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DISPLACEMENT OF RIGID RETAINING WALLS DURING EARTHQUAKES

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ABSTRACT

Rigid retaining walls experience displacements in sliding and rotation during an earthquake. A widely used displacement based method of design considers sliding displacements only (Richard and Elms 1978). Prakash, Wu, and Rafnsson (1995) developed design charts for such walls considering both sliding and rocking displacements. Both soil nonlinearity and frequency dependent stiffness and radiation damping of the base soil and backfill have been considered. Two retaining walls which suffered failure during the Kobe and Hokkaido-Nansi-Oki earthquakes have been analyzed based on these techniques, which show promise in prediction of the real behavior.

KEYWORDS

Retaining wall, seismic displacements, Kobe earthquake, Hokkaido-Nansi-oki earthquake, design of wall.

INTRODUCTION

Conventional static design of rigid retaining walls requires estimating the earth pressure behind a wall and choosing the wall geometry to satisfy specified factors of safety against sliding, overturning and bearing capacity failure based upon earth pressure theories of Coulomb and Rankine. The factors of safety will be decreased due to increased dynamic pressure under earthquake loading. The magnitude of these dynamic increases was studied by Okabe (1926) and Mononobe and Matsuo (1929). The earth pressure can be defined as an active condition when the earthquake acceleration is towards the backfill and the wall moves away from it. The walls moved laterally, tilted and settled in several past earthquakes. Table 1 shows listing of damage of walls during Hokkaido-Nansi-Oki, Northridge and Kobe earthquakes.

Richards and Elms (1978) proposed a design procedure which considers displacements of walls in sliding only. Rafnsson (1991) developed solutions to predict horizontal movements at the top of a retaining wall under dynamic loading due to simultaneous sliding and rocking motion. Rafnsson's work has been advanced to develop design charts for permissible displacement of walls 4m - 10m high and for 21 backfill and base soils (Wu 1995, Prakash *et al.* 1995).

Two retaining walls which experienced damage during recent earthquakes have been analyzed. Their predicted behavior shows that the procedure developed and the model used is capable of predicting the real behavior.

MODEL USED

Rafnsson (1991) used a mathematical model which considered displacements both in the active case (Fig. 1) and passive case (not shown). Nonlinear behavior of soil is included in defining the following properties, both at the base as well as the backfill:

(1) Soil stiffness in sliding.
(2) Material damping in sliding.
(3) Geometrical damping in sliding.
(4) Soil stiffness in rocking.
(5) Material damping in rocking.
(6) Geometrical damping in rocking.

Table 1. Failure and movement of retaining structures during recent earthquakes.

Earthquake	M	Year	Place	Wall type	H	A_{hmax}	Base soil	Report damage
Hokkaido-Nansi-Oki (Chung, 1995)	6.7	1993	Kamiiso		4m	0.2g		rotate 1 or 2 degree
			Oshamanbe	Combination of retaining and sea walls	?	0.2g		rotate 2 - 3 degree settle 0.1m
Northridge (Dames and Moore, 1994, Stewart *et al.*, 1994)	6.7	1994	King harbor Marian	Concrete *RW	1.5m	0.2g	Holocene sediments	move outward 5 - 6m
			Universal Center Drive Crossing	Southern-most crib wall	12m	0.35g	Rock-undifferentiated consolidated sedimentary basement rocks	5cm settlement
			Woodland Hills	Battered concrete crib walls	9m	0.6g	Holocene sediments	complete failure
			Woodland Hills	Conventional concrete RW	5m	0.6g	Holocene sediments	complete failure
Great Hanshin Awaji-(Kobe) (Tateyama *et al.*, 1995)	7.2	1995	Kobe city area	Masonry RW	4m	0.5g	Gravel and sand	complete failure
				Lean-type unreinforced concrete RW	2.6m	0.5g	Gravel and sand	complete overturning
					5m	0.5g	reclaimed land	totally overturned

A_{hmax} = maximum horizontal acceleration
* RW = retaining wall

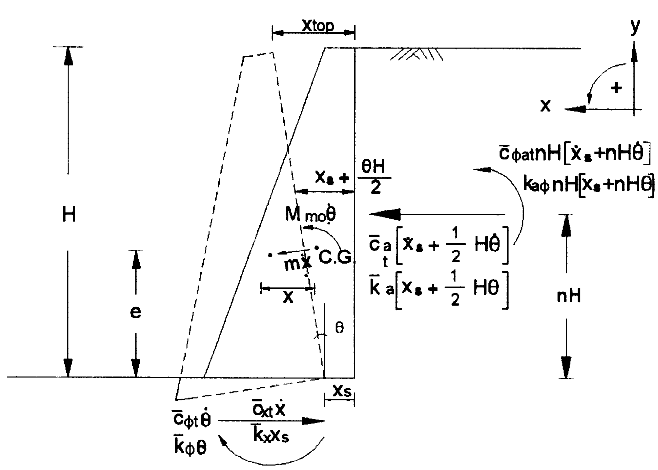

Fig. 1 Mathematical model for stiffness and damping constants for the active case (Rafnsson 1991).

The equations of motion for active case and horizontal sliding and rotation were written as:

$$m\ddot{x}_S + c_X \dot{x}_S + k_X x_S + me\ddot{\theta} - c_{HS}\dot{\theta} - k_{HS}\theta = P_X(t) \quad (1)$$

$$M_{mo}\ddot{\theta} + c_R\dot{\theta} + k_R\theta - c_{HR}\dot{x} - k_{HR}x_S = M_X(t) \quad (2)$$

In the above equations, m represents the mass of the wall, M_{mo} - the mass moment of inertia, x_s the horizontal displacement, θ the angular rotation, and c the dynamic damping. Subscripts "HS" and "HR" represent total damping for backfill in sliding and rocking respectively, subscript "x" sliding, and "R" rotation. The stiffness (k) and damping (ξ) in several modes are both strain and frequency dependent and have been presented elsewhere (Rafnsson 1991 and Rafnsson and Prakash 1991).

The displacements are computed using non-linear soil modulus and material damping (Fig. 2, 3). In this model, the sinusoidal motion is used. The wall moves away from the backfill (active condition) when the earthquake acceleration is towards the backfill, and it moves toward the backfill (passive condition) when the earthquake acceleration is away from the backfill. The latter is however negligible as compared to the former. The wall is assumed to rotate about its heel (Fig. 1). The mass of the backfill material participating in the wall motion is neglected.

Prakash's *et al.* (1995) solution computes first base width based for required factors of safety (Fang, 1991) under static loading as:

sliding	≥ 1.5	bearing capacity	≥ 2.5
overturning	≥ 1.5	eccentricity	≤ B/6

G_{max} is computed using Hardin and Black (1969) expression for maximum shear modulus as:

$$G_{max} = 3230\ OCR^k \frac{(2.973-e)^2}{1+e} \bar{\sigma}_o^{\frac{1}{2}} \qquad (kN/m^2) \quad (3)$$

where OCR is overconsolidation ratio, e is void ratio, $\bar{\sigma}_o$ is mean effective pressure (kN/m^2) and k is a constant and depends upon plasticity index of clays. Displacements are then computed for given input motion.

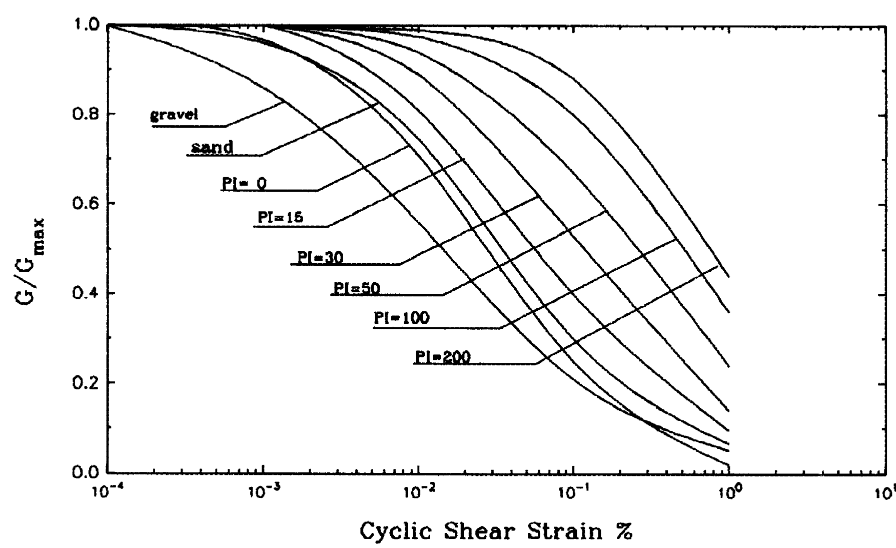

Fig. 2 Average values of G/G_{max} versus shear strain (γ) for different types of soils (After Seed and Idriss 1970, for sand; Seed, Wong, Idriss and Tokimatsu 1986, for gravel; Vucetic and Dobry 1991, for PI=0-200).

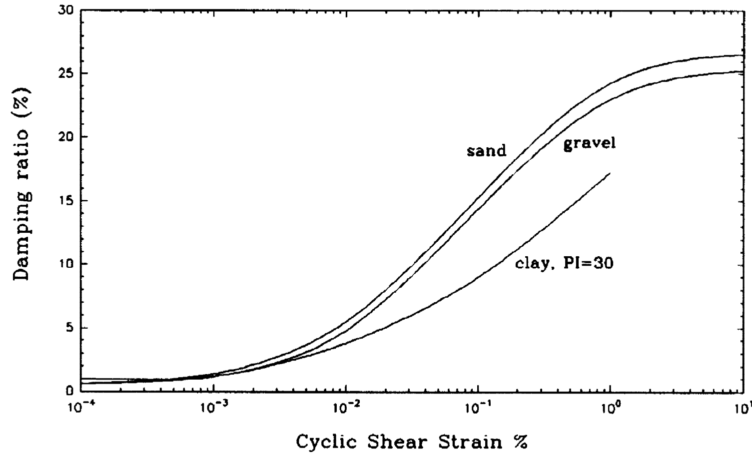

Fig. 3 Average values of damping ratio (ζ) vs. shear strain (γ) for gravel, sand and clay (PI=30, After Seed and Idriss 1970, for sand; Seed, Wong, Idriss and Tokimatsu 1986, for gravel; Vucetic and Dobry 1991, for PI=30).

ANALYSIS OF WALLS IN RECENT EARTHQUAKES

Wall in Kobe Earthquake

A 4m high masonry retaining wall with base soil as gravel and sand (see Table 1) experienced complete failure during the Kobe earthquake (M=7.2, $\alpha_{hmax} = 0.5$) of January 17, 1995 on the JR Tohkaido line (Tateyama, *et al.* 1995). The equivalent number of cycles for M=7.2 at $0.65\alpha_{hmax}$ ($\alpha_h = 0.33$) is 13 (Seed *et al.* 1983). No details of the section and soil properties except a mention of alluvial fan deposit consisting of mainly gravel and sand are listed. Therefore, several analyses will be performed based on assumed data. The assumed soil properties at the time of the construction are as in Table 2.

Table 2. Two-sets of soil properties for analysis.

Set No.	Location	USSC	γ_t kN/m^3	w%	void ratio	Gs	$\phi°$	$\delta°$	α_{hmax}	α_h in analysis
1st	Backfill	GP	21.2	8	0.35	2.7	35	23.3	0.5	0.33
	Base soil	GP	21.2	8	0.35	2.7	35	23.3	0.5	0.33
2nd	Backfill	SP	20.23	8	0.6	2.7	30	23.3	0.5	0.33
	Base soil	SP	20.23	8	0.6	2.7	30	23.3	0.5	0.33

a. Based on the above properties and for factors of safety listed previously, the base width for 4 m high wall is 1.9590m for first set and 2.6839m (Fig. 4) for the second set of soil properties is in Table 2. The response of the 2 sections is shown in the Fig. 5. The displacements for 13 cycles of 0.33g acceleration are 21.7cm and 29.3cm for sections of Figure 4a an 4b respectively.

b. Further analyses will be performed by using the provisions of EUROCODE-8 which include:

E7. Dynamically (highly) pervious soil below the water table - earth pressure coefficient.
The following parameters apply:

$$\gamma^* = \gamma - \gamma_w \tag{4}$$

$$\tan\psi = \frac{\gamma_d}{\gamma - \gamma_w} \times \frac{\alpha_h}{1 \mp \alpha_v} \tag{5}$$

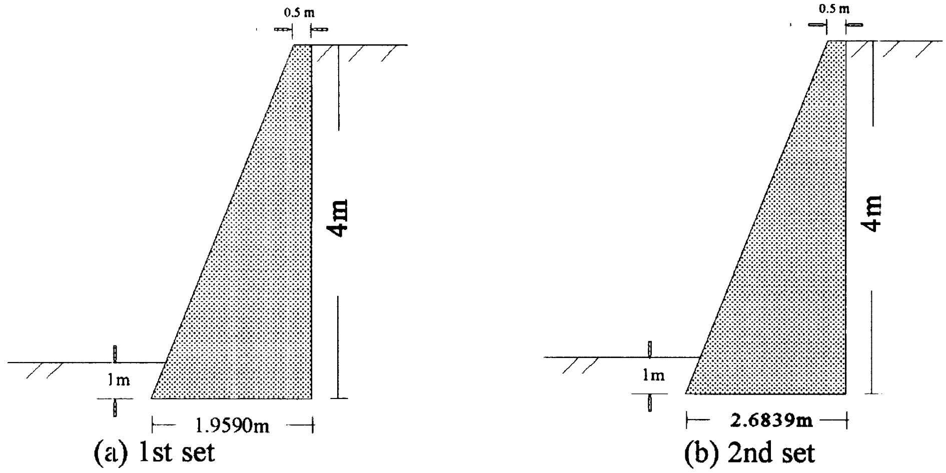

Fig. 4 Section of the wall during Kobe earthquake.

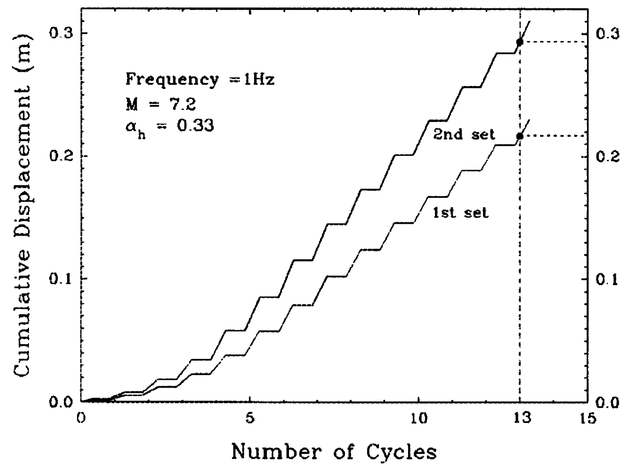

Fig. 5 The computed displacement for the walls designed by static FOS.

$$E_{wd} = 7/12\ k_h \gamma_w H'^2 \tag{6}$$

E_{wd} is hydrodynamic water force where H′ is the height of the water table from the base of the wall.

c. It is assumed that the water table is at the base of the wall. The effect of the E_{wd} in (b) can be neglected. It is further assumed that the base soil of the 1st set (Table 2) may have large variation as shown in Table 3.

Table 3. Parameters of the 1st set base soil used in further analysis of wall during Kobe earthquake.

ϕ	35	34	33	32	31	30	29	28	27
e	0.35	0.4	0.45	0.5	0.55	0.6	0.65	0.7	0.75
γ_{sat} kN/m^3	22.16	21.72	21.31	20.93	20.57	20.23	19.92	19.62	19.34

As shown in the Table 3, the strength of base soil and γ_{sat} decrease and void ratio increases. Fig. 6 shows the effect of the base soil strength on cumulative displacements for section in Fig. 4a. This analysis shows that the displacements would increase from 23.69cm to 48.34cm if ϕ decrease from 35° to 27°. α_h for these computation is 0.33 (or ψ = 29.63°).

d. It is reasonable that angle ψ may change during on earthquake as shown in Table 4. Therefore for ϕ =35° and ψ varying from 9° to 34°, the increase in cumulative displacements is shown in Fig. 7 by 'a' and for ϕ = 30° and ψ varying from 11.6° to 29.88° by 'b'.

Tables 4. Values of ψ at different α_h for various of ϕ.

Set	ϕ	γ_d kN/m^3	α_h(g)	0.1	0.15	0.2	0.25	0.28	0.3	0.33	0.4
1st	35°	19.63	ψ° (eq5)	9.78	14.49	19.02	23.31	25.76	27.34	29.63	34.58
2nd	30°	16.56	ψ° (eq5)	11.6	17.11	22.31	27.16	29.88	(31.63)	(34.10)	(39.38)

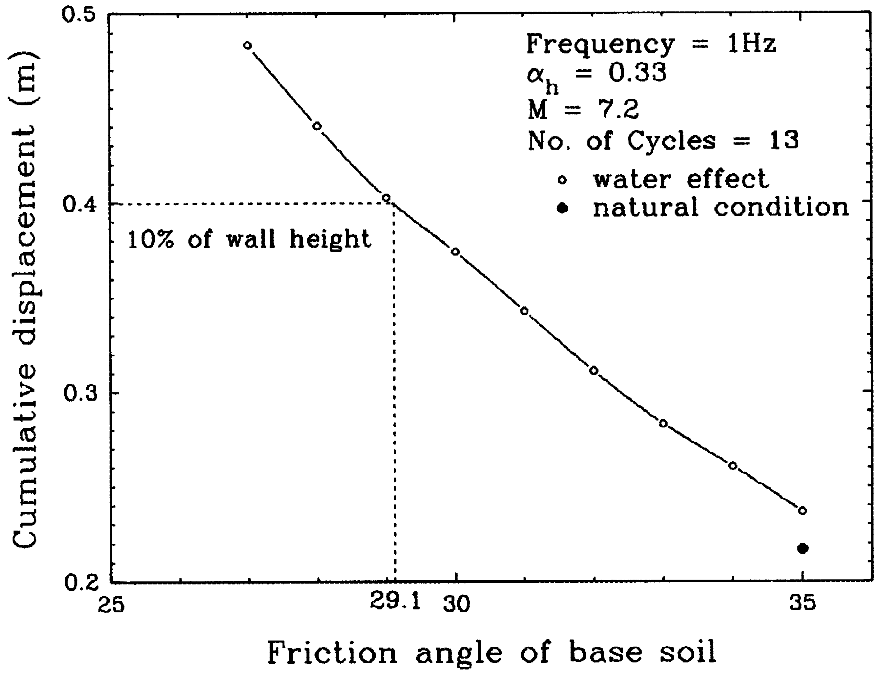

Fig. 6 The effect of base soil strength on cumulative displacement of soil properties in Table 3.

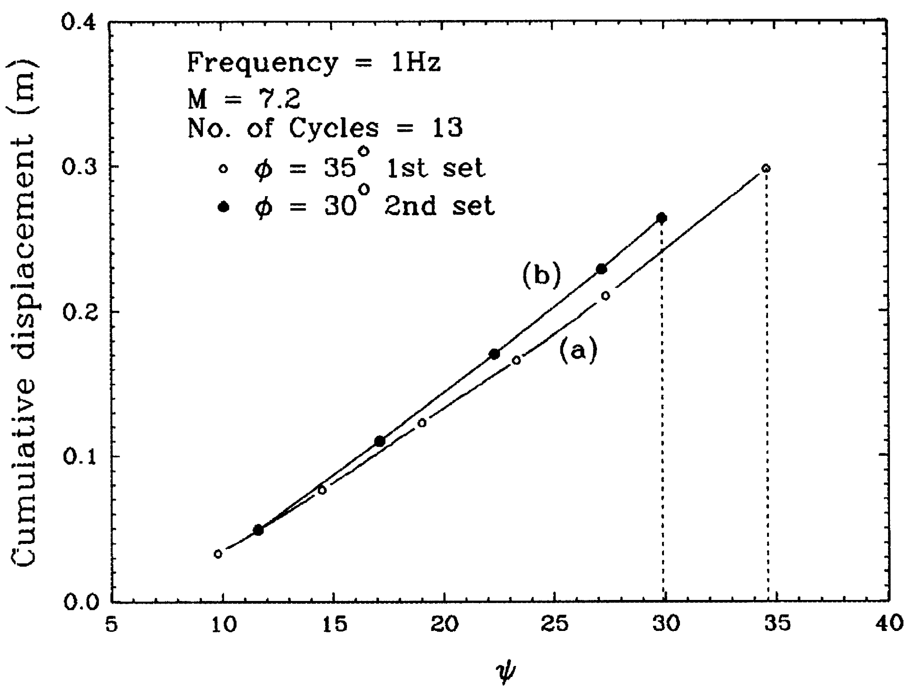

Fig. 7 Effect of varying ψ on cumulative displacements for different properties (a) first, (b) second set (Table 4).

Wall in Hokkaido-Nansi-oki Earthquake

For the 4m high wall in Hokkaido-Nansi-Oki (HNO) earthquake of 1993 (see Table 1) with M=6.7, N=9.7 and α_{hmax} =0.2, α_h for design is 0.133 (2/3 α_{hmax}). Using ψ from eq. 5, the displacements of this wall for different base soils and backfill with 35° (ϕ angle) are given in Table 5.

Table 5. Displacements of 4m high wall with varying *base* soil properties in HNO earthquake.

ϕ °	35	34	33	32	31	30	29	28	27
e	0.35	0.4	0.45	0.50	0.55	0.6	0.65	0.7	0.75
γ_{sat} (kN/m^3)	22.16	21.72	21.31	20.93	20.57	20.23	19.92	19.62	19.34
Base width (m)	1.9590	2.0503	2.1483	2.2523	2.3609	2.4781	2.6032	2.7348	2.8779
Cumulative Displacement (m)	0.0289	0.0323	0.0352	0.0384	0.0421	0.0462	0.0512	0.0571	0.0637
Cumulative rocking degree	0.21	0.22	0.23	0.24	0.25	0.26	0.27	0.29	0.30
$\frac{\text{cumulative disp.}}{\text{height of wall}}$ (°)	0.41	0.46	0.50	0.55	0.60	0.66	0.73	0.82	0.91

DISCUSSION

1. The wall sections in Fig. 4 for the 2 set of properties in Table 2 have been used for analysis in this case. If no corrections for the ψ due to submergence is made, the cumulative displacements are 21.7cm and 29.3cm for 13 cycles of 0.33g motion for the first and second set of properties respectively (Fig. 5). The displacements are 5% and 7.5% of the height of wall and may not constitute failure.

2. If the soil properties of the base are as in Table 3, the displacements for $\phi \leq 29.1°$ (Fig. 6) result in displacements more than 10% of the wall height, which may not be acceptable and may constitute failure.

3. For base $\phi = 30°$ and $\alpha_h = 0.33$, the value of ψ as shown in the Table 4 is 34.10°. According to Mononobe and Okabe method, ϕ should be greater than ψ, otherwise the method fails. In fact the size of the failure wedge becomes infinite with horizontal boundary from the heel (Prakash 1981). Therefore, these set of values result in failure in any case. Also, for this case, the maximum ψ for which the present analysis can be used is 29.88 for $\alpha_h = 0.28$ in 2nd set of soil property (Table 4). The displacement of the wall for 13 cycles is 26.39cm i.e. about 6.6% of wall height . As α_h exceeds 0.28 (or $\psi \geq \phi$) the displacements will increase suddenly. Therefore failure is reached.

4. The wall in HNO earthquake had rotation by 1° to 2°. Since the walls actually experience sliding and rotation, the rotation is computed based on cumulative displacement (Table 5). The rotation is of the order 0.41° to 0.91°.

CONCLUSION

The present method was used to analyze two walls in recent earthquakes.

1. In Kobe earthquake, the wall failed. According to the present method, the wall displacements could be more than 10% of the wall height, which may constitute failure. Therefore, the displacement computed do indicate large displacements leading to failure.

2. In HNO earthquake, the measured rotation of the wall is 1° to 2°. The compute rotation is 0.41° to 0.91°.

Therefore, it may be concluded that the method shows promise in predicting both displacements and failure of rigid retaining walls.

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