Paper No. 239. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

SYSTEM IDENTIFICATION OF A LONG-SPAN BRIDGE

R. Y. TAN and H.LIU

Center for Earthquake Engineering Research
National Taiwan University , Taipei , Taiwan , 10764

ABSTRACT

On the basis of the modal analysis and an effective modal sweep, the system dynamic parameters of a particular bridge are identified. The bridge is a continuous prestressed concrete structure with the total span of 800 meters and a horizontal radius of curvature 750 meters. In the analysis , the bridge is subjected to three-component excitations. Meanwhile , the initial conditions are also deemed as the parameters to be identified . The numerical results show that the identified response is close to the measured one. Thus, the dynamic characteristics of the bridge may be explored even with insufficient response records.

KEYWORDS

System identification; long-span bridge; multi-component excitations; filtering process; modal sweep; iterative identification; modal parameters.

INTRODUCTION

Bridge structures are often subjected to dynamic loadings. In recent years, analytical structural models and numerical schemes have been well developed to perform the dynamic analysis. The usefulness of such analytical solutions is, however, limited by the degree of realistic representation of the formulated mathematical models. Obviously, a logical prelude to determine the dynamic properties and to evaluate the safety of bridges is the performance of system identification on the basis of measured field data. This is particularly urgent for an important and complex bridge structure.

The main problem in performing such an identification arises from the fact that the response is usually measured only at a few locations in the bridge. This limits the degree to which the dynamic properties of a bridge can be resolved. In view of insufficiency of the response data, the modal identification procedure is regarded as the most popular one since parameters of a linear system, e.g. modal frequencies and damping ratios, can be determined on the basis of a single measurement (Beck *et al.* , 1980). It is noted, however, that the modal interference should be avoided to ensure the accuracy of the estimates.

This paper intends to establish a numerical scheme for carrying out the identification of a long-span bridge assuming that the response data are available.Based on the modal sweep concept and the band-pass filtering process, an iterative identification algorithm is employed to estimate the modal parameters of the bridge. The initial conditions, which account for the effects of truncation of the measured record, are also deemed as the parameters to be identified. In addition, a more realistic dynamic model is used in which the bridge is multiple-supported and subjected to three-component excitations.

MOTION EQUATION

A bridge is supported at more than one point and is subjected to different and multicomponent excitations. This will induce the quasi-static stresses that must be considered in addition to the dynamic effects resulting from the inertial force. The equations of motion of a MDOF bridge subjected to support excitation can be written in partitioned matrix form as follows:

$$\begin{pmatrix} \underset{\sim}{m} & \underset{\sim}{m}_g \end{pmatrix} \begin{pmatrix} \ddot{\underset{\sim}{v}}^t \\ \ddot{\underset{\sim}{v}}_g \end{pmatrix} + \begin{pmatrix} \underset{\sim}{c} & \underset{\sim}{c}_g \end{pmatrix} \begin{pmatrix} \dot{\underset{\sim}{v}}^t \\ \dot{\underset{\sim}{v}}_g \end{pmatrix} + \begin{pmatrix} \underset{\sim}{k} & \underset{\sim}{k}_g \end{pmatrix} \begin{pmatrix} \underset{\sim}{v}^t \\ \underset{\sim}{v}_g \end{pmatrix} = \underset{\sim}{0} \tag{1}$$

in which all components of support displacement are listed in the vector $\underset{\sim}{v}_g$. Similarly, all the superstructure nodal response components are listed in the vector $\underset{\sim}{v}^t$, where the superscript t denotes that they are total nodal displacements. In addition, $\underset{\sim}{m}$, $\underset{\sim}{c}$ and $\underset{\sim}{k}$ represent the mass matrix, damping matrix and stiffness matrix, respectively, while $\underset{\sim}{m}_g$, $\underset{\sim}{c}_g$ and $\underset{\sim}{k}_g$ refer to the forces in the response degrees of freedom due to motions of the supports.

The total displacement vector is expressed as the combination of a quasi-static displacement vector $\underset{\sim}{v}^s$ plus a relative displacement vector $\underset{\sim}{v}$; thus

$$\underset{\sim}{v}^t = \underset{\sim}{v}^s + \underset{\sim}{v} \tag{2}$$

The quasi-static displacement can be expressed as

$$\underset{\sim}{v}^s = \underset{\sim}{r}\, \underset{\sim}{v}_g \tag{3}$$

where $\underset{\sim}{r}$ is an influence coefficient matrix which represents the nodal responses due to unit support motions. Since the entire velocity-dependent part of the effective input is negligible (Clough and Penzien, 1993), substituting Eqs.(2) and (3) into Eq.(1) leads to

$$\underset{\sim}{m}\ddot{\underset{\sim}{v}} + \underset{\sim}{c}\dot{\underset{\sim}{v}} + \underset{\sim}{k}\underset{\sim}{v} = -\left(\underset{\sim}{m}\underset{\sim}{r} + \underset{\sim}{m}_g \right) \ddot{\underset{\sim}{v}}_g \tag{4}$$

MODAL ANALYSIS

When the damping matrix is orthogonal with respect to the mode shapes, modal orthogonality enables us to transform Eq.(4) into a set of N uncoupled differential equations. To do so, let

$$\underset{\sim}{v} = \underset{\sim}{\phi}\, \underset{\sim}{y} \tag{5}$$

in which $\underset{\sim}{\phi}$ = mode-shape matrix = ($\underset{\sim}{\phi_1}$ $\underset{\sim}{\phi_2}$ .... $\underset{\sim}{\phi_n}$), and $\underset{\sim}{y}$ = modal responses = (y_1 y_2 .... y_n $)^T$. Based on the normal-coordinate transformation, the equation of motion for each mode can be written as

$$\ddot{y}_n + 2\xi_n\omega_n\dot{y}_n + \omega_n^2 y_n = -\underset{\sim}{p_n^T}\underset{\sim}{\ddot{v}_g} \tag{6}$$

where ω_n is the modal frequency, ξ_n is the modal damping ratio, and $\underset{\sim}{p_n}$ is the modal participation factor whose transpose is given by

$$\underset{\sim}{p_n^T} = \underset{\sim}{\phi_n^T}\left(m\underset{\sim}{r} + \underset{\sim}{m_g}\right) / \left(\underset{\sim}{\phi_n^T}\, m\, \underset{\sim}{\phi_n}\right) \tag{7}$$

If the available measured responses are collected from the s-th degree of freedom, Eq.(6) can be premultiplied by the s-th component of $\underset{\sim}{\phi_n}$, ϕ_{sn}, and becomes

$$\ddot{v}_{sn} + 2\xi_n\omega_n\dot{v}_{sn} + \omega_n^2 v_{sn} = -\underset{\sim}{p_{sn}^T}\underset{\sim}{\ddot{v}_g} \tag{8}$$

In Eq.(8), $v_{sn} = \phi_{sn}y_n$ = the modal displacement at the s-th DOF due to the contribution of the n-th mode, and $\underset{\sim}{p_{sn}} = \phi_{sn}\underset{\sim}{p_n}$ = the effective participation factor. It is noted that the acceleration at the s-th DOF is

$$\ddot{v}_s = \sum_{n=1}^{N}\ddot{v}_{sn} \tag{9}$$

Eqs.(8) and (9) are two governing equations needed in the modal identification of a linear bridge system with classical damping.

IDENTIFICATION ALGORITHM

Let the modal displacement be (Tan *et al.* ,1993)

$$v_{sn} = v_{snf} + v_{sni} \tag{10}$$

in which v_{snf} refers to the force response, while v_{sni} refers to the response under a nonzero initial condition. Therefore, v_{sn} is a function of ξ_n, ω_n, $\underset{\sim}{P_{sn}}$ and the initial conditions.

An output-error approach is utilized to perform the identification on the modal basis. The predicted response from the n-th frequency bandwidth is given by

$$\left(\ddot{v}_{sn}\right)_m = \int_{-\infty}^{\infty} h_n(\tau)\ddot{v}_{sn}(t-\tau)d\tau \tag{11}$$

where $h_n(\tau)$ is the weighting function of the filter. Accordingly, the measure-of-fit is defined by

$$e = \int_0^t \left[\left(\ddot{v}_{sn}\right)_r - \left(\ddot{v}_{sn}\right)_m\right]^2 dt \tag{12}$$

in which $(\ddot{v}_{sn})_r$ is the recorded modal acceleration response, contributed by the n-th mode. It is obtained by filtering the "modified response" through the same filter used in Eq.(11). The modified response herein is nothing but the response calculated by substracting the contribution of all the other modes from the recorded response. The minimization of e yields the parameter values.

If the number of modes involved is M, the above procedure consists of M single-mode identifications, which constitute one "sweep" or one cycle of iteration. To compare the convergence and accuracy of the iterative identification process, the error index is defined as

$$EI = \frac{\int_0^t \left[(\ddot{v}_s)_r - \sum_{n=1}^{M} \ddot{v}_{sn} \right]^2 dt}{\int_0^t \left[(\ddot{v}_s)_r \right]^2 dt} \tag{13}$$

where $(\ddot{v}_s)_r$ is the recorded acceleration response. Continue a number of sweeps until the error index converges. Numerical study reveals that only a few sweeps are required.

IDENTIFICATION RESULTS

The bridge considered and shown in Fig.1 is currently under construction and is one of the most important and special bridge in Taiwan. It is a continuous prestressed concrete structure with the total span of about 800 meters long and a horizontal radius of curvature about 750 meters. Center span of the bridge, arch-shaped with 160 meters long, is constructed using prestressed concrete box girders. The piers supporting the center span are of inverted triangular shape with hollow centers. Due to many special features of this bridge, long-term static and dynamic monitoring systems have been implementing.

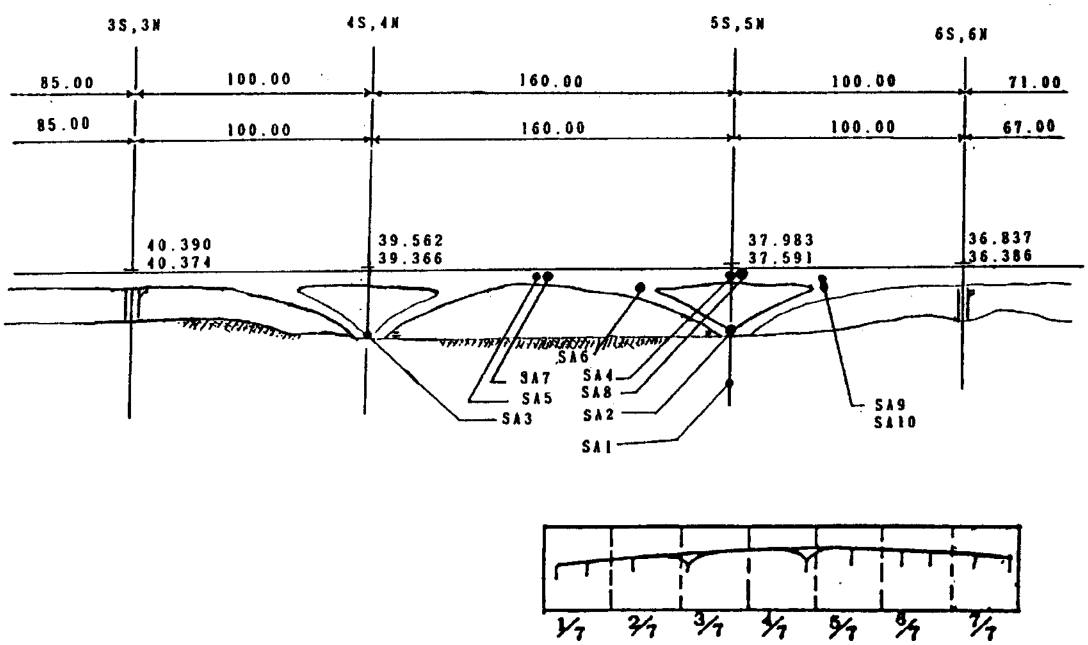


Fig.1 Bridge model and locations of measuring instruments

Before the field data are obtained, the bridge responses subjected to the three-component excitations of a local earthquake are calculated and treated as the measured responses. A total of 207 3-D beam elements are used in the analysis(Chang *et al.* , 1993). The damping ratio is 0.05 for each mode. The response at the mid-span of the bridge is adopted since several measuring instruments are implemented there.

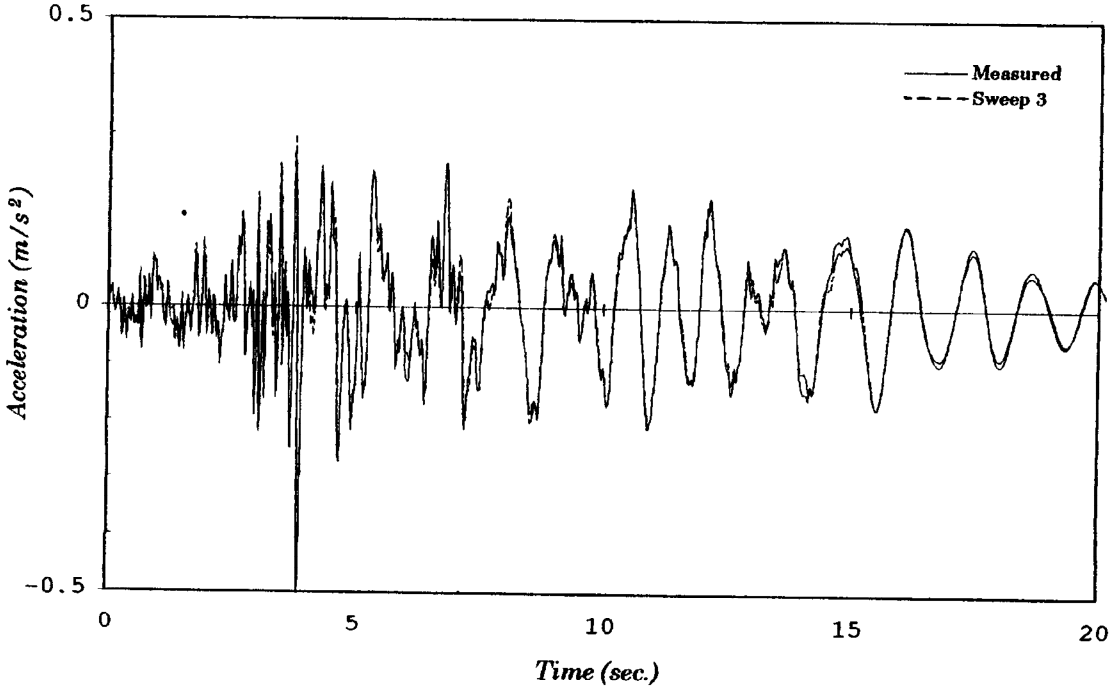
To estimate the parameter values in the y-direction(transverse direction), the y-component response at the center of the bridge together with the y-component inputs at the supports are used. Table 1 summarizes the identification result after three sweeps. A rapid convergence of parameter values is seen. In fact, all three input components can be utilized to obtain a better estimate. The estimated parameters based on the multicomponent inputs are listed in Table 2. The effective participation factors contributed by x and z directions are smaller, as expected. The error index is reduced. As shown in Fig. 2, the identified response and the transfer function can be compared with the corresponding measured ones. Two curves are very close.

Table 1 Estimated parameters in y-direction based on y-component input

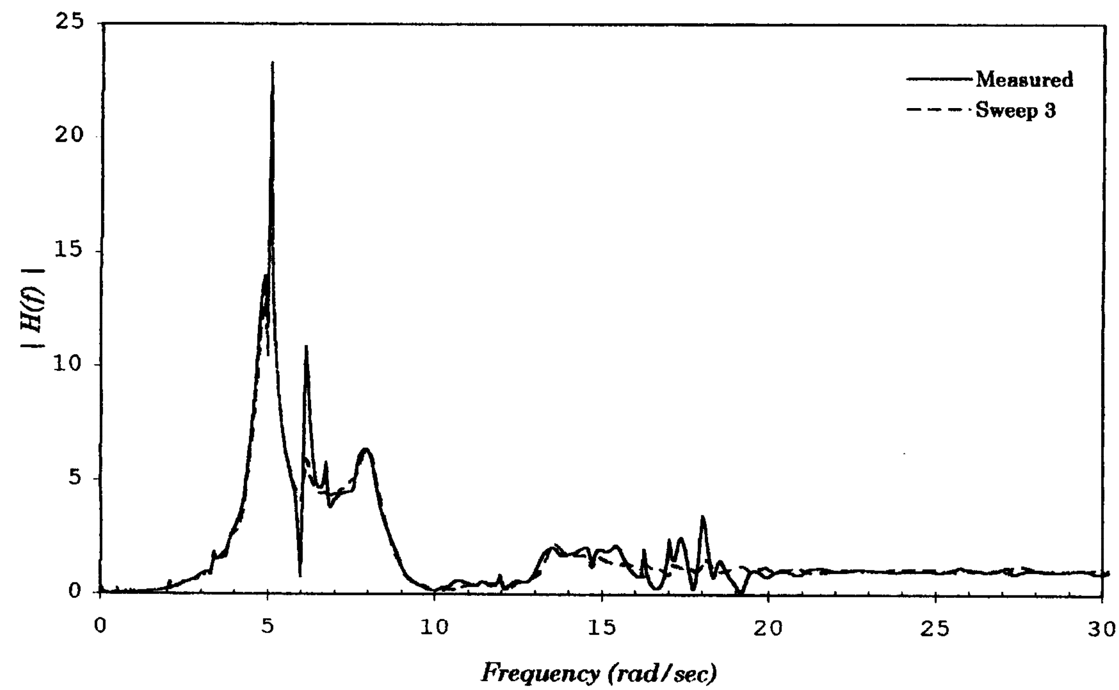
	Sweep 1				Sweep 2		
mode	ξ_n	ω_n	p_{sn}	mode	ξ_n	ω_n	p_{sn}
1	0.0517	4.97	-1.55	1	0.0515	4.95	-1.54
2	0.0516	8.15	0.59	2	0.0491	8.17	0.54
3	0.0403	13.47	-0.13	3	0.0436	13.46	-0.14
	EI=2.774%				EI=2.802%		
	Sweep 3				Theoretical values		
mode	ξ_n	ω_n	p_{sn}	mode	ξ_n	ω_n	
1	0.0508	4.95	-1.47	1	0.0500	4.93	
2	0.0496	8.15	0.55	2	0.0500	8.15	
3	0.0360	13.48	-0.11	3	0.0500	13.57	
	EI=2.419%						

Table 2 Estimated parameters in y-direction based on multicomponent inputs

mode	ξ_n	ω_n	p_{snx}	p_{sny}	p_{snz}
		Sweep 1			
1	0.0567	4.95	0.17	-1.71	-0.01
2	0.0471	8.16	-0.01	0.57	-0.04
3	0.0491	13.59	0.04	-0.17	0.00
	EI=5.025%				
		Sweep 2			
1	0.0522	4.94	0.24	-1.66	-0.02
2	0.0503	8.15	-0.02	0.55	-0.04
3	0.0465	13.60	0.05	-0.17	-0.01
	EI=5.020%				
		Sweep 3			
1	0.0504	4.93	0.09	-1.48	0.12
2	0.0493	8.15	-0.02	0.56	-0.05
3	0.0352	13.53	0.01	-0.11	0.02
	EI=1.988%				



(a) time domain



(b) transfer function

Fig.2 Comparison between identified response in y-direction and measured one

Similarly, the parameter values in the x-direction (longitudinal direction) can be acquired on the basis of the x-component response and the multicomponent inputs. The outcome of identification process is tabulated in Table 3. The comparison of the identified response and the measured one is shown in Fig. 3. In the time domain, two are virtually indistinguishable. This indicates that a single-mode response is sufficient to represent the response.

Table 3 Estimated parameters in x-direction based on multicomponent inputs

		Sweep 1			
mode	ξ_n	ω_n	p_{snx}	p_{sny}	p_{snz}
1	0.0532	6.38	-0.95	-0.01	0.02
	(0.0500)	(6.35)			
	EI=3.785%				
	( ) : theoretical value				

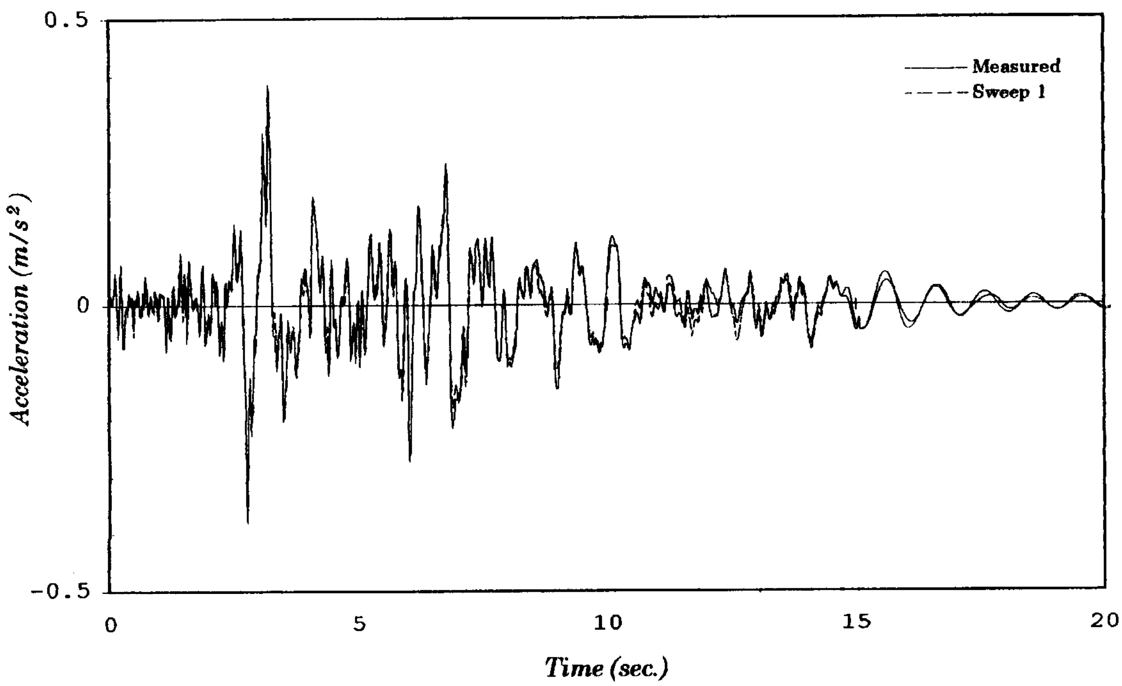


Fig.3 Comparison between identified response in x-direction and measured one

The parameters in the z-direction (vertical direction) are also identified. The estimates after three sweeps are listed in Table 4. It shows that the first three modes are related with the y-direction and only the fourth mode is contributed by the z-direction. Therefore, without the utilization of all three input components, it is unlikely to identify the parameter values in the vertical direction. The identified response is contrasted with the measured one in Fig.4.

Table 4 Estimated parameters in z-direction based on multicomponent inputs

		Sweep 3			
mode	ξ_n	ω_n	p_{snx}	p_{sny}	p_{snz}
1	0.0514	4.93	-0.02	0.11	0.01
2	0.0514	7.32	0.00	-0.45	-0.09
3	0.0467	8.13	0.03	0.28	0.02
4	0.0510	17.98	0.04	0.48	-0.70
	(0.0500)	(18.18)			
	EI=1.192%				
	( ):theoretical value				

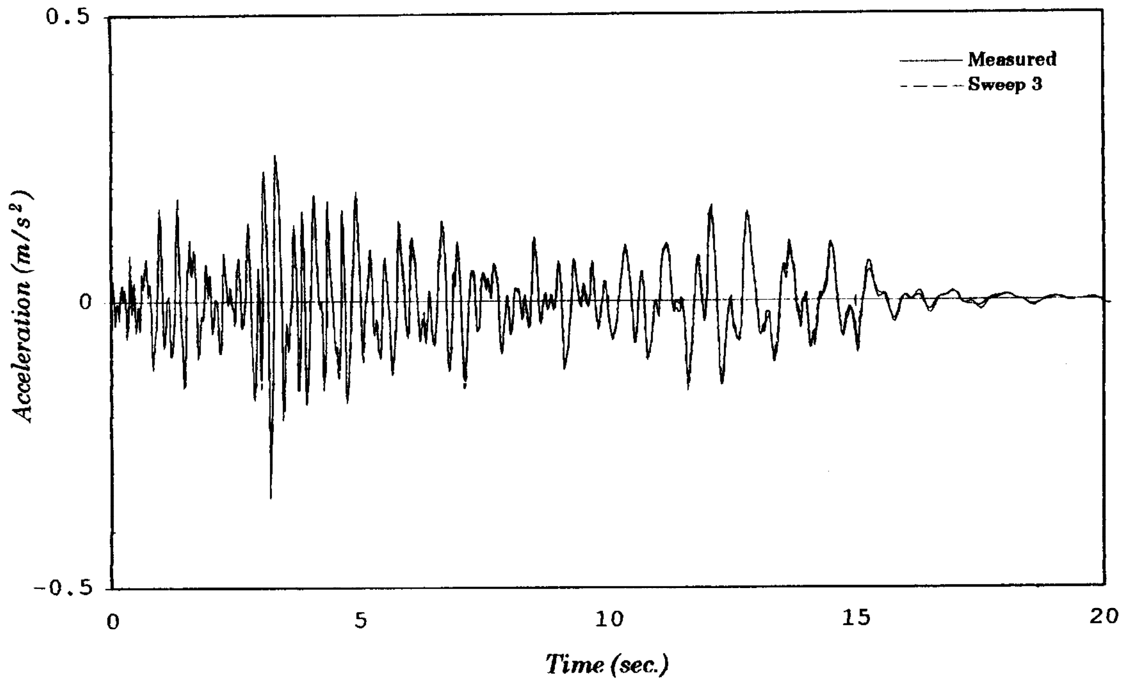


Fig.4 Comparison between identified response in z-direction and measured one

CONCLUSIONS

A practical algorithm is devised for systematically determining the best estimate of the dynamic parameters of a long-span bridge. This method has been set within the framework of the output-error approach in which parameter values are assessed by minimizing a measure-of-fit between the filtered structural output and the model output.The application of filtering process and modal sweep make it possible that the identification is performed on the modal basis.This is generally more tractable from the viewpoint of the limitation of recorded data and the numerical efficiency. The study confirms that multicomponent excitations must be considered to obtain a better estimate of the system parameters.In conclusion,the proposed identification algorithm provides a useful tool to investigate the dynamic properties of a bridge.

REFERENCES

Beck,J.L. and P.C. Jennings (1980). Structural identification using linear models and earthquake records. *Earthquake Engng Struct. Dyn,8*, 145~160.

Chang,K.C. and *et al.* (1993). *Structural monitoring and analysis of Bit-Tan Bridge,* Technical Report ,National Expressway Engineering Bureau, Taiwan.

Clough,R.W. and J.Penzien (1993). *Dynamics of Structures*, McGraw-Hill,New York.

Tan,R.Y. and W.M.Cheng (1993) System identification of a non-classically damped linear system. *Computer &Structures,46,*67~75.