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DYNAMICS OF HORIZONTAL SETBACK BUILDINGS WITH FLEXIBLE FLOOR DIAPHRAGMS

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ABSTRACT

Dynamics of a class of horizontal setback buildings with flexible floor diaphragms has been studied by developing a "separable model." The mass and stiffness matrices of such structures have been written in terms of direct products of matrices. The separable buildings have two types of natural modes of vibration: (a) those which involve in-plane floor deformation, and (b) those in which the floors do not undergo in-plane deformation. Further, spatially-uniform ground motion does not excite the modes involving in-plane floor deformation. Therefore, the problems associated with diaphragm flexibility, *e.g.*, stress concentration, are minimal in separable buildings. Most general conditions for a building to be separable have been obtained.

KEY WORDS

Direct Product, Dynamics of Buildings, Earthquake Analysis, Flexible Diaphragm, Floor Diaphragms, Horizontal Setback Buildings, Seismic Design, Separable Model

INTRODUCTION

Past earthquake performance of horizontal setback buildings clearly shows that horizontal setback buildings are vulnerable to damage caused by in-plane flexibility of the floor diaphragms. Floor diaphragm deformation causes two difficulties: (a) it alters the lateral load distribution to the vertical elements over what one would obtain assuming the floors to be rigid in their own plane, and (b) it causes stress concentration at reentrant corners of the building. It is of interest to study the dynamics of such buildings and to see in what manner the in-plane floor deformations can be eliminated or minimized.

Maybee *et al.* (1966) introduced the "separable model" for uniform and regular rectangular multistorey buildings. This model has now been extended to more general class of rectangular buildings, and to multistorey buildings with plan shapes such as L, T, U, H, +, or a combination thereof (Jain and Jain, 1993). This paper summarises the results on setback buildings. The model does not require the horizontal setback building to have identical wings even though some constraints on mass and stiffness properties are imposed. Suggestions are made on how to configure horizontal setback buildings so as to make them "separable" and thereby avoid stress concentration at the re-entrant corners. The technique applied here allows treatment of a very general class of buildings, as against the rather regular and uniform configurations studied earlier, *e.g.*,

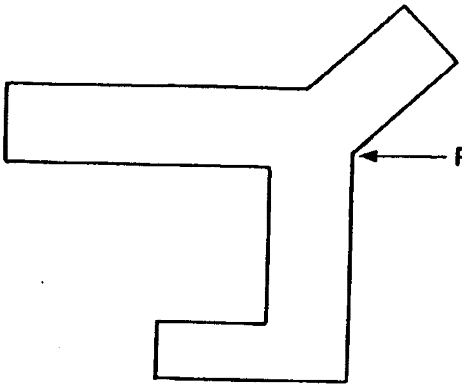

Fig. 1. Plan of horizontal setback building (number of wings $p = 4$)

symmetric V-shaped buildings treated by Jain and Mandal (1992) and symmetric Y-shaped buildings treated by Jain and Mandal (1995).

BUILDING MODEL

The multistorey building with horizontal setbacks may consist of several wings as shown in Fig. (1). Each wing may be long and narrow, with a number of transverse frames (or shear walls). Earthquake-induced vibrations of the building involve longitudinal and transverse motions of each wing. In the analytical model presented here, the floor is treated as a beam with bending deformation only or with bending and shear deformations, depending on aspect ratio of the floor. If the resisting elements consist of moment resisting frames the building is modeled as a grid with beams (representing floors) in the horizontal direction and frames in the vertical direction. The mass is lumped at the floor-frame intersections. Where building contains shear walls rather than frames, the walls are also treated as beams (with bending only or with bending and shear deformations depending on aspect ratio of the wall). Torsional stiffness of the floors and frames is neglected. It is assumed that: (a) ground motion is uniform at different points at base, *i.e.*, spatially-uniform ground motion; and (b) there is no torsion component in the ground motion. The building is assumed to be linear elastic and fixed at the base.

Let the building be of n storeys, with number of wings in the building as p. Also, let the s^{th} wing have $(q_s\text{-}1)$ transverse frames. Axial deformation of floors is negligible and width of a wing is usually small. Hence, for each wing only one displacement per floor in the longitudinal direction needs to be considered. Thus, the translational degrees of freedom at a given floor of the s^{th} wing will be q_s. For dynamic analysis, the rotational degrees of freedom can be condensed suitably.

The multistorey building studied herein is characterized by the following properties.

(a) The lumped mass matrices of all the floors are proportional to each other; *i.e.*, the mass matrix of i^{th} floor is $\mathbf{M}_i^{fl} = C_{mi}^{fl}\mathbf{M}_o^{fl}$, where C_{mi}^{fl} is the mass proportionality constant for the i^{th} floor, and $\mathbf{M}_o^{fl}$ is the characteristic mass matrix for the floors. The lumped mass matrix for a floor is defined by lumping the floor and the frame mass at the floor-frame intersections. Define matrix $\mathbf{D}^{fl}$ as a diagonal matrix $(n \times n)$ of the mass proportionality constants (C_{mi}^{fl}) of the floors.

(b) The stiffness matrices for all the floors are proportional to each other; *i.e.*, the stiffness matrix of the i^{th} floor is $\mathbf{B}_i^{fl} = C_{ki}^{fl}\mathbf{B}$, where C_{ki}^{fl} is the stiffness proportionality constant for the i^{th} floor, and $\mathbf{B}$ is the characteristic stiffness matrix for the floors. Define matrix $\mathbf{C}^{fl}$ as a diagonal matrix ($n \times n$) of the stiffness proportionality constants ($C_{ki}{}^{fl}$) of the floors.

(c) The lumped mass matrices of all the frames are proportional to each other; *i.e.*, the mass matrix of j^{th} frame is $\mathbf{M}_j{}^{fr} = C_{mj}{}^{fr}\mathbf{M}_o{}^{fr}$, where $C_{mj}{}^{fr}$ is the mass proportionality constant, and $\mathbf{M}_o{}^{fr}$ is the characteristic mass matrix for the frames.

(d) The stiffness matrices for all the frames are proportional to each other; *i.e.*, the stiffness matrix of j^{th} frame is $\mathbf{A}_j{}^{fr} = C_{kj}{}^{fr}\mathbf{A}^*$, where $C_{kj}{}^{fr}$ is the stiffness proportionality constant, and $\mathbf{A}^*$ is the characteristic stiffness matrix for the frames.

EQUATIONS OF MOTION

Mass matrix ($\mathbf{m}_{sl}$) and stiffness matrix ($\mathbf{k}_{sl}$) for the s^{th} wing in its local coordinate system can be expressed in the direct product (see Appendix) form as follows (Jain and Jain, 1993):

$$\mathbf{m}_{sl} = m_o(\mathbf{D}_s{}^{fr} \otimes \mathbf{D}^{fl}) \tag{1}$$

$$\mathbf{k}_{sl} = \mathbf{C}_s{}^{fr} \otimes \mathbf{A}^* + \mathbf{B}_s \otimes \mathbf{C}^{fl} \tag{2}$$

Here $\mathbf{D}^{fl}, \mathbf{A}^*$, and $\mathbf{C}^{fl}$ are $n \times n$ matrices and are same for all wings of the building. Also, m_o is the characteristic lumped mass, $\mathbf{D}_s{}^{fr}$ is a diagonal matrix ($q_s \times q_s$) containing mass proportionality constants ($\mathbf{C}_{mj}{}^{fr}$) for the frames of wing s, matrix $\mathbf{C}_s{}^{fr}$ is a diagonal matrix ($q_s \times q_s$) containing stiffness proportionality constants ($C_{kj}{}^{fr}$) for different frames of wing s, and $\mathbf{B}_s$ is the characteristic stiffness matrix ($q_s \times q_s$) in local coordinates of the floors in wing s.

The mass and stiffness matrices of the wing s are now to be transformed from local coordinate system to the global coordinate system. Let $\mathbf{u}_s{}^i$ and $\mathbf{v}_s{}^i$ denote the displacement vectors for the i^{th} floor of the s^{th} wing in the local and global coordinate systems, respectively. These are related such that

$$\mathbf{u}_s{}^i = \mathbf{R}_s\mathbf{v}_s{}^i \tag{3}$$

where $\mathbf{R}_s$ is the transformation matrix. All the floors of the wing have similar degrees of freedom. Hence, degrees of freedom for the entire s^{th} wing can be transformed from local system ($\mathbf{u}_s$) to the global system ($\mathbf{v}_s$) as

$$\mathbf{u}_s = (\mathbf{R}_s \otimes \mathbf{I}_n)\mathbf{v}_s \tag{4}$$

Where $\mathbf{I}_n$ is the ($n \times n$) identity matrix. The mass matrix ($\mathbf{m}_{sg}$) and stiffness matrix ($\mathbf{k}_{sg}$) for the s^{th} wing in the global coordinate system become

$$\mathbf{m}_{sg} = (\mathbf{R}_s \otimes \mathbf{I}_n)^T \mathbf{m}_{sl}(\mathbf{R}_s \otimes \mathbf{I}_n) = m_o(\mathbf{R}_s{}^T\mathbf{D}_s{}^{fr}\mathbf{R}_s \otimes \mathbf{D}^{fl}) \tag{5}$$

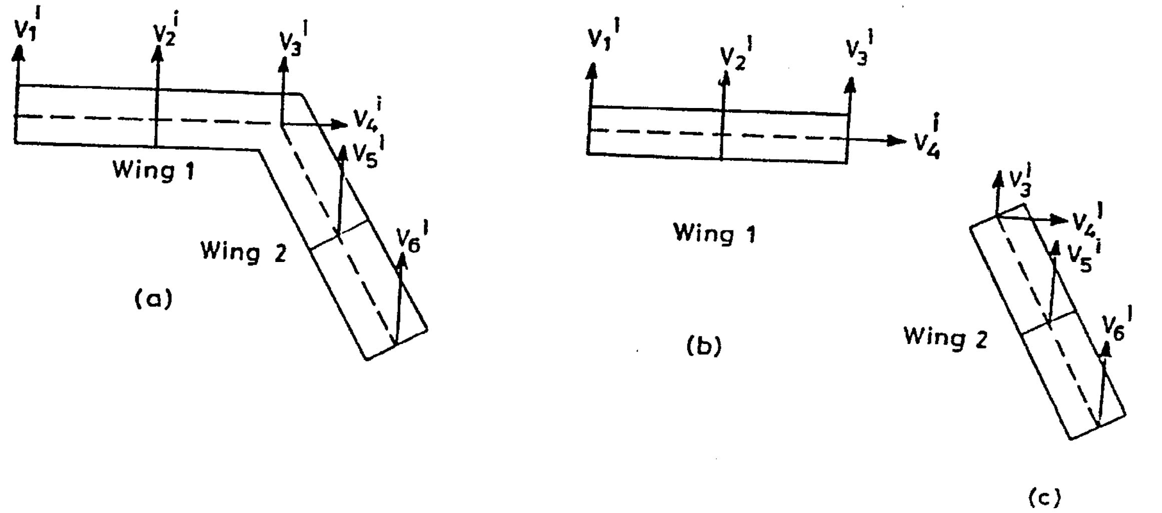

Fig. 2. Degrees of freedom in global coordinate system of (a) *i*th floor of entire building, (b) *i*th floor of wing 1, and (c) *i*th floor of wing 2

$$\mathbf{k}_{sg} = (\mathbf{R}_s \otimes \mathbf{I}_n)^T \mathbf{k}_{sl} (\mathbf{R}_s \otimes \mathbf{I}_n) = \mathbf{R}_s^T \mathbf{C}_s^{fr} \mathbf{R}_s \otimes \mathbf{A}^* + \mathbf{R}_s^T \mathbf{B}_s \mathbf{R}_s \otimes \mathbf{C}^{fl} \tag{6}$$

The mass and stiffness matrices for different wings are then assembled to give the overall mass matrix (**m**) and stiffness matrix (**k**) of the building (Fig. 2). Let $\mathbf{v}^i$ be the displacement vector for the i^{th} floor of the entire building. The $\mathbf{v}_s^i$ and $\mathbf{v}^i$ vectors are related through the locator matrix $\mathbf{L}_s$ as

$$\mathbf{v}_s^i = \mathbf{L}_s \mathbf{v}^i \tag{7}$$

Number of rows in $\mathbf{L}_s$ is equal to the degrees of freedom of the i^{th} floor of the s^{th} wing, while the number of columns in $\mathbf{L}_s$ is equal to the total degrees of freedom in the typical floor of the entire building. Since all the floors of the building have identical degrees of freedom, the global coordinates $\mathbf{v}_s$ of the s^{th} wing are related with the global coordinates (**v**) of the entire building as

$$\mathbf{v}_s = (\mathbf{L}_s \otimes \mathbf{I}_n)\, \mathbf{v} \tag{8}$$

Mass and stiffness matrix of the entire building can then be assembled as

$$\mathbf{m} = \sum_{s=1}^{p} (\mathbf{L}_s \otimes \mathbf{I}_n)^T \mathbf{m}_{sg} (\mathbf{L}_s \otimes \mathbf{I}_n) = m_o \left(\sum_{s=1}^{p} \mathbf{L}_s^T \mathbf{R}_s^T \mathbf{D}_s^{fr} \mathbf{R}_s \mathbf{L}_s \right) \otimes \mathbf{D}^{fl} \tag{9}$$

or, $$\mathbf{m} = m_o (\mathbf{D}^{fr} \otimes \mathbf{D}^{fl}) \tag{10}$$

$$\mathbf{k} = \sum_{s=1}^{p} (\mathbf{L}_s \otimes \mathbf{I}_n)^T \mathbf{k}_{sg} (\mathbf{L}_s \otimes \mathbf{I}_n) \tag{11}$$

or, $\mathbf{k} = (\sum_{s=1}^{p} \mathbf{L}_s{}^T \mathbf{R}_s{}^T \mathbf{C}_s{}^{fr} \mathbf{R}_s \mathbf{L}_s) \otimes \mathbf{A}^* + (\sum_{s=1}^{p} \mathbf{L}_s{}^T \mathbf{R}_s{}^T \mathbf{B}_s \mathbf{R}_s \mathbf{L}_s) \otimes \mathbf{C}^{fl}$ (12)

or, $\mathbf{k} = (\mathbf{C}^{fr} \otimes \mathbf{A}^* + \mathbf{B}^* \otimes \mathbf{C}^{fl})$ (13)

Where

$$\mathbf{D}^{fr} = \sum_{s=1}^{p} \mathbf{L}_s{}^T \mathbf{R}_s{}^T \mathbf{D}_s{}^{fr} \mathbf{R}_s \mathbf{L}_s \tag{14}$$

$$\mathbf{C}^{fr} = \sum_{s=1}^{p} \mathbf{L}_s{}^T \mathbf{R}_s{}^T \mathbf{C}_s{}^{fr} \mathbf{R}_s \mathbf{L}_s \tag{15}$$

$$\mathbf{B}^* = \sum_{s=1}^{p} \mathbf{L}_s{}^T \mathbf{R}_s{}^T \mathbf{B}_s \mathbf{R}_s \mathbf{L}_s \tag{16}$$

The eigen value problem of the entire building may now be written as

$$(\mathbf{C}^{fr} \otimes \mathbf{A}^* + \mathbf{B}^* \otimes \mathbf{C}^{fl})\mathbf{v} = m_o \omega^2 (\mathbf{D}^{fr} \otimes \mathbf{D}^{fl})\mathbf{v} \tag{17}$$

Where ω is the natural frequency and **v** is the mode shape of the entire building.

EIGEN VALUE SOLUTION

Consider the case when (a) the stiffness and mass proportionality constants for the frames become proportional

$$\mathbf{D}^{fr} = c^{fr} \mathbf{C}^{fr} \tag{18}$$

and also (b) the stiffness and mass proportionality constants for the floors become proportional

$$\mathbf{D}^{fl} = c^{fl} \mathbf{C}^{fl} \tag{19}$$

Physical significance of conditions (18) and (19) is that the natural frequencies of all the frames, with appropriate lumped mass, must be the same; and the natural frequencies of all the floors in the building must be the same. Further, if the mass matrix of any frame and the stiffness matrix for the same frame are taken as the characteristic mass and stiffness matrices for the frames, the value of c^{fr} will be unity. Similarly, if mass and stiffness matrices of the same floor are taken as the characteristic mass and stiffness matrices for the floors, then the value of c^{fl} will be unity. This is assumed in all further discussions.

It turns out that under the conditions (18) and (19), the building becomes separable; *i.e.*, the eigenvalue solution of the entire building can be obtained from the eigenvalue solution of a typical floor and of a typical frame. For instance, the eigenvalue problem for a typical frame and a typical floor can be formulated as

$$\mathbf{A}^* \mathbf{T} = m_o \tau^2 \mathbf{C}^{fl} \mathbf{T} \tag{20}$$

$$\mathbf{B}^* \mathbf{S} = m_o \sigma^2 \mathbf{C}^{fr} \mathbf{S} \tag{21}$$

Here, τ and σ are the natural frequency of the frame and the floor, respectively. Similarly, $\mathbf{T}$ and $\mathbf{S}$ are the corresponding mode shapes of the frame and the floor, respectively. Substitution of $\mathbf{v} = \mathbf{S} \otimes \mathbf{T}$ into Eq. (17) and application of Eqs. (20) and (21) shows that the building is indeed separable, and that the natural frequencies and mode shapes of the entire building are given by

$$\omega^2 = \sigma^2 + \tau^2 \tag{22}$$

$$\mathbf{v} = \mathbf{S} \otimes \mathbf{T} \tag{23}$$

Thus, when these conditions are satisfied the building may be treated as a separable building. Frequencies of the separable building are simply the square roots of all the possible sums of the square of the frequencies of the typical floor and the square of the frequencies of the typical frame. Similarly, the eigenvectors are the tensor product of the eigenvectors of the typical floor and the eigenvectors of the typical frame.

MODAL PARTICIPATION FACTORS

Since the floors have free-free boundary conditions, the first two floor modes consist of rigid body translation and rigid body rotation with zero natural frequency. Thus, the first mode of the floors is a unit vector

$$\mathbf{S}_1 = \mathbf{1} \tag{24}$$

By orthogonality condition on floor modes,

$$\mathbf{S}_m^T \mathbf{D}^{fr} \mathbf{1} = 0 \qquad m \neq 1 \tag{25}$$

Let the combination of m^{th} floor mode and n^{th} frame mode be termed as mn^{th} mode of the building, *i.e.*,

$$\mathbf{v}_{mn} = \mathbf{S}_m \otimes \mathbf{T}_n \tag{26}$$

The modal participation factor for spatially uniform ground motion is

$$P_{mn} = \frac{\mathbf{v}_{mn}^T \mathbf{m} \mathbf{1}}{\mathbf{v}_{mn}^T \mathbf{m} \mathbf{v}_{mn}} \tag{27}$$

Consider the numerator

$$\mathbf{v}_{mn}^T \mathbf{m} \mathbf{1} = m_o (\mathbf{S}_m \otimes \mathbf{T}_n)^T \ (\mathbf{D}^{fr} \otimes \mathbf{D}^{fl}) \ (\mathbf{1} \otimes \mathbf{1}) \tag{28}$$

$$= m_o (\mathbf{S}_m{}^T \mathbf{D}^{fr} \mathbf{1}) \otimes (\mathbf{T}_n{}^T \mathbf{D}^{fl} \mathbf{1}) \tag{29}$$

$$= 0 \text{ when} \qquad m \neq 1 \tag{30}$$

Thus, the participation factor is zero for all modes other then those involving rigid body translation of floors. Hence, under the stipulated conditions, the modes of vibrations involving in-plane floor vibrations are not excited by ground motion.

CONCLUSIONS

It is found that for a building to become "separable", it must satisfy the following conditions: (a) the lumped mass matrices of the floors must be proportional; (b) the lumped mass matrices of the frames must be proportional, (c) the stiffness matrices of the floors must be proportional, (d) the lateral stiffness matrices of the frames must be proportional, (e) the ratio of stiffness and mass proportionality constants for all the floors must be the same, and (f) the ratio of stiffness and mass proportionality constants for all the frames must be the same. While considering the lumped mass matrices for the frames and the floors, both the frame and the floor mass is lumped at the floor-frame intersections. Such a buildings have two types of natural modes of vibration: (a) those which involve in-plane floor deformation, and (b) those in which the floors do not undergo in-plane deformation. Further, spatially-uniform ground motion does not excite the modes involving in-plane floor deformation.

Given a plan configuration of a building from architectural considerations, the structural engineer can adjust the stiffness properties of the floors and the frames to make the building separable (or as nearly separable as possible). This will avoid, or minimize, the problems associated with the in-plane floor deformations. This is a much better solution over providing separation joints between different wings of the building. Separation joints spoil aesthetics of a building, are leaky, and make the building prone to damage due to pounding between the adjacent wings.

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APPENDIX: DIRECT PRODUCT OF MATRICES

Let **A** and **B** be rectangular matrices of order $(p \times q)$ and $(r \times s)$, respectively. The direct product, also termed as Kronecker product or tensor product, $\mathbf{A} \otimes \mathbf{B}$ of **A** and **B** is the rectangular matrix of order $(pr \times qs)$ defined by (Halmos, 1958, Lynch *et al.*, 1964)

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}\mathbf{B} & a_{12}\mathbf{B} & . & . & a_{1q}\mathbf{B} \\ a_{21}\mathbf{B} & a_{22}\mathbf{B} & . & . & a_{2q}\mathbf{B} \\ . & . & . & . & . \\ . & . & . & . & . \\ a_{p1}\mathbf{B} & a_{p2}\mathbf{B} & . & . & a_{pq}\mathbf{B} \end{bmatrix}$$

Each element $a_{ij}\mathbf{B}$ is the product of the scalar a_{ij} with the matrix **B**. There are no restrictions on the sizes of matrices **A** and **B**. Some of the formal rules of operating with direct products are as follows

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}$$
$$(\mathbf{A} + \mathbf{C}) \otimes \mathbf{B} = \mathbf{A} \otimes \mathbf{B} + \mathbf{C} \otimes \mathbf{B}$$

$$(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = \mathbf{AC}\otimes\mathbf{BD}$$
$$(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D})(\mathbf{E}\otimes\mathbf{F}) = \mathbf{ACE}\otimes\mathbf{BDF}$$
$$(\mathbf{A}\otimes\mathbf{B})^T = \mathbf{A}^T\otimes\mathbf{B}^T$$

In the above equations **1** is a unity matrix,