# Analytical Study of Active Vibration Control on Tall Buildings

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## **ABSTRACT**

In this study, active vibration control analyses are performed on a three story building model by using both optimal control theory and fuzzy control theory. The analytical results by the fuzzy control theory are almost nearly similar to those by the optimal control theory.

#### **KEYWORDS**

Active vibration control; Fuzzy control theory; Optimal control theory; Tall buildings.

## INTRODUCTION

In recent years, there have been many studies on vibration control for tall buildings and the pylons of suspension bridges and so on. The methods of vibration control for structures is generally divided into passive control and active control. The passive control method is difficult to control in several of the vibration modes of structures, and the active control method has the ability to control a comparatively wide frequency range. Infrastructures are usually acted upon by relatively random excitation forces, active control is probably advantageous for vibration control in civil structures. The applicability of control theory and operation time for real time control are important problems to make an effective active control system. Formerly, active control was unpractical, because it's operation time was too slow for real time control. Recently, it has become possible to apply active control in practical use, as the performance of microprocessors has improved greatly. Many studies have been done on control theory. As a consequence, optimal control theory, fuzzy control theory,  $H^{\infty}$  control theory and others have been observed.

In this study, active vibration control analyses are performed on a three story building model by using both optimal control theory and fuzzy control theory (Obata et al., 1995). The fuzzy control theory is superior in its robustness, but for this system it is difficult to set up membership functions and to design control rules. As

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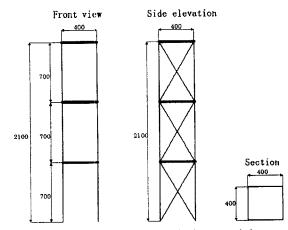


Fig. 1. Three story building model Table 1. Vibration characteristics

	1st. mode	2nd. mode	3rd. mode
ω (rad / s)	8. 943	26.026	37. 997
f (Hz)	1. 423	4. 242	6.047
T (s)	0.703	0.241	0.165
ξ (%)	0.360	0. 250	

Table 2. Parameters for analysis

story	Mass (kg)	stiffness constant $(kg / cm^2)$
upper	24.5	9248. 0
middle	21.5	9248.0
lower	19.2	9248.0

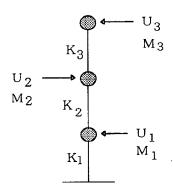


Fig. 2. Three degree of freedom lumped mass system

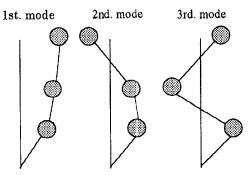


Fig. 3. Natural mode shapes

for the optimal control theory, it's robustness is less than that of the fuzzy theory, but the method of design for the vibration control system has been established. A three story building model that assumes a three degree of freedom lumped mass system is used. The time history response analyses are done by using the Newmark- $\beta$  method. The action position and effectiveness of the control force for active control of free vibration and forced vibration excited by an earthquake force are investigated in this study.

# MODELING OF STRUCTURE AND DYNAMIC RESPONSE ANALYSIS

The three story building model that assumes a three degree of freedom lumped mass system is used. The general view of the three story building model is shown in Fig. 1, and the three degree of freedom lumped mass system is shown in Fig. 2. The natural frequencies and damping constants of the building model were measured by a ambient vibration test. The vibration characteristics of the structure are summarized in Table 1, the parameters used by analysis are shown in Table 2, and Fig. 3 are shown the natural mode shapes of the building model.

In general, the equation of motion of multi-degree of freedom system is given by Eq. 1.

$$[\mathbf{M}] \ddot{\mathbf{x}}(t) + [\mathbf{C}] \dot{\mathbf{x}}(t) + [\mathbf{K}] \mathbf{x}(t) = \{\mathbf{F}(t)\} + \{\mathbf{U}(t)\}$$

$$(1)$$

 $\mathbf{M}$ : Mass matrix  $\mathbf{C}$ : Damping matrix  $\mathbf{K}$ : Stiffness matrix  $\mathbf{F}(t)$ : Force vector

 $\mathbf{U}(t)$ : Control force vector

The control force vector U(t) of Eq. 1 is used in the control theory. Various methods have been used for in the time history response analysis of a multi-degree of freedom system. In this study, the numerical analyses are done by using the Newmark- $\beta$  method. The damping matrix C is used for Rayleigh's damping matrix in this analysis.

#### CONTROL THEORY

# OPTIMAL CONTROL THEORY

The control system by the optimal control theory is given by Eq. 2 and Eq. 3 (Shiraishi, 1987).

$$\left\{ \dot{\mathbf{X}}_{s} \right\} = \left[ \mathbf{A} \right] \left\{ \mathbf{X}_{s} \right\} + \left[ \mathbf{B} \right] \left\{ \mathbf{U}_{s} \right\}$$

$$\left\{ \mathbf{Y}_{s} \right\} = \left[ \mathbf{D} \right] \left\{ \mathbf{X}_{s} \right\}$$

$$\left\{ \mathbf{X}_{s} \right\} : \text{State vector} \quad \left\{ \mathbf{U}_{s} \right\} : \text{Control vector}$$

$$\left[ \mathbf{A} \right] : \text{System matrix} \quad \left[ \mathbf{B} \right] : \text{Control matrix}$$

$$\left\{ \mathbf{Y}_{s} \right\} : \text{Output vector} \quad \left[ \mathbf{D} \right] : \text{Output matrix}$$

Eq. 2 is the state equation and Eq. 3 is the output equation. When Eq. 1 is transformed to Eq. 2, the following state equation is obtained.

$$\begin{cases}
\dot{\mathbf{X}}_{1} \\
\dot{\mathbf{X}}_{2}
\end{cases} = \begin{bmatrix}
0 & \mathbf{I} \\
-\mathbf{M}^{-1} \cdot \mathbf{K} & -\mathbf{M}^{-1} \cdot \mathbf{C}
\end{bmatrix} \begin{Bmatrix} \mathbf{X}_{1} \\
\mathbf{X}_{2}
\end{Bmatrix} + \begin{Bmatrix} 0 \\
-\mathbf{M}^{-1}
\end{Bmatrix} \begin{Bmatrix} \mathbf{U}_{s}
\end{Bmatrix}$$

$$\mathbf{X}_{1} = \mathbf{X}(t) \quad \mathbf{X}_{2} = \dot{\mathbf{X}}(t)$$

$$\mathbf{U}_{s} = \mathbf{F}(t) + \mathbf{U}(t)$$
(4)

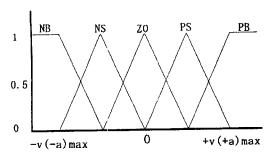
The displacement and velocity of each story are expressed in the state vector  $\{\mathbf{X}_s\}$  of Eq. 4. The control vector  $\{\mathbf{U}_s\}$  expresses the control force that is given in the product of state vector  $\{\mathbf{X}_s\}$  and the feedback gain  $[\mathbf{F}_s]$  as shown in Eq. 5. The feedback gain  $[\mathbf{F}_s]$  is determined by the minimum value of the evaluation function  $J_d$  of Eq. 6 (Ono, 1992, Ooyama, 1993).

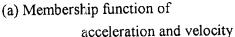
$$\{\mathbf{U}\} = -[\mathbf{F}_s] \cdot \{\mathbf{X}_s\} \tag{5}$$

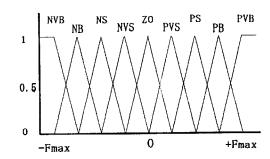
$$J_d = \sum_{i=0}^{\infty} \left( \mathbf{X}_s^i \mathbf{Q}_d \mathbf{X}_s + \mathbf{U}^i \mathbf{R}_d \mathbf{U} \right) \ge 0$$
 (6)

 $[\mathbf{F}_s]$ : Feedback gain

Matrixes  $[\mathbf{Q}_d]$  and  $[\mathbf{R}_d]$  of Eq. 6 are the weight matrix of the state vector  $\{\mathbf{X}_s\}$  and the control vector  $\{\mathbf{U}_s\}$ . It is thought that the first term of the right side of Eq. 6 expresses the state of the energy of the structure system in this study. Feedback gain  $[\mathbf{F}_s]$  is determined by Eq. 6. Weight matrix  $[\mathbf{Q}_d]$  is used by the mass matrix and stiffness matrix of Eq. 1,  $[\mathbf{R}_d]$  is used by the unit matrix. And the maximum value of the control force set to 6N.







(b) Membership function of

control force

Fig. 4. Membership functions

Table 3. Control rules

	NB	NS	zo	PS	PB
NB	PVB	PVB	PB	PVS	NVS
NS	PVB	PB	PS	ZO	NS
ZO	PB	PS	zo	NS	NB
PS	PS	ZO	NS	NB	NVB
PS	PVS	NVS	NB	NVB	NVB

Table 4. Values of  $|V_{MAX}|$  and  $|a_{MAX}|$ 

Middle story control	$ V_{\text{MAX}} $ $(cm / \text{sec})$	$\begin{vmatrix} a_{MAX} \end{vmatrix}$ $(gal)$
1st. mode	1.05	15.8
2nd. mode	1.05	15.8
3rd. mode	1.18	17.6

### **FUZZY CONTROL THEORY**

It is necessary to obtain the membership functions and fuzzy IF-THEN rules for fuzzy control (Terano et al., 1987). In this study, two antecedent parts and one consequent part of the fuzzy IF-THEN rule are used. As for the parameters, the antecedent parts are acceleration and velocity, and the consequent part is control force. The control rule examples are as follows (Obata et al., 1994, 1995, Saito et al., 1994):

## Examples

rule 1: If the acceleration is Negative Big and the velocity is Negative Big,

then the control force is Positive Very Big.

IF a is NB and v is NB then u is PVB

rule 2: If the acceleration is Positive Big and the velocity is Zero, then the control force is Negative Big.

IF a is PB and v is ZO then u is NB

rule 25: If the acceleration ...

PVB: Positive Very Big, PB: Positive Big, PS: Positive Small, PVS: Positive Very Small, ZO: Zero,

NVS: Negative Very Small, NS: Negative Small, NB: Negative Big, NVB: Negative Very Big.

In this study, the reasoning rules are used 25, and the number of discreteness of the base sets are 17. The membership functions use a triangle type. The membership functions are shown in Fig.4, and the rule table is shown in Table 3.

When the control force acts on the upper story, a good damping effect is obtained at about the first mode. If the membership function that is the same as the first mode is used in the third mode, it is difficult to obtain an adequate damping effect at about the third mode. As for the reason, the amplitude of the third mode is smaller than that of the other modes at the upper story. In this study, the best vibration control is achieved by using the membership function that corresponds to each mode. Concretely, the generation sensitivity of the control force is adjusted by changing  $v_{max}$  and  $a_{max}$  of the membership function. The examples of  $v_{max}$  and  $a_{max}$  are shown in Table 4. And the maximum value of the control force is set at 6N which is the same as optimal control.

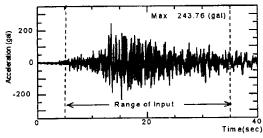
#### RESULTS AND DISCUSSION

The time history response analyses of the 12 cases are done by using both fuzzy control and optimal control. The analytical cases are shown in Table 5. Upper story control, middle story control and lower story control described in Table 5 express the action position of the control force. An initial displacement in which the first to third mode predominate is given, the active control is performed for the free vibrations shown in Table 5. The input earthquake wave used for the forced vibration analysis is the strong ground motion of the Kushiro-oki earthquake that occurred on January 15,1993. The acceleration record wave of the bridge axial direction observed in the Chiyoda-oohashi is used for analysis, and it is converted into a maximum acceleration of 100 gal. The input earthquake wave is shown in Fig. 6, and the Fourier spectrum of the input wave is shown in Fig. 7. Figs. 8~11 express the analysis results. The thin lines in Figs. 8~11 express the results in non-control cases.

The control force acted efficiently in the first mode for about 3 sec after control started from the response of Fig.8(a), and Fig.9(a). After about 3 sec, it is thought that the control force acts for the higher vibration mode that remained. The characteristics of the fuzzy control theory and optimal control theory are compared as follows. The component of third mode acceleration remains in the optimal control cases. The response wave in the fuzzy control cases are smoother than the optimal control cases, and the component of second mode remains. The damping effect of the optimal control acts efficiently for the first and second modes as the result.

Table 5. Analytical cases

Control point	Free vibration	forced vibration
	1st. mode	_ Kusiro-oki
Upper story	2nd. mode	earthquake
	3rd. mode	(Chiyoda-oohashi)
	1st. mode	Kusiro-oki
Middle story	2nd. mode	earthquake
	3rd. mode	(Chiyoda-oohashi)
	1st. mode	Kusiro-oki
Lower story	2nd. mode	earthquake
	3rd. mode	(Chiyoda-oohashi)



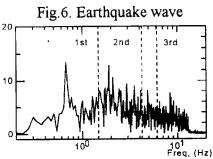
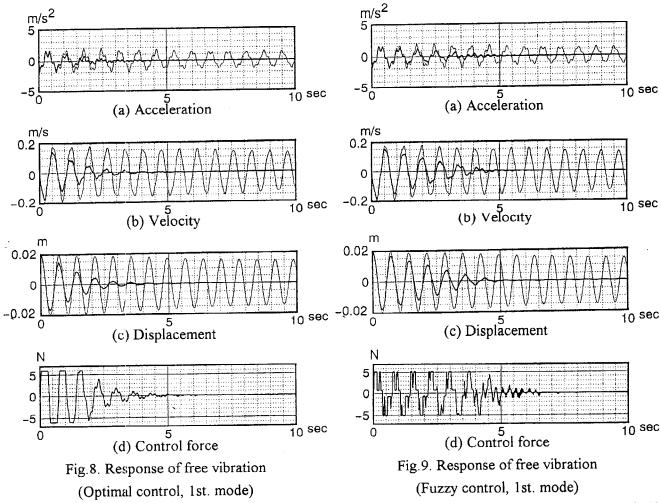


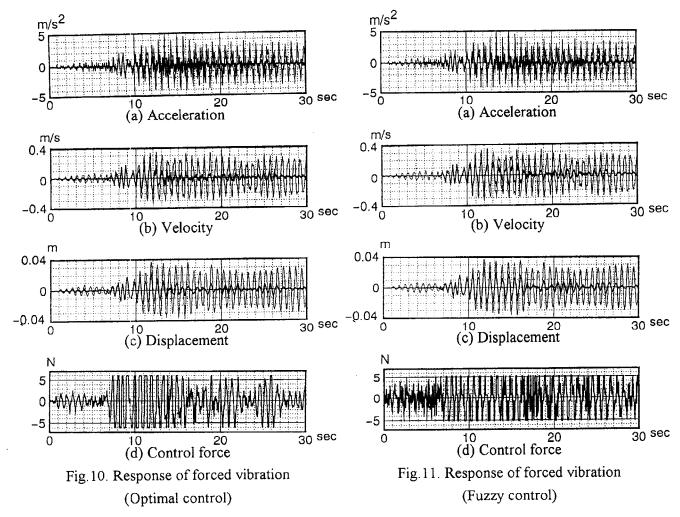
Fig. 7. Fourier spectrum of earthquake wave



It is considered that the control force is effective for the first and third modes in the case of fuzzy control. The damping characteristics of optimal control are like an exponential function from Fig.8(b), and in the fuzzy control cases become linearly damping from Fig.9(b). This reason is presumed to be an influence of a difference of the characteristics of the control theory and the parameters that are used in each theory.

It is compared about the displacement response of Fig.8(c) and Fig.9(c). The response of the structure displacement stops at about 5 sec in both optimal control and fuzzy control. Therefore, it is thought that the displacement response of the analysis result exhibits an almost similar vibration control effect. The Time history of the output of the control force in shown in Fig.8(d) and Fig.9(d). The output time of the maximum control force of the optimal control is about 2 sec after the start. It is understand that the output time is about 4.2 sec in the case of fuzzy control from Fig.9(d). In the optimal control, the value of response velocity approximately 2.0 sec that stops outputting the maximum control force at abut 0.04m/sec from Fig.8(b). The value of the velocity at the stopping time of the output of the maximum control force is about 0.03 m/sec in the fuzzy control case. The response velocity output of both theories are approximately the same level. In order to obtain the same damping effect, the optimal control is effectively with small control force than the fuzzy control.

The analytical results in the case of forced vibration excited by an earthquake wave are shown in Fig.10 and Fig.11. The maximum value of response of displacements are almost nearly the same for in both theories. The maximum displacement value is reduced to about 65% by optimal control, and it is reduced to about 70% by



fuzzy control. The time at which the displacement more than 0.01m for the optimal control is approximately 5.4sec, and approximately 8.5sec for fuzzy control. Therefore, it is considered for the vibration depression effect due to control theories that optimal control is a bit better than fuzzy control. The control force of both theories are compared by using Fig.10(d) and Fig.11(d). In the optimal control case, the parameters of feedback are displacement and velocity. The output history of control force receives an influence of the response of displacement and it becomes the control force characteristics that corresponds to the displacement characteristics. In fuzzy control, the parameters are acceleration and velocity. Even if the displacement response became small, the control force reacts to the response of acceleration sensitively. The output characteristics show that a fluctuation became rapid.

#### CONCLUSIONS

The major conclusions obtained in this study are summarized as follows: The fuzzy control theory and the optimal control theory have an almost nearly similar control effect as per the analysis results. As for the analyses for free vibration control, the vibration is restrained in approximately 5 sec by both methods. The response values of the structure by an earthquake force are much decreased. There is a little difference in the damping characteristics for both control theories. The fuzzy control has a tendency towards a linear damping effect, and on the other hand, the optimal control has a logarithmic damping effect. The optimal control design is comparatively easier than the fuzzy control theory, because the design method has been established

theoretically for optimal control. And, a lot of experience and knowledge of vibration control for setting the control rules and the membership functions are required to make the fuzzy control system.

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