11 WCEE

Copyright © 1996 Elsevier Science Ltd
Paper No. 231. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

SIMPLE METHOD FOR AXIAL AND LATERAL IMPEDANCES OF PILE GROUPS

HISASHI NOZOE

Department of Environmental Design, Faculty of Environmental Studies,
Hiroshima Institute of Technology
2-1-1, Miyake, Saeki, Hiroshima, 731-51, Japan

ABSTRACT

A simple method of analysis based on the physically approximate modelling is presented for computing the axial and lateral impedances of pile groups installed in a homogeneous surface stratum lying on rigid bedrock, where the impedances play an essentially important role in coupling to structures with pile foundation exposed to seismic excitation. The presented simple method for dynamic impedances of pile groups possesses the remarkable features that the behaviour of the wide range from stiff and short piles to flexible and long piles is considered in the simple expression with exponents, computation of multi-order inverse matrix or characteristic equation is not required, and the presented mothod is easily executed by the simple closed-form expressions and is applicable to various arrangement of floating piles in a group. The verification of the presented simple method is performed from comparison with the rigorous solution for the dynamic impedances of pile groups oscillating harmonically. The simple method is sufficiently accurate and readily useful in practical application without much computational effort.

KEYWORDS

Pile group; axial and lateral impedances; impedance group factor; pile-soil-pile system; dynamic interaction factor; simple method of analysis; physically approximate modelling; simple closed-form expression; floating pile; stiff and short pile; flexible and long pile

INTRODUCTION

In recent years, in order to predict responses of structures with pile foundation exposed to seismic excitation, a amount of work has been done on estimation for dynamic impedances of pile groups which play an important role in coupling to superstructures. It is desirable in practical situation that the dynamic responses of structures with pile foundation are readily predicted in consideration of the complex pile-soil-pile interaction. Simplified estimations for the dynamic impedances of pile groups have been performed mainly in connection with 1) the dynamic Winkler spring along the individual single pile and 2) the pile-to-pile dynamic interaction as follows:

1) The dynamic Winkler springs along the individual single pile are necessary for evaluation of subgrade reactions which are expressed as the product of spring constant and soil displacement on the pile circumference. The Winkler assumption has been early introduced for single piles, and the Winkler spring constants have been analytically obtained from the dynamic plane strain solution of the soil without the static state by Novak *et al.* (1978). The Winkler spring constants obtained consistently in the static and dynamic states have been also estimated from physical and analytical approximation based on the dynamic Kelvin's solution by Nozoe *et al.* (1992a, b). Several researchers have presented simple approximations of distributed springs and dashpots.

2) The pile-to-pile dynamic interaction affects substantially the behaviour of pile groups. The interaction factors are defined as the ratio of the displacements of receiver pile to source pile through the response of the soil. The interaction factors have been approximately estimated as the soil displacements at the axis of the receiver pile on the basis of the radial propagation of the cylindrical waves, and a simple method for computing the dynamic impedances of pile groups from the results of single piles has been proposed by Dobry *et al.* (1988). Afterward, Gazetas *et al.* (1991) have pointed out that the assumption of synchronous wave emission introduced into the interaction factor is unsatisfied for much longer and softer piles. Makris *et al.* (1992) have indicated that the interaction factor as the response of the receiver pile to be infinitely long becomes smaller than the interaction

factor as the response of the soil at the axis of the pile. The interaction factors are approximately expressed in the form involving the ratio of the rigidities of soil to pile by Hijikata *et al.* (1994).
The above researchers have differently developed simple methods for the dynamic impedances of pile groups on the basis of the method of Dobry *et al.* (1988). On these simple methods, as the number of pile in the group n is larger, the computaional effort of the inverse matrix of the n th order is used up more costly.
By contrast simplified methods for the dynamic impedances of pile groups have been presented employing the Winkler springs and the interaction factors estimated analytically on the basis of the plane strain solution by Nogami (1983) and of the Kelvin's solution by Nozoe *et al.* (1992a). Since the characteristic equation of the n th order must be solved, much computational effort is also used up.
In this paper, a simple method for the axial and lateral impedances of pile groups is presented physically and analytically, and adapted for flexible and long piles as well as stiff and short piles without computation of the inverse matrixthe of the n th order or without solution of the characteristic equation of the n th order.

DESCRIPTION OF MODEL AND FORMULATION

An analytical model of a pile–soil–pile system is illustrated in Fig. 1 as the cylindrical and Cartesian coordinates. The model is considered for a floating pile group installed in a surface stratum lying on rigid bedrock under vertical and horizontal vibrations. The soil deposit is elastic, homogeneous and isotropic with the linear hysteretic damping. Each pile in the group is identical and assumed to be elastic rod and beam. The pile j is subjected to the hormonic loadings exp($i\omega t$) at the pile head.

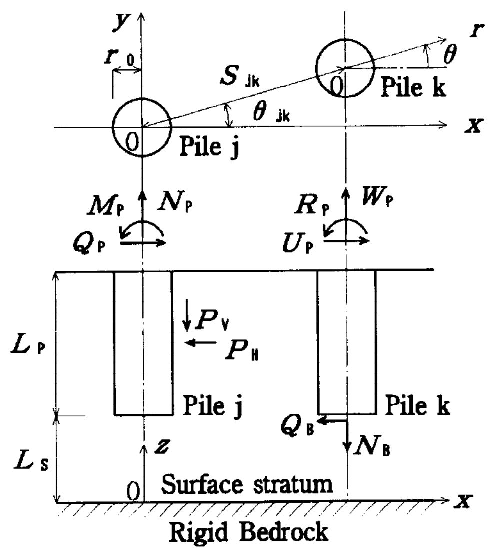


Fig. 1. Model of pile–soil–pile system

The equations of motion and the constitutive relationships with respected to the vertical and horizontal displacements W_{Pj} and U_{Pj}, respectively, and the rotational angle R_{Pj} are expressed as

$$\frac{dN_{Pj}}{dz_P} - P_{Vj} = -\rho_P A_P \omega^2 W_{Pj} \; ; \frac{dW_{Pj}}{dz_P} = \frac{N_{Pj}}{E_P A_P} \tag{1}$$

$$\frac{dQ_{Pj}}{dz_P} - P_{Hj} = -\rho_P A_P \omega^2 U_{Pj} \; ; \frac{dU_{Pj}}{dz_P} = -R_{Pj} \tag{2a}$$

$$\frac{dM_{Pj}}{dz_P} - Q_{Pj} = 0 \qquad ; \frac{dR_{Pj}}{dz_P} = \frac{M_{Pj}}{E_P I_P} \tag{2b}$$

where i is the imaginary unit, the time factor exp($i\omega t$) is abbreviated for convenience, and $z_P = z - L_S$. N_{Pj} and Q_{Pj} are axial and shear forces, respectively, and M_{Pj} is bending moment. ρ_P is mass density of pile, E_P is Young's modulus of pile, A_P is cross sectional area, and I_P is second moment of area. P_{Vj} and P_{Hj} are the total soil reactions per unit length along the shaft of the pile in the z and x directions, respectively. Herein the shearing deformatation and the rotational inertia force of the pile are neglected to be substantially small.

The total soil displacements W_j and U_j on the pile circumference, the total soil reactions P_{Vj} and P_{Hj} along the pile shaft, and the total soil reactions N_{Bj} and Q_{Bj} at the pile tip in the z and x directions, respectively, can be obtained from superposing the behaviours of the soil in the solitary pile j–to–soil system and in the other solitary pile k–to–soil system:

$$W_j = W_{jj} + \sum_{k \neq j} W_{jk} \quad \text{and} \quad U_j = U_{jj} + \sum_{k \neq j} U_{jk} \tag{3}$$

$$P_{Vj} = P_{Vjj} + \sum_{k \neq j} P_{Vjk} \quad \text{and} \quad P_{Hj} = P_{Hjj} + \sum_{k \neq j} P_{Hjk} \tag{4}$$

$$N_{Bj} = N_{Bjj} + \sum_{k \neq j} N_{Bjk} \quad \text{and} \quad Q_{Bj} = Q_{Bjj} + \sum_{k \neq j} Q_{Bjk} \tag{5}$$

Rotatinal quantities of the soil, herein, are not taken into account because the quantities may be negligibly small.
By solving Eqs. (1), (2a) and (2b) to take account of Eqs. (3) to (5), of the continuity condition of the displacements between the pile j and the soil, i.e. $W_{Pj} = W_j$ and $U_{Pj} = U_j$, and of the boundary conditions at the pile head and tip, the impedance matrix $[K]$ referred to the rigid massless cap of the pile group is obtained from the following definition:

$$\begin{Bmatrix} N_C \\ Q_C \\ M_C \end{Bmatrix} = \begin{bmatrix} K^G_{VV}, & 0, & 0 \\ 0, & K^G_{HH}, & K^G_{HR} \\ 0, & K^G_{RH}, & K^G_{RR} \end{bmatrix} \begin{Bmatrix} W_C \\ U_C \\ R_C \end{Bmatrix}; \; K_{HR} = K_{RH} \tag{6}$$

where N_C, Q_C and M_C are vertical, horizontal and moment loadings harmonically acting at the middle point of the cap, respectively, and then W_C, U_C and R_C occur as the corresponding responses of the cap.

PHYSICALLY APPROXIMATE MODELLING

In the boundary-value problem of the above analytical model, the exact solution of the soil reaction and the pile-to-pile interaction through the soil as three dimensional continuum requires the huge computational process and may be rejected in the practical application. Thus the surrounding soil is assumed to be the Winkler type medium introduced by Novak *et al.* (1978) as a physically approximate modelling for the soil. The soil reaction acting on the pile is simply expressed as the product of the dynamic Winkler spring constant and the displacement of the soil. The soil reactions P_{Vjj} and P_{Hjj} along the pile shaft, and N_{Bjj} and Q_{Bjj} at the pile tip occur due to the soil displacements W_{jj} and U_{jj} with the vertical and horizontal motions of the solitary pile j. That is

$$P_{Vjj} = K_{CV} W_{jj} \quad \text{and} \quad P_{Hjj} = K_{CH} U_{jj} \tag{7}$$

$$N_{Bjj} = K_{BV} W_{jj} \quad \text{and} \quad Q_{Bjj} = K_{BH} U_{jj} \tag{8}$$

where K_{CV}, K_{CH}, K_{BV} and K_{BH} are complex spring constants, of which real and imaginary parts are modelled as spring and dashpot to be dependent on frequency, respectively. K_{CV} and K_{CH} are analytically derived in the closed form from the solutions approximately satisfied the conditions of free surface and circular cross section to remain during motion on the basis of the dynamic Kelvin's solutions by Nozoe *et al.* (1992a, b). K_{BV} and K_{BH} are also obtained as the simple expressions calibrated from Kausel's semi-analytical formulae for a rigidly circular foundation on a stratum over rigid bedrock.

By contrast the motion of the other source pile k affects into the behaviour of the receiver pile j due to the pile-to-pile dynamic interaction through the soil. The soil reactions P_{Vjk} and P_{Hjk} along the pile shaft, and N_{Bjk} and Q_{Bjk} at the pile tip due to the motion of the other solitary pile k are approximately derived from the equations of motion and the constitutive relationships for a soil column, which replaces the pile j for no reflection of the incident wave on the receiver pile j, as well as Eqs. (1) and (2) of the pile:

$$P_{Vjk} = E_S A_S \frac{d^2 W_{jk}}{d z_P^2} + \rho_S A_S \omega^2 W_{jk} \tag{9a}$$

$$P_{Hjk} = -E_S I_S \frac{d^4 U_{jk}}{d z_P^4} + \rho_S A_S \omega^2 U_{jk} \tag{9b}$$

$$N_{Bjk} = E_S A_S \frac{d W_{jk}}{d z_P} \quad \text{and} \quad Q_{Bjk} = -E_S I_S \frac{d^3 U_{jk}}{d z_P^3} \tag{10}$$

where ρ_S is mass density of soil, E_S is Young's modulus of soil, A_S is circular cross section and I_S is second moment of circular cross section. These approximate estimations can be utilized for $a_0 = \omega r_0 / V_S \leqq 0.5$ in which V_S is shear wave velocity of soil.
The soil displacements W_{jk} and U_{jk} on the circumference of the receiver pile j are approximately evaluated at the axis of the soil culumn replacing the pile j from the wave emission of the soil due to the soil displacements W_{kk} and U_{kk} on the circumference ($r = r_0$) of the source pile k, and are expressed as

$$W_{jk} = T_{Vjk} W_{kk} \quad \text{and} \quad U_{jk} = T_{Hjk} U_{kk} \tag{11}$$

Although the interaction factors T_{Vjk} and T_{Hjk} have been derived from the same solutions as the above estimated Winkler spring constants by Nozoe *et al.* (1992a), herein, the interction factors are utilized to be partially calibrated the simple expressions adopted by Dobry *et al.* (1988) for the sake of the more simplicity. Thus the interaction factors are expressed by locating the axis of the pile k at the origin and the axis of the pile j at $(r, \theta) = (S_{jk}, \theta_{jk})$:

$$T_{Vjk} = \phi(S_{jk}) \quad \text{and} \quad T_{Hjk} = \phi(S_{jk}) \cos^2\theta_{jk} + \psi(S_{jk}) \sin^2\theta_{jk} \tag{12}$$

where also $T_{Vjk} = 1$ and $T_{Hjk} = 1$.
The attenuation functions $\phi(r)$ and $\psi(r)$ in Eqs. (12) are simply expressed as

$$\phi(r) = \left[\frac{r}{r_0}\right]^{1/2} \exp[-(\xi + i)\kappa_L(r - r_0)] \tag{13a}$$

$$\psi(r) = \left[\frac{r}{r_0}\right]^{1/2} \exp[-(\xi + i)\kappa_S(r - r_0)] \tag{13b}$$

where wave numbers: $\kappa_L = \omega / V_{La}$ and $\kappa_S = \omega / V_S$, Lysmer's analog wave velocity: $V_{La} = 3.4/[\pi(1 - \nu_S)] V_S$, and shear wave velocity: $V_S = (\mu / \rho_S)^{1/2}$. μ and ν_S are shear modulus and Poisson's ratio of soil, respectively. ξ is ratio of hysteretic damping in the soil and defined as complex modulus $\mu^* = \mu(1 + 2i\xi)$. The wave emissions propagate approximately with V_{La} as the compression–extension wave in the direction of horizontal loading, and with V_S both in the perpendicular to the direction of horizontal loading and in the radial direction under vertical loading.

DYNAMIC IMPEDANCES OF PILE GROUPS

Under the above physical approximation of the analytical model, the equations of motion of the pile j are expressed by the soil displacements W_{jj} and U_{jj} due to the motion of the solitary pile j in account of the continuity condition between the displacements of the pile j and the total soil displacements, then the pile–soil–pile interaction problem arrives at the eigen–value problem with respected to W_{jj} and U_{jj}.
Now the soil displacements W_{jj} and U_{jj} are assumed as the following functions.

$$W_{jj} = \overline{A}_{Vj} \exp(\overline{\lambda}_V z_P) \quad \text{and} \quad U_{jj} = \overline{A}_{Hj} \exp(\overline{\lambda}_H z_P) \tag{14}$$

By superposing Eqs. (14) over the group of n piles according with Eqs. (3) in account of Eqs. (11), the total soil displacements W_j and U_j are obtained:

$$W_j = \sum_{k=1}^{n} T_{Vjk} \overline{A}_{Vj} \exp(\overline{\lambda}_V z_P) \quad \text{and} \quad U_j = \sum_{k=1}^{n} T_{Hjk} \overline{A}_{Hj} \exp(\overline{\lambda}_H z_P) \tag{15}$$

Moreover by substituting Eqs. (15) into Eqs. (1), (2a) and (2b) in account of $W_{Pj} = W_j$ and $U_{Pj} = U_j$, the complex characteristic equations of the n th order yield to

$$(\lambda_V^2[\overline{B}_V] + [\overline{C}_V])\{\overline{A}_{Vj}\} = \{0\} \quad \text{and} \quad (\lambda_H^4[\overline{B}_H] + [\overline{C}_H])\{\overline{A}_{Hj}\} = \{0\} \tag{16}$$

From analysis of Eqs. (16), the eigen–values of the l th order: $\overline{\lambda}_{Vl}$ and $\overline{\lambda}_{Hl}$, and the corresponding eigen–vectors $\{\overline{A}_{Vjl}\}$ and $\{\overline{A}_{Hjl}\}$ are determined. Thus the solutions of the soil displacements W_{jj} and U_{jj} are expressed as

$$W_{jj} = \sum_{l=1}^{n} \overline{A}_{Vjl} W(\overline{\lambda}_{Vl} z_P) \quad \text{and} \quad U_{jj} = \sum_{l=1}^{n} \overline{A}_{Hjl} U(\overline{\lambda}_{Hl} z_P) \tag{17}$$

where $W(\overline{\lambda}_{Vl} z_P)$ and $U(\overline{\lambda}_{Hl} z_P)$ are general solutions of the l th order, and the including integral constants are determined from the boundary conditions of the pile head and tip.
The dynamic impedances of pile groups can be computed by the above simplified method.

Since the above simplified method for the dynamic impedances of pile groups is compelled to solve the complex eigen–value problem of the n th order, its computation becomes troublesome as the number of pile n in the group increases. With the intention of computing easier, herein, the physically approximate modelling assumptions are introduced that the subgrade reactions along the shaft and at the tip of the receiver pile j are omitted to be negligibly small, and the inertia of pile is ignored in the interesting low frequency range. Consequently the above eigen–value problem corresponding to the soil displacements W_{jj} and U_{jj} becomes identical with those corresponding to the pile displacements W_{Pj} and U_{Pj}.
By assuming also the pile displacements W_{Pj} and U_{Pj} as the following functions,

$$W_{Pj} = A_{Vj} \exp(\pm \lambda_V z_P) \quad \text{and} \quad U_{Pj} = A_{Hj} \exp[\pm(1 \pm i)\lambda_H z_P] \tag{18}$$

the characteristic equations with respect to the vectors $\{A_{Vj}\}$ and $\{A_{Hj}\}$ are reduced as

$$\{A_{Vj}\} = (\lambda_V/\alpha)^2[T_{Vjk}]\{A_{Vk}\} \quad \text{and} \quad \{A_{Hj}\} = (\lambda_H/\beta)^4[T_{Hjk}]\{A_{Hk}\} \tag{19}$$

where $\alpha = (K_{CV}/E_P A_P)^{1/2}$, $\beta = [K_{CH}/(4 E_P I_P)]^{1/4}$, and $E_P A_P$ and $E_P I_P$ are axial and flexural rigidities, respectively.
By solving Eqs. (19), the pile displacements W_{Pj} and U_{Pj} are also expressed as

$$W_{Pj} = \sum_{l=1}^{n} A_{Vjl} W(\lambda_{Vl} z_P) \quad \text{and} \quad U_{Pj} = \sum_{l=1}^{n} A_{Hjl} U(\lambda_{Hl} z_P) \tag{20}$$

The general solutions of the l th order: $W(\lambda_{Vl} z_P)$ and $U(\lambda_{Hl} z_P)$ with the pile groups become the same with the individual single piles. The additional simplifying assumptions are introduced that the pile head is rotationally restrained, and the

subgrade reactions of Eqs. (8) at the tip of the source pile j are ignored. Consequently the solution respectively satisfied with the boundary conditions at both head and tip of the pile with regard to each mode can be obtained as the trivial solution.
For the pile head, the axial and lateral displacements, and axial and shear forces are simply expressed:

$$\{W^{o}_{Pj}\} = [A_{Vjl}]\{W^{o}_{l}\} \quad \text{and} \quad \{N^{o}_{Pj}\} = [A_{Vjl}]\{K_{Vl}\, W^{o}_{l}\} \tag{21a}$$

$$\{U^{o}_{Pj}\} = [A_{Hjl}]\{U^{o}_{l}\} \quad \text{and} \quad \{Q^{o}_{Pj}\} = [A_{Hjl}]\{K_{Hl}\, U^{o}_{l}\} \tag{21b}$$

where K_{Vl} and K_{Hl} for each mode are equivalent to the axial and lateral impedances.
By forcing $\{W^{o}_{Pj}\} = \{W_C\}$, $\{U^{o}_{Pj}\} = \{U_C\}$ and $\{R^{o}_{Pj}\} = \{R_C\} = \{0\}$ from the condition of the rigidly capped pile group, each modal displacement at the pile head yields from Eqs. (21a) and (21b):

$$\{W^{o}_{l}\} = [A_{Vjl}]^{-1}\{W_C\} \quad \text{and} \quad \{U^{o}_{l}\} = [A_{Hjl}]^{-1}\{U_C\} \tag{22}$$

where $[\ \ ]^{-1}$ indicates inverse matrix.
From the equilibrium between the external loadings and the sum of resistant forces of the pile head in the group,

$$N_C = \{1\}^T\{N^{o}_{Pj}\} = \{1\}^T(K_{V1}[I] + \underline{[A_{Vjl}](K_{Vl} - K_{V1})][A_{Vjl}]^{-1}})\{W_C\}$$
$$\fallingdotseq \{1\}^T(K_{V1}[I])\{1\}W_C = n\,K_{V1}\,W_C \tag{23a}$$
$$Q_C = \{1\}^T\{Q^{o}_{Pj}\} = \{1\}^T(K_{H1}[I] + \underline{[A_{Hjl}](K_{Hl} - K_{H1})][A_{Hjl}]^{-1}})\{U_C\}$$
$$\fallingdotseq \{1\}^T(K_{H1}[I])\{1\}U_C = n\,K_{H1}\,U_C \tag{23b}$$

where $\{1\}$ and $\{1\}^T$ are unit vector and its transpose of the n th order, and all components are unity. $[I]$ is unit matrix of the n th order.

For computing the axial and lateral impedances of pile groups by simpler method than the above simplified method, the eigen-value and vector of the first order ($l = 1$) are adopted only. The underlined terms in Eqs. (23a) and (23b) are neglected to be small, and subscript l is abbreviated for convenience.
To begin with, in order to satisfy with the condition of the rigidly capped pile group: $\{W_{Pj}\} = \{W_C\}$, and $\{U_{Pj}\} = \{U_C\}$, the eigen-vectors are set as

$$\{A_{Vj}\} = \{1\} \quad \text{and} \quad \{A_{Hj}\} = \{1\} \tag{24}$$

By substituting Eqs. (24) into the right hand of Eqs. (19), respectively, the first approximate eigen-vectors can be obtained as

$$\{A_{Vj}\} \fallingdotseq (\lambda_V/\alpha)^2\{T_{Vj}\} \quad ; \quad T_{Vj} = \sum_{k=1}^{n} T_{Vjk} \tag{25a}$$

$$\{A_{Hj}\} \fallingdotseq (\lambda_H/\beta)^4\{T_{Hj}\} \quad ; \quad T_{Hj} = \sum_{k=1}^{n} T_{Hjk} \tag{25b}$$

If the eigen-vectors of Eqs. (25a) and (25b) coincide with the assumed eigen-vectors of Eqs. (24), the solutions result in correctness and the eigen-values become constant with no relation to the pile. However, in the case that the eigen-vectors of Eqs. (25a) and (25b) are approximate vectors, the eigen-values differ from each pile and can not be determined as constant. Therefore by superposing each component of the first approximate eigen-vectors of Eqs. (25a) and (25b), respectively, and also of the assumed eigen-vectors of Eqs. (24), the equations of the condition approximately satisfied at the pile head as a whole of the pile group are derived:

$$\sum_{j=1}^{n} A_{Vj} = (\lambda_V/\alpha)^2 \sum_{j=1}^{n} T_{Vj} = n \quad \text{and} \quad \sum_{j=1}^{n} A_{Hj} = (\lambda_H/\beta)^4 \sum_{j=1}^{n} T_{Hj} = n \tag{26}$$

From the above equations, the approximate eigen-values are determined as constant:

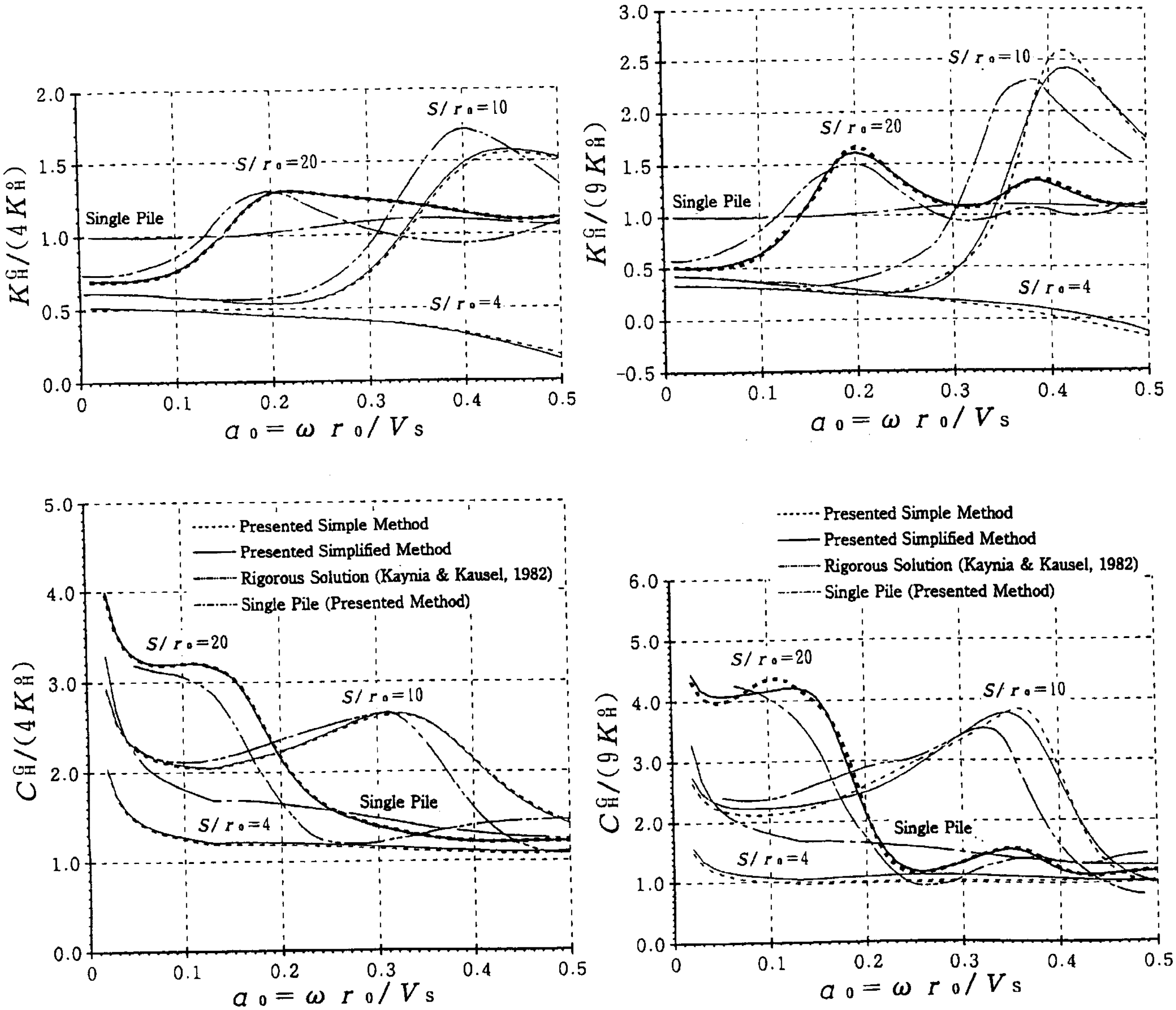
$$\lambda_V/\alpha = (n/\sum_{j=1}^{n} T_{Vj})^{1/2} \quad \text{and} \quad \lambda_H/\beta = (n/\sum_{j=1}^{n} T_{Hj})^{1/4} \tag{27}$$

Alternatively, the axial and lateral impedances of single piles sufficiently depend on the characteristic parameters αL_P and βL_P. Thus the impedances of single piles can be simply expressed as follows:

$$K_{VV} = \alpha E_P A_P (\alpha L_P)^{mv} \quad \text{and} \quad K_{HH} = 4\beta^3 E_P I_P (\beta L_P)^{mh} \tag{28}$$

where L_P is pile length. The exponents 'mv' and 'mh' are equal to unity for the short-stiff pile behaving as rigid pile: Re(αL_P) < 0.3 and Re(βL_P) < 0.5, and tend to zero for the long-flexible pile behaving as infinitely long pile: Re(αL_P) and Re(βL_P) > 3. For the other pile, $0 <$ 'mv' and 'mh' < 1.
Since K_{V1} and K_{H1} of the first mode can be expressed as well as the impedances of single piles by Eqs. (28), the axial and lateral impedances of pile groups can be obtained from Eqs. (23a) and (23b). That is

$$K^{G}_{VV} = n\,\lambda_V E_P A_P (\lambda_V L_P)^{mv} \quad \text{and} \quad K^{G}_{HH} = 4n\,\lambda_H^3 E_P I_P (\lambda_H L_P)^{mh} \tag{29}$$



a) 2 × 2 square pile group

b) 3 × 3 square pile group

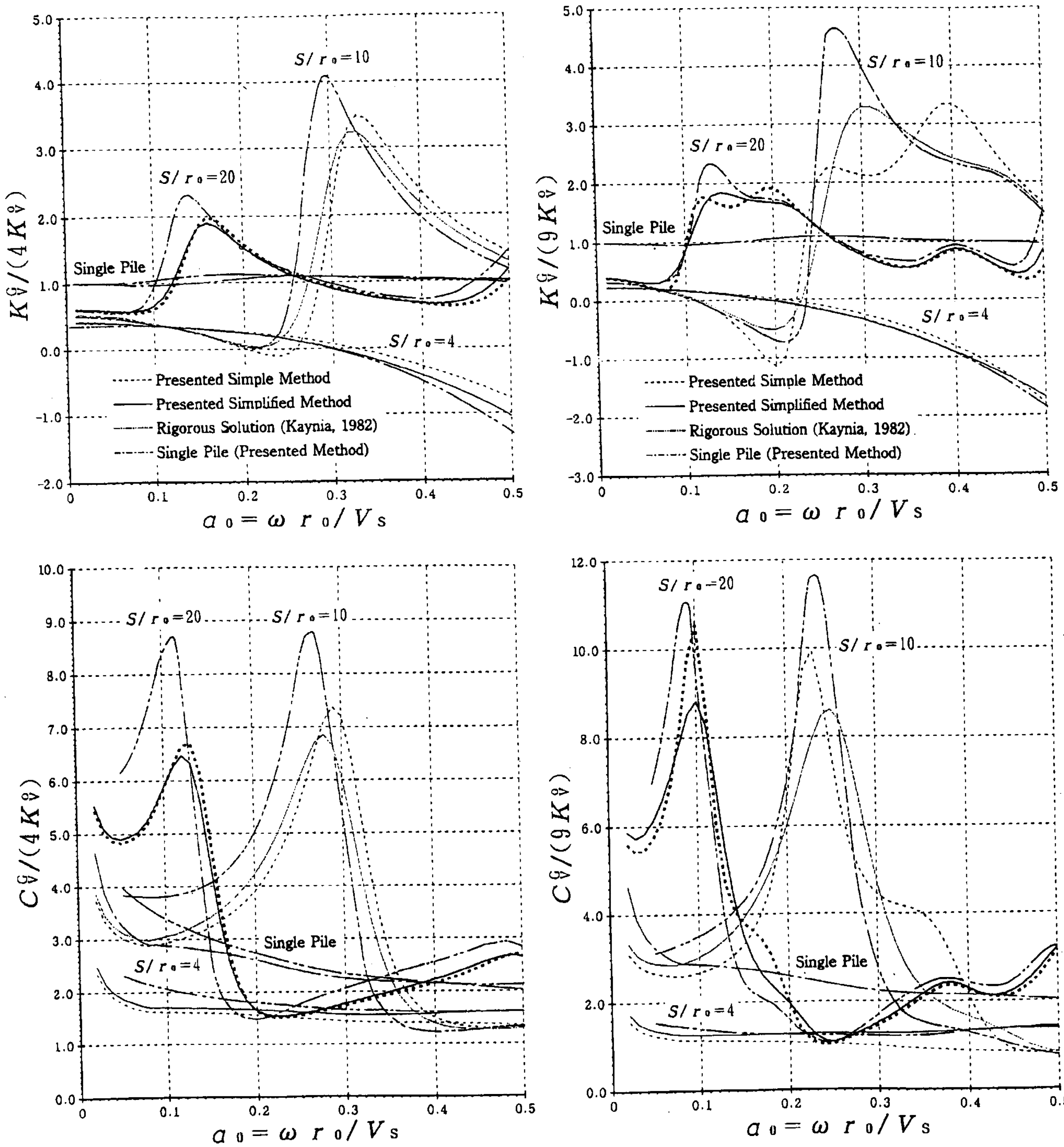
Fig. 2. Lateral impedance group factors with frequency

For the purpose of ready computaion, the exponents 'mv' and 'mh' are made use of the values determined from the real part of the static impedances, i.e. spring constants, of single piles.
The impedance group factors are defined as the ratio of the impedances of the pile group to the individual single pile. That is

$$K^{G}_{VV}/(n\,K_{VV}) = (\lambda_{V}/\alpha)^{(mv+1)} = (n/\sum_{j=1}^{n} T_{Vj})^{(mv+1)/2} \tag{30a}$$

$$K^{G}_{HH}/(n\,K_{HH}) = (\lambda_{H}/\beta)^{(mh+3)} = (n/\sum_{j=1}^{n} T_{Hj})^{(mh+3)/4} \tag{30b}$$

The presented simple method for the axial and lateral impedances of pile groups is justified only for regular polygon pile groups oscillating axially, and for 1 × 2, 2 × 1 and 2 × 2 pile groups oscillating laterally within the category of the above simplified method. Additionally, the presented simple method coincides with Dobry's simple method for the short-stiff pile: i.e. exponents 'mv' = 'mh' = 1. It is explained from the presented simple method that the interacrion factor adoped by Dobry *et al.* (1988) is unsatisfied for the long-flexible pile: i.e. exponents 'mv' = 'mh' = 0. This point of view for much longer and softer pile has been also discussed by Gazetas *et al.* (1991) and Makris *et al.* (1992).
The presented simple method is applicable to various arrangement of piles in a group and utilizable for short-stiff piles to long-flexible piles indicated by the exponents 'mv' and 'mh'.



a) 2 × 2 square pile group

b) 3 × 3 square pile group

Fig. 3. Axial impedance group factors with frequency

RESULTS OF IMPEDANCES AND DISCUSSIONS

For verification of the simplified method and the simple method proposed herein, comparison of the presented two methods with the rigorous solution of Kaynia *et al.* (1982) adapted from Dobry *et al.* (1988) is shown in Figs. 2 and 3 for a 2 × 2 and a 3 × 3 square pile group in a homogeneous halfspace. Each pile is rigidly capped. The analytical parameters are as follows: Poisson's ratio $\nu_s = 0.4$ and the hysteretic damping ratio $\xi = 0.05$ of soil, the slenderness ratio $L_P / r_o = 30$, the ratio of Young's moduli of pile to soil $E_P / E_s = 1000$, and the ratio of mass densities of soil to pile $\rho_s / \rho_P = 0.7$, and the closest spacing of the axis-to-axis of the pile $S / r_o = 4$, 10 and 20. From the exponents 'mv' = 0.99 and 'mh' = -0.01, the pile is classified as the short-stiff pile under the vertical vibration and as the long-flexible pile under the horizontal vibration.

The axial and lateral impedances of pile groups are expressed as follows:

$$K^{G}_{VV} = K^{G}_{V} + i\, a_0 C^{G}_{V} \quad \text{and} \quad K^{G}_{HH} = K^{G}_{H} + i\, a_0 C^{G}_{H} \tag{31}$$

The impedance group factors in these figures are presented as the ratio of the spring constants of the pile group K^{G}_{V} and K^{G}_{H}, and as the ratio of the damping coefficients of the pile group C^{G}_{V} and C^{G}_{H} to the sum of the static spring constants of the individual single pile $n\,K^{o}_{V}$ and $n\,K^{o}_{H}$, respectively.

For the 2 × 2 pile group, the results of the presented simple method, which is analytically justified in this case, agree well with those of the presented simplified method for both axial and lateral impedances. The presented simple method is effective in the low frequency range, i.e. $a_0 < 0.5$. It is seen that the introduced assumptions, i.e. the influences of the subgrade reactions along the shaft and at the tip of the receiver pile and at the source pile tip on the impedances are negligibly small and the effect of the inertia of pile can be ignored, are appropriate for the simple method. Alternatively the results of the presented two methods for the axial and lateral impedances agree well with those of the rigorous solution, and the presented simple method is excellent as well as the presented simplified method.

For the 3 × 3 pile group, the presented two methods for the lateral impedance are in satisfactory agreement, and the results of the presented two methods for the lareral impedance are sufficiently accurate in comparison with those of the rigorous solution. It is seen that the presented simple method for the lateral impedance is reasonable for the long–flexible pile, i.e. exponent 'mh' = 0. On the other hand, though the frequency, at which the difference between the presented two methods for the axial impedance is obseved, exsists partially, the results of the presented two methods for the axial impedance almost differ from those of the rigorous solution, and the presented two method are generally applicable. It is seen that the presented simple method for the axial impedance is also reasonable for the short–stiff pile, i.e. exponent 'mv' = 1.

CONCLUSIONS

A simple method for computing the axial and lateral impedances of pile groups has been presented in the physical and analytical procedure as well as a simplified mehod. Although the simplified method is compelled to solve the characteristic equations of the n th order, the simple method does not require computing the characteristic equation or the inverse matrix and is easily executed by the simple closed–form expressions. The presented simple method is applicable to various arrengement of floating piles in a group, and is taken account of the behaviour of the wide range from stiff and short piles to flexible and long piles in the simple expressions with exponents. The verification of the presented simple method is performed from comparison with the rigorous solution. Consequently the simple method is sufficiently accurate and readily useful in practical application without much computational effort. This application to the preliminary design and so on may offer a good insight into the prediction of responses of structures with pile foundation exposed to seismic excitation.

REFERENCES

Dobry, R. and Gazetas, G. (1988). Simple Method for Dynamic Stiffness and Damping of Floating Pile Groups. Geotechnique, Vol. 38, No. 4, pp.557–574.

Gazetas, G. and Makris, N. (1991). Dynamic Pile–Soil–Pile Interaction, Part I: Analysis of Axial Vibration. Earthquake Engineering and Structural Dynamics, Vol. 20, pp. 115–132.

Hijikata, K., Yagishita, F. and Tomii, Y. (1994). Simple Method for Dynamic Impedance of Pile Group. Journal of Structural and Construction Engineering, Transactions of Architectural Institute of Japan, No. 455, pp.73–82.

Makris, N. and Gazetas, G. (1992). Dynamic Pile–Soil–Pile Interaction, Part II: Lateral and Seismic Response, Earthquake Engineering and Structural Dynamics, Vol. 21, pp. 145–162.

Nogami, T. (1983). Dynamic Group Effect in Axial Responses of Grouped Piles. Journal of Geotechnical Engineering Division, ASCE, Vol. 109, No. 2, pp.228–243.

Novak, M., Nogami, T. and Aboul–Ella, F. (1978). Dynamic Soil Reaction for Plane Strain Case. Journal of Engineering Mechanics Division, ASCE, Vol. 104, No. EM4, pp.953–959.

Nozoe, H. and Fukusumi, T. (1992a). A Simplified Estimation of Impedances for Pile Groups under Vertical and Horizontal Vibrations. Proc. of Tenth WCEE, Vol. 3, pp.1771–1776.

Nozoe, H., Kusakabe, K. and Fukusumi, T. (1992b). A Simplified Estimation for Characteristics of Single Piles under Vertical and Horizontal Vibrations. Journal of Structural and Construction Engineering, Transactions of Architectural Institute of Japan, No. 438, pp. 49–64.