraction tool, but it can also furnish a probabilistic measure of the average statistical characteristics of earthquake ground motion records corresponding to a spatially distributed region. Such a representation could be a valuable tool for government agencies and risk managing authorities interested in quantifying the average seismic risk over a spatially extended area, as opposed to a specific location.

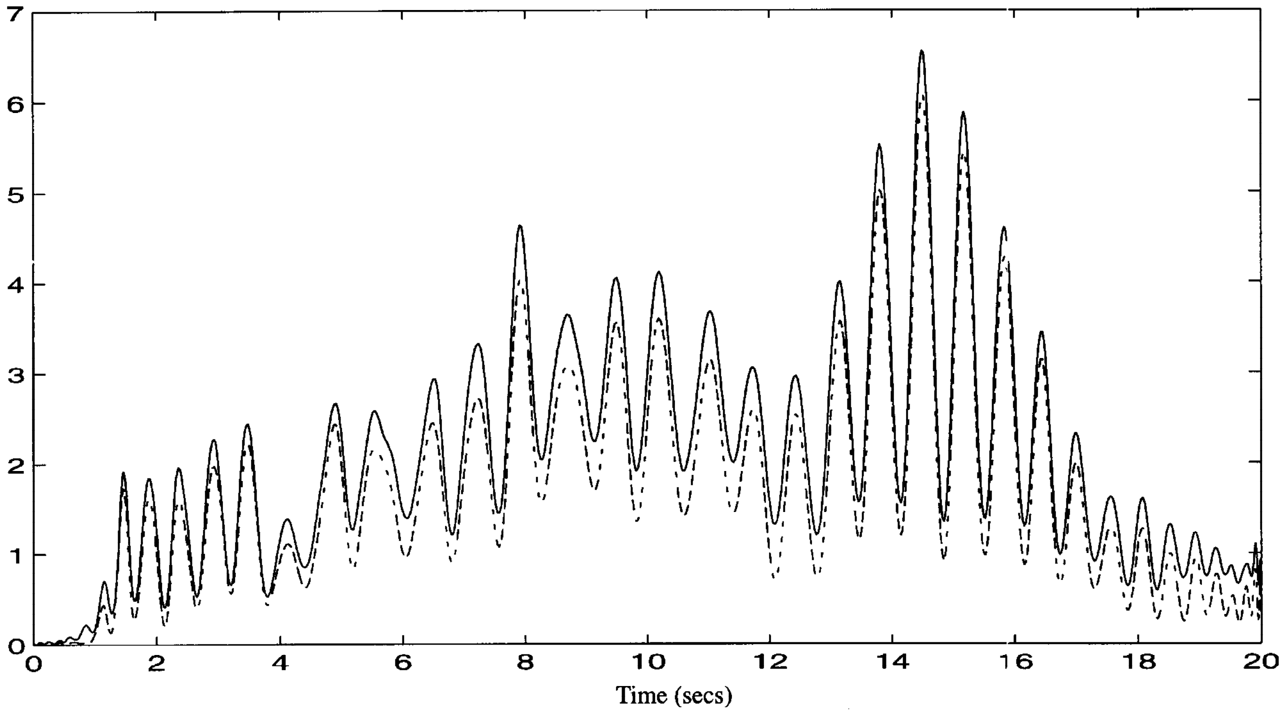

Fig. 7 Comparison between approximate and exact diagonal of the response covariance matrix.

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