Paper No. 235. (quote when citing this article)
11 WCEE
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

HEALTH MONITORING SYSTEM OF BUILDING BY USING WAVELET ANALYSIS

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ABSTRACT

It is very important to estimate the damage rate of building caused by low cycle fatigue under their dynamic loading such as seismic loading from the point of view of guarantee of their safety and making the scheme of their maintenance. Therefore, the health monitoring system in-service to monitor the damage rate of building by the low cycle fatigue is proposed to assure the safety of the structures. In this paper, the use of the wavelet transform of measured record of response of building during earthquake is proposed to extract the low cycle fatigue signals from the noisy detected signals. The model of low cycle fatigue is considered to be that of the reducing stiffness of structural member having impulse forces occurred at random intervals.The response of model structure is analyzed for impluse forces and noise. The wavelet analysis is applied to the record of response and it is proved to be able to extract the assumed signals of fatigue from this response.

KEYWORDS

Health monitoring; wavelet analysis; low cycle fatigue; impulse force; Daubechies wavelet; identification

INTRODUCTION

It is very important to estimate the damage rate of building caused by low cycle fatigue under their dynamic loading such as seismic loading from the point of view of guarantee of their safety and making the scheme of their maintenance. Such the damage is cumulated by low cycle fatigue and the decision for replacing and maintenance of building is made according the damage rate. Therefore, it is expected that the health monitoring system in-service will be used to monitor the damage rate of building by the low cycle fatigue. In the health monitoring system, the seismic response signals of building are monitored and cumulative damage rate is estimated by using these signals. However, the monitored signals may be contaminated by some noises.

With this circumstance as background, the authors intend to apply the wavelet transform to the health monitoring system. The wavelet transform provides the multi-resolution analysis and the frequency information locally in time in contract to the conventional Fourier transform. As the wavelet transform provides short windows at high frequency and long windows at low frequency, it is the so called "constant Q filtering " such as the band pass filter (Chui, 1992). Therefore, the wavelet transform will be useful for the design of filter to remove the noise component from the signals.

In this paper, the wavelet transform is proposed to extract the fatigue signals from the noisy detected signals. The model of fatigue is considered to be that of the reducing stiffness of structural member having impulse forces generated at random intervals. The structural response by these impulse forces will be analyzed under noisy condition. The wavelet transform is applied to the calculated responses and it is proved to be able to extract the assumed signals of fatigue from these responses, namely monitored signals. Finally, the wavelet analysis is applied to system identification for estimating the reducing stiffness of structural member.

CONTINUOUS WAVELET TRANSFORM

The continuous wavelet transform of arbitrary function x(t) in $L^2(\mathbf{R})$ is defined as follows (Daubechies, 1992).

$$(Tx)(a,b) = |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} x(t)\, \overline{\psi\left(\frac{t-b}{a}\right)}\, dt \tag{1}$$

where $a \neq 0$ and $b \in \mathbf{R}$ are the dilation parameter and translation parameter, respectively. $\psi(t)$ is called analyzing wavelet or mother wavelet. $\overline{\psi(t)}$ is the complex conjugate of $\psi(t)$. Generally, as the analyzing wavelet, the compactly supported function in both time domain and frequency domain is used. Also, the wavelet expansion of x(t) is defined by the following equation.

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (Tx)(a,b)\, \psi\left(\frac{t-b}{a}\right) \frac{1}{a^2}\, da\, db \tag{2}$$

The necessary condition for $\psi(t)$ to be an analyzing wavelet is

$$C_\psi := \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|}\, d\xi \; < \infty \tag{3}$$

where $\hat{\psi}(\omega)$ is the Fourier transforms of $\psi(t)$. By applying the Parseval Identity, the relationship among x(t), $\hat{x}(\omega)$ and wavelet transform (Tx)(a,b) of x(t) is obtained as follows.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\hat{x}(\omega)|^2 d\xi = \frac{1}{C_\psi} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{|(Tx)(a,b)|^2}{a^2}\, da\, db \tag{4}$$

DISCRETE WAVELET TRANSFORM

In the wavelet transform, a function or signal is expanded by the basis compactly supported in both time domain and frequency domain. However, this basis is generally the oblique system and over-complete system. Therefore, for the actual computation of wavelet transform, the dilation parameter a and translation parameter b should be discretized. This wavelet transform using this discrete wavelet is called the discrete wavelet transform. For some very special choices of ψ, the discretized wavelets $\{\psi_{j,k}\}$ constitute an orthonormal basis for $L^2(\mathbf{R})$. By using the orthonormal bases, the wavelet expansion of a function x(t) and the coefficients of wavelet expansion are defined as follows (Daubechies, 1992, Yamada and Ohkitani, 1991 and Sasaki and Maeda, 1993).

$$x(t) = \sum_j \sum_k \alpha_{j,k} \psi_{j,k}(t) \tag{5}$$

$$\alpha_{j,k} = \int_{-\infty}^{\infty} x(t)\, \overline{\psi_{j,k}(t)}\, dt = \langle x(t), \psi_{j,k}(t) \rangle \tag{6}$$

where $\alpha_{j,k}$ is the coefficients of wavelet expansion and $\psi_{j,k}$ is the discrete basis generated by dilating and translating an analyzing wavelet ψ. The symbol of $<\,,\,>$ stands for the inner product. Integers j and $k \in \mathbf{Z}$ are the dilation parameter and the translation parameter, respectively. Generally, by choosing the dilation parameter a to be $a_j = 2^j$, $\psi_{j,k}$ is expressed by

$$\psi_{j,k}(t) = 2^{j/2} \psi(2^j t - k) \tag{7}$$

GENERATION OF ORTHONORMAL WAVELET

In order to make $\{\psi_{j,k}\}$ a complete orthonormal basis, some methods to generate the analyzing wavelet $\psi(t)$ compactly supported in time domain and frequency domain are proposed by Daubechies (1988) and Meyer

(1989). In this study, using the proposed method by Daubechies, the analyzing wavelet with compact support in time domain is generated. The generation of this wavelet is based on the multi-resolution analysis (Daubechies, 1992).

First, in the multi-resolution analysis, the scaling function or father wavelet $\phi(t)$ and the analyzing wavelet $\psi(t)$ in the central closed subspace V_0 can be written in terms of the orthonormal basis $\phi_{1,n}$ in V_1 as follows.

$$\phi(t) = \sqrt{2} \sum_n h_n \, \phi(2t - n) \tag{8}$$

$$\psi(t) = \sqrt{2} \sum_n (-1)^n h_{1-n} \, \phi(2t - n) \tag{9}$$

where a sequence $\{h_n\}$ satisfies the following relations.

$$\left.\begin{array}{l} h_n = 0 \quad , n < 0 \;\text{ or }\; n > 2N \\ \sum_n h_n = \sqrt{2} \\ \sum_n (-1)^n h_{1-n} n^m = 0 \quad , \; 0 \leq m \leq N-1 \end{array}\right\} \tag{10}$$

The way to ensure real-valued compact support for the analyzing wavelet $\psi(t)$ is to choose the scaling function $\phi(t)$ with compact support. For this reason, the sequence $\{h_n\}$ in Eq.(10) is required to be a finite real-valued sequence. Therefore, for the arbitrary integer $N \geqq 2$, the finite sequence is determined by Daubechies (1988), so that the support of $\phi(t)$, the moment of p-th order of $\psi(t)$ and the regularity of $\phi(t)$ and $\psi(t)$ can satisfy the following conditions.

$$\left.\begin{array}{l} \mathrm{supp}\phi = [0,\; 2N-1] \\ \int_{-\infty}^{\infty} t^p \, \psi(t) \, dt = 0 \quad , \; 0 \leq p \leq N-1 \\ \phi(t)\,,\; \psi(t) \in C^{\lambda(N)} \end{array}\right\} \tag{11}$$

$C^{\lambda(N)}$ represents the space consisting of the functions that are $\lambda(N)$ times continuously differentiable. For an integer N, $\lambda(N)$ is approximated by 0.3485N. Also, the support of the analyzing wavelet, $\mathrm{supp}\psi$ is $[1-N, N]$. The examples of $\phi(t)$ and $\psi(t)$ obtained by such a method are shown in Fig.1 for the previous integer N=3 and 7. From these examples, it is shown that the regularity of both $\phi(t)$ and $\psi(t)$ clearly increases with N.

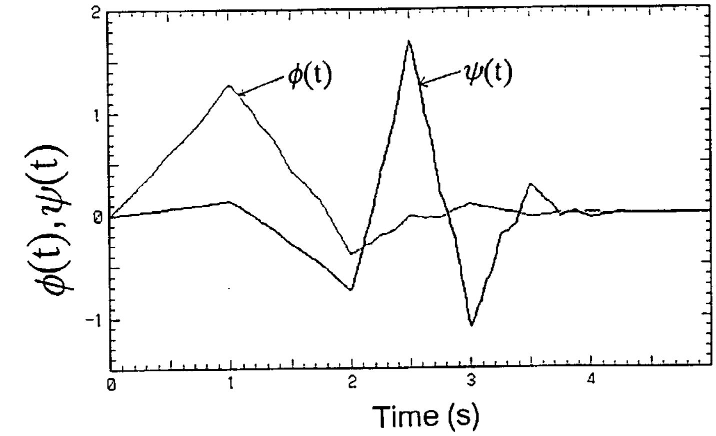
DETECTION OF FATIGUE SIGNALS BY WAVELET ANALYSIS

In this paper, the phenomenon of low cycle fatigue is considered to be the reducing stiffness of structural member of building having impulse forces generated at random intervals. It is the target to detect the time occurrence for these impulse forces to be generated by applying the wavelet transform to the measured response of building.

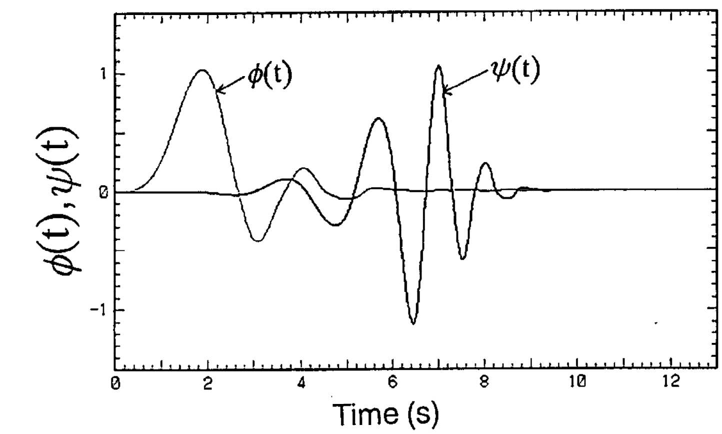
The single degree of freedom structural model as shown in Fig. 2 is considered, which is subjected by the seismic force and the impulse forces are generated at random intervals due to the fatigue of its stiffness or damping. The sum f(t) of these impulse forces and seismic force is considered to be the input force for the structure. Here, the input force f(t) for the model in Fig.2 is generated as the sum of the filtered white noise and some impulses. The parameters of the single degree of freedom system used for a filter are assumed to be the natural circular frequency $\omega_f = \pi/4$ rad/s and damping ratio $\zeta_f = 0.5$. Keeping the root mean square value of filtered white noise constant and varying the amplitude of impulse, the input forces f(t) are generated for several ratios of signal to noise. The sampling time interval of input force and response is selected to be $\Delta t = 2^{-3}$ (s). The occurrence time of impulse is set to t = 32 and 80 (s). The example of input force f(t) is shown in Fig.3. This is for the ratio of the amplitude of impulse S and the root mean square value of filtered white noise N_0, $S/N_0 = 1.5$. The equation of motion of model in Fig. 2 is given by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{12}$$

where, as the parameters of model, the natural circular frequency $\omega_0 = \sqrt{k/m} = \pi$ rad/s and damping ratio $\zeta_0 =$

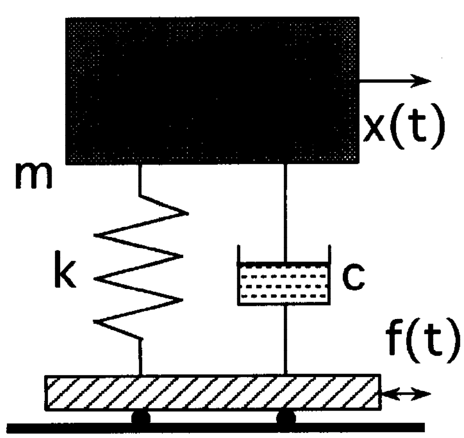


(a) N=3



(b) N=7

Fig.1. Scaling function $\phi(t)$ and analyzing wavelet $\psi(t)$



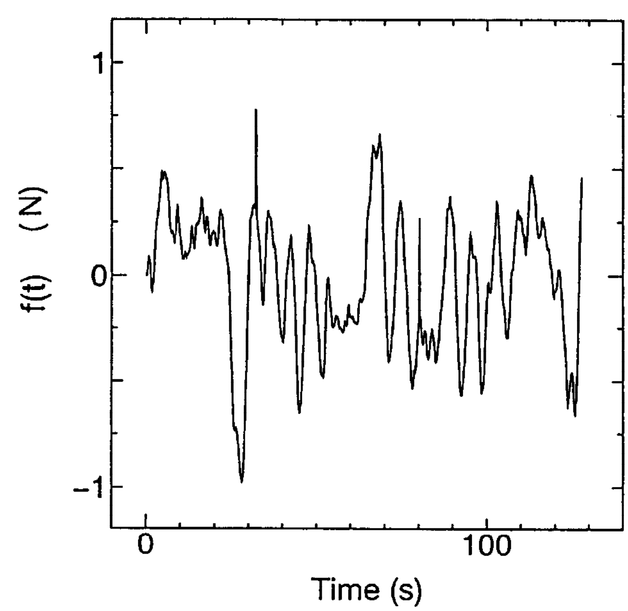


Fig.2. Analytical model

Fig. 3. Input force f(t)

$c/2\sqrt{mk}=0.05$ are used. The response of model subjected to the input force in Fig. 3 is shown in Fig.4. It is

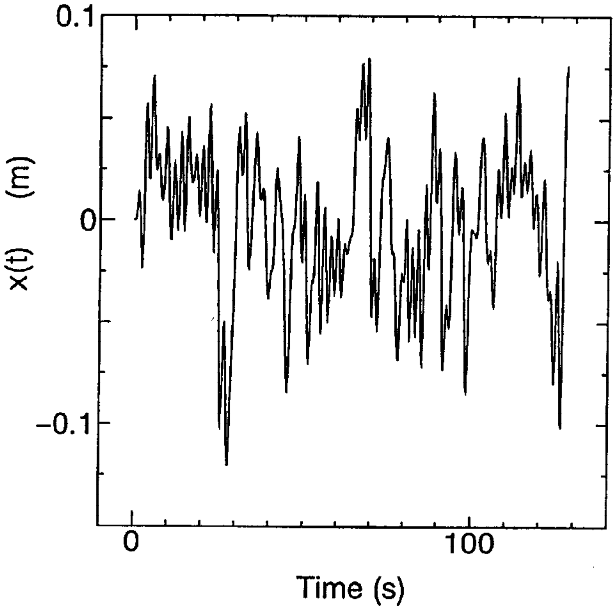


Fig. 4. Response x(t)

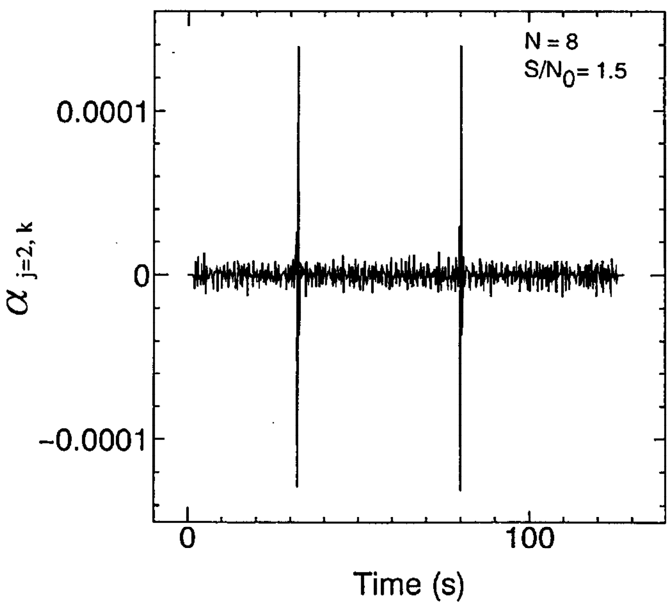
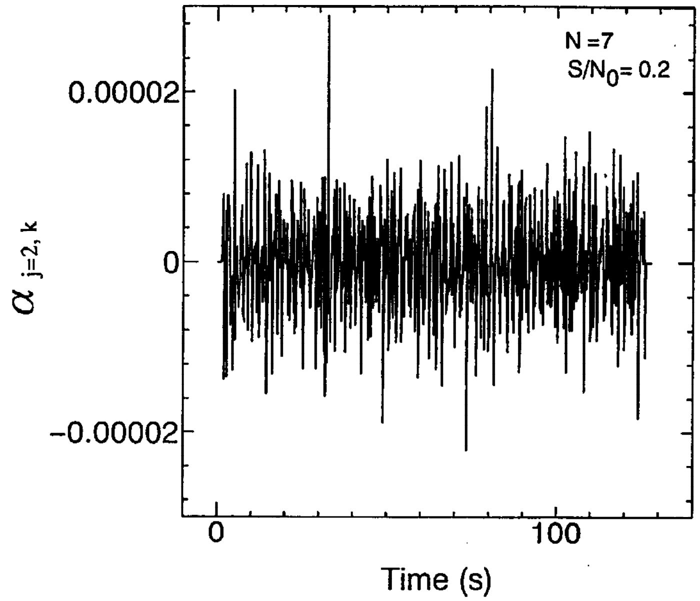


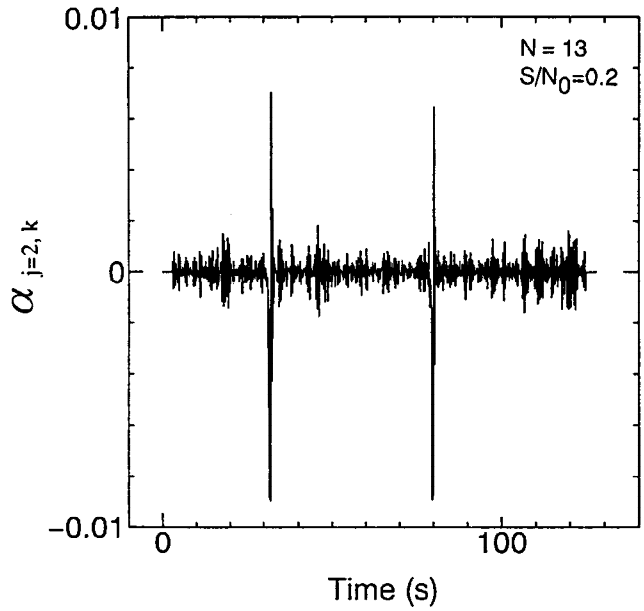
Fig. 5. Coefficients of wavelet expansion of x(t) (j=2)

very difficult to identify the occurrence time t = 32 and 80 (s) of impulses added in the input force by inspecting only this response. Next, using the Daubechies wavelet, the coefficients of wavelet expansion of x(t) obtained by Eq.(6) are shown in Fig.5. This examples is for a wavelet with the previous integer N = 8 and j = 2. In case of higher frequency range for j = 2 corresponding to $5.6\omega_q$, the occurrence time of impulse, namely the signals about progressing the fatigue can be clearly observed because the coefficient of wavelet expansion shows large variation at time t = 32 and 80 (s). In the case where the impulses generated at random interval are added to the input force, namely, in the case where the input force has discontinuity, the input force includes the higher frequency component by this discontinuity. Therefore, the large value of j should be used for getting this higher frequency component.

Using the coefficient of wavelet expansion of response x(t) for j = 2, the limitation for estimation of occurrence time of impulse for S/N_0 is examined. In this study, the case where the peak value of coefficient of wavelet expansion is greater than 4 times its root mean square value is defined as the detectable region for impulse. Taking the ratio of the amplitude of impulse S and the root mean square value of filtered white noise N_0, S/N_0 = 0.2, the calculated coefficient of wavelet expansion of x(t) is shown in Fig.6. These examples are for wavelets with integer N=7 ans 13. From this figure, it is shown that the variation of coefficient of wavelet



(a) N = 7



(b) N = 13

Fig. 6. Coefficients of wavelet expansion of x(t)

expansion at time t = 32 and 80 (s) become remarkable with the increase of N. Furthermore, taking integer N for generating an analyzing wavelet on abscissa and the ratio of the amplitude of impulse S and the root mean square value of filtered white noise N_0 on ordinate, the limitation curve for estimation of impulse is drawn in Fig. 7. In this figure, the upper region of curve is detectable one, while the lower is undetectable one. From this figure, it is clear that the limitation curve shows a decrease with N. And for $N \geqq 6$, the occurrence time of impulse can be detectable even in case of $S/N_0=0.7$.

IDENTIFICATION OF REDUCING STIFFNESS OF STRUCTURE

Next, when the stiffness of structure subjected to white noise input excitation shown in Fig.2 decreases due to the low cycle fatigue, the reduction of its stiffness can be estimated by using the wavelet analysis. First, let write Laplace transform of input excitation f(t) and Laplace form of relative displacement response x(t) as F(s) and X(s), respectively. Using Laplace transform E(s) of equation error e(t), its relationshop is shown in Fig. 8 and in the time domain the next equation is obtained(Tabaru *et al.*, 1993).

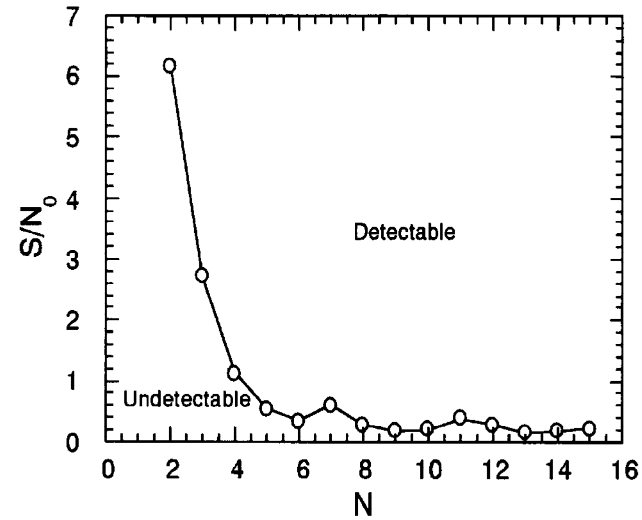


Fig.7. Limitation curve for estimating impulse

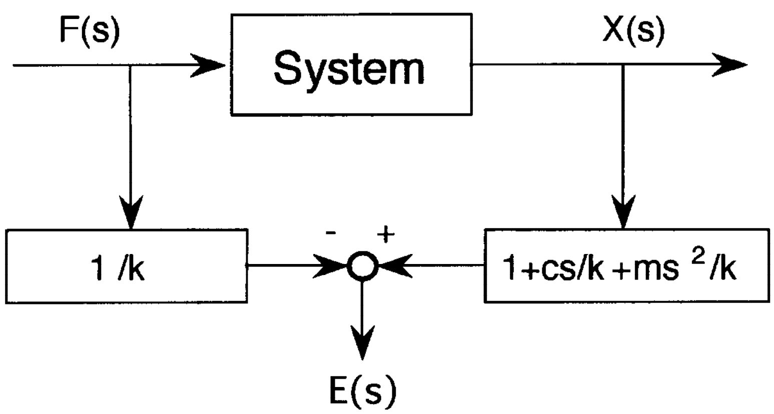


Fig. 8. Block diagram for system identification

The corresponding equation of model is

$$x(t) + \frac{c}{k}\dot{x}(t) + \frac{m}{k}\ddot{x}(t) = \frac{1}{k}f(t) + e(t) \tag{13}$$

Taking the wavelet transform of both sides of this equation, the next equation is obtained as

$$(Tx)(a,b) + \frac{c}{k}(T\dot{x})(a,b) + \frac{m}{k}(T\ddot{x})(a,b) = \frac{1}{k}(Tf)(a,b) + (Te)(a,b) \tag{14}$$

On the other hand, by the properties of the wavelet transform and using integration by parts, the wavelet transform of n*th* derivative $x^{(n)}$ isgiven by

$$(Tx^{(n)})(a,b) = (-1)^n \frac{1}{a^{(n)}\sqrt{a}} \int_{-\infty}^{\infty} \overline{\psi^{(n)}\left(\frac{t-b}{a}\right)}\, x(t)\, dt \tag{15}$$

Therefor, by only monitoring the displacement response x(t) without first derivative response $\dot{x}(t)$ and second derivative response $\ddot{x}(t)$, their wavelet transforms are calculated. In this study, the parameters c/k and m/k on the left-hand side are identified so that the square of wavelet transform of e(t) on the right-hand side of Eq.(14) is minimized by wavelet analysis using Daubechies wavelet. As the error performance index, the discretized square-error criterion is used.

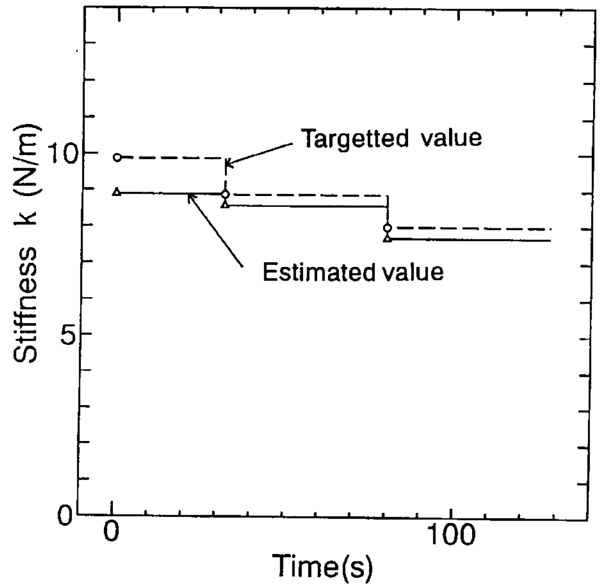


Fig.9. Identified stiffness (N=6)

$$I = |(Te)(a,b)|^2 = \sum_j \sum_k |\langle \psi_{j,k}, e(t) \rangle|^2 \qquad (16)$$

For the first time, under the condition in which the stiffness of structure decreases by 10 % at t = 32 and 80 (s), the response x(t) is simulated numerically. Then, the reducing stiffness is tried to identify as shown in Fig.9. This example is for Daubechies wavelet with N = 6.In this figure, the dashed line denotes the targetted value and the rigid line denotes the estimated value by the wavelet analysis. In the first region from zero to 32 (s) of this figure, the difference between both values becomes large because of small sampling data. However, it is clear that the error decreases by 4 % and the wavelet analysis can give the good estimation for the reducing stiffness in another two regions.

CONCLUSIONS

In this study, in order to detect low cycle fatigue signals from the noisy monitored signals under the seismic condition, the wavelet transform is applied to the response of structure. Especially, the model of fatigue is considered to be impulse forces occurring at random intervals by the reduction of stiffness of structural member. Mainly, the identification of occurrence time of impulse and identification of the reducing stiffness are discussed. The results obtained herein are summarized as follows.

(1)The wavelet analysis using the orthonormal wavelet with larger N can give the good estimation for the occurrence times of impulse, namely fatigue signal, under the noisy condition.

(2)The identification technique using wavelet analysis has proven to be accurate for the parameter such as reducing stiffness.

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