



CALCULATING BUILDING SEISMIC PERFORMANCE RELIABILITY: A BASIS FOR MULTI-LEVEL DESIGN NORMS

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ABSTRACT

An explicit equation for the probability of exceeding any nonlinear performance level criterion is derived and inverted to produce a deformation-based, strength-formatted design equation that meets a specified probabilistic performance goal. The model accounts for variability in yield-level capacity, ductility capacity, and record-to-record differences in post-yield behavior, as well as for the seismic hazard per se. The development follows a SDOF model but the extension and application to MDOF systems are shown too.

KEYWORDS:

probabilistic; performance; nonlinear; multi-degree of freedom; design equations; ductility

INTRODUCTION

The objective of this paper is to provide a convenient explicit probabilistic basis for new seismic design and evaluation guidelines and codes. As has been pointed out (Bertero and Bertero, 1994), this objective implies that, on top of traditional code approaches, the procedure must be able to incorporate variability and uncertainty in the many elements of the seismic problem, e.g., times, locations, sizes and ground motions of future earthquakes, dynamic responses of complex structures, and behavior and capacity of structural elements and systems. Further the method must be able to be responsive to the current demand that new structural norms incorporate a range of performance levels. The research literature is replete with probabilistic analyses of various parts of this problem using often very esoteric concepts of random vibration theory even for limited (e.g., linear and stationary) models of the structure and the accelerograms, or using simulation of (practically) prohibitively large numbers of samples of events. The proposed procedure makes use only of traditional and available tools: static and dynamic analyses of conventional linear and non-linear structural models, a limited number of recorded accelerograms, standard probabilistic hazard analysis results (e.g., maps of 1000-year mean return period spectral accelerations), and elementary "second-moment" measures of variability and uncertainty in certain variables (such as story-drift capacity) to be discussed. The last items, second-moment measures, are parallel to those used in existing probability-based LRFD building codes.

For clarity we shall first develop the basic probabilistic model under the assumption that the structure is a single-degree-of-freedom oscillator with a bilinear force-deformation relationship. Then we shall describe the modifications, interpretations, and computations necessary to extend the procedure to a more realistic building model.

We shall begin with the underlying probabilistic model and then provide explicit equations for the seismic "reliability" of the structure. These equations can be used to assess the safety with respect to any specified performance level; where we presume that the levels of performance of interest can be defined in terms of levels of some response measure such as ductility or story drift. The same equations will then be "inverted" to give the design capacity (in force and/or ductility terms) that must be provided to insure specified levels of reliability with respect to the various performance levels.

THE BASE PROBABILISTIC MODEL

We start by finding the complement of the reliability, the probability of "failure", i.e., failure to achieve the performance level of interest. Consider initially a representation of the building in terms of a single degree of freedom (SDOF) oscillator with a bilinear non-linear restoring force subjected to a seismic demand in the form of a random base acceleration history with random spectral acceleration level, S_a (measured at the initial or "reference" natural frequency and damping of the oscillator). For convenience we shall work with displacement measured in terms of ductility. Then let "failure" be associated with a (random) ductility response (or "demand"), μ_D , being greater than the (random) ductility capacity μ_C . The ductility capacity is random because the ductility at which, say, structural repair becomes necessary may well vary from structure to structure. The failure probability is:

$$p_f = P[\mu_D \geq \mu_C] \quad (1)$$

where the notation $P[\dots]$ is read "the probability that". Expanding the equation (1) by conditioning on the capacity $\mu_C = \mu$, i.e., some specific value and (2) by switching from displacement to spectral acceleration terms through the factor $F_R(\mu)$ defined below, we get

$$p_f = \int P[S_a \geq F_R(\mu) \cdot S_{a_{ref}} | \mu_C = \mu] f_{\mu_C}(\mu) d\mu \quad (2)$$

This equation says, for a given ductility (or displacement) capacity μ , failure occurs if the future (random) spectral acceleration demand exceeds a certain (also random) factor $F_R(\mu)$, defined below, times the (perhaps also random) spectral acceleration yield capacity $S_{a_{ref}}$ which is the ground motion level at which the oscillator would be on the verge of yielding (for a SDOF oscillator, this a structural property, yield force divided by mass). The factor $F_R(\mu)$ depends on that given ductility capacity. To consider the variability in ductility capacity it is then necessary, as the equation indicates, to consider all possible values of μ and weight them by their likelihoods, which are given by the probability density function $f_{\mu_C}(\mu)$.

It is helpful to (closely) approximate this result by the simpler equation involving four random variables in the following, final product form:

$$p_f \approx P[S_a \geq F_R(\hat{\mu}_C) \cdot S_{a_{ref}} \cdot \varepsilon_{\mu_C}] \quad (3)$$

This, our *basic underlying equation*, says failure occurs if the spectral acceleration demand exceeds a factor times the yield spectral acceleration times a random "dummy" term ε_{μ_C} with a unit mean. Now the factor is presumed to depend only on the *median* value of the ductility capacity $\hat{\mu}_C$. The dummy variable with unit mean is introduced to capture the variability implied by the random ductility capacity. Formally our notation is: (1) the notation \hat{X} is used for median of random variable X ; (2) the "reference spectral acceleration" $S_{a_{ref}}$ is (for this SDOF case) the elastic spectral acceleration associated with the yield (force) resistance or elastic capacity of the oscillator, R_{yld} , i.e., $R_{yld} = m \cdot S_{a_{ref}}$ (where m = mass); (3) $F_R(\mu)$ is a (random) non-linear response factor in common use in the earthquake community (it is closely related to the "R-factor" in current US seismic codes, where it is used as a demand or response *reduction* factor, i.e., on the left hand side of the inequality; by definition $F_R(\mu)$ is the factor by which a specific acceleration time history capable of causing incipient first yield must be

scaled up¹ to produce a maximum (non-linear) displacement characterized by ductility level μ , where μ is the maximum displacement divided by the oscillator's yield displacement; because it varies from record to record it $F_R(\mu)$ is a *random record property* (it also depends, of course, on the characteristics of the oscillator in question; and (4) ε_{μ_c} is a random variable that captures the variability in the ductility capacity, μ_c , *measured in spectral acceleration terms*; for example, for oscillators of moderate period it can be assumed that the median value of the scaling factor $\hat{F}_R(\mu)$ is approximately equal to μ in which case the coefficient of variation of ε_{μ_c} is simply equal to that of μ_c itself² and $\hat{\varepsilon}_{\mu_c} = 1$.

To obtain explicit analytical results, in this work we shall make two *mild* assumptions about the form of the probability distributions in Eq. 3. We assume (1) that R the product of the three random variables on the right hand or "capacity" side has a *lognormal distribution* with a median $\hat{R} = \hat{F}_R(\hat{\mu}_c) \cdot \hat{S}_{a_{ref}} \cdot \hat{\varepsilon}_c$ and a coefficient of variation (δ) which is at least approximately equal to

$$\delta_R \approx \sqrt{\delta_{F_R}^2 + \delta_{S_{a_{ref}}}^2 + \delta_{\varepsilon_c}^2} \quad (4)$$

and (2) that (at least in the region of interest) the complementary cumulative distribution function (CCDF) of the left hand side, the demand spectral acceleration, can be represented as

$$P[S_a > x] = H(x) \approx k_0 x^{-k_1} \quad (5)$$

The notation $H(x)$ reflects the fact that this function is commonly referred to as the site's "seismic hazard curve" for the spectral acceleration (associated with the oscillators natural frequency). We presume this curve is readily available in some form or another.³ The "region of interest" here is the range of values of x such that $H(x)$ is in the range of approximately $10p_f$ to p_f . Then, it can be found by substitution into Eq. 2 and completion of the square in the exponent that

$$p_f = H(\hat{R}) e^{1/2(k_1 \sigma_{\ln R})^2} \approx H(\hat{R}) e^{1/2(k_1 \delta_R)^2} \quad (6)$$

where $\sigma_{\ln R}^2 = \ln(\delta_R^2 + 1) \approx \delta_R^2$ for lognormal variables.

This simple explicit equation says that the failure probability (the annual probability of exceeding the ductility capacity) is simply the hazard curve evaluated at the median capacity (which is the exact answer for very small δ_R) times an exponential "correction factor" that depends simply on the product of the variability of the three "capacity" variables and the "slope" (in log-log terms) of the hazard curve. Some values of the correction factor are shown in Table 1. The log-log slope k_1 can be easily estimated graphically from a_R , the ratio of the demand levels one order of magnitude apart in accedence ratio (i.e., $a_R = \hat{R}/R'$ where R' is the demand at $H(R') = 10H(\hat{R})$; then $k_1 = 1/\log_{10}(a_R)$). This equation has already found use in the development of seismic criteria for the U.S. Department of Energy (DOE, 1994).

¹Alternatively $F_R(\mu)$ is the amount the yield force must be reduced (below the value that implies incipient yield) to produce ductility μ .

²If more generally $F_R(\mu) \approx \mu^b$ for values of μ greater than one, then the coefficient of variation of ε_{μ_c} is approximately that of μ_c raised to the power b , which is typically less than one.

³In cases where the hazard is described only by a set of maps displaying contours of spectral acceleration at different mean return periods (where mean return period is $1/H(x)$) the hazard curve can be estimated by plotting the reciprocal of these return periods versus the corresponding site spectral accelerations. Estimation of the slope k_1 facilitated by plotting on log-log graph paper.

Observing Table 1, one finds that for a large range of practical distribution parameter values the correction factor is not significantly different from unity; in these regions, e.g., for cases involving the onset of damage or even for cases of near-collapse in low hazard regions, the failure probability is dominated by the hazard curve and one may say with some accuracy that the failure probability is simply, say, "about twice" the probability that the median capacity is exceeded $H(\hat{R})$. Elsewhere the correction factor is still less than an order of magnitude; here it is necessary to understand that the estimation of the failure probability depends on explicit recognition of the value of the product of the slope of the hazard curve and the capacity variability $k_1 \delta_R$. For very steep demand slopes and very broad capacity distributions, however, the correction factor is so large that this approach is simply not an effective one. These are cases when, unlike the others, the failure probability may unfortunately be quite sensitive to the estimation of the capacity probability distribution in its lower tail. The applications of what follows therefore are limited in practice to the case, common in the seismic situation, of comparatively large demand variability (relative to capacity variability). If these conditions do not hold numerical integration is necessary with the attendant loss of explicitness here and below.

COV of R δ_R	σ_{lnR}	Hazard Curve Slope					
		k_1	10.32	5.68	4.11	3.32	2.51
		a_R	1.25	1.5	1.75	2.0	2.5
0	0		1.0	1.0	1.0	1.0	1.0
0.2	0.197		8.1	1.9	1.4	1.25	1.1
0.4	0.385		2700	10.9	3.5	2.2	1.6
0.6	0.55		12.8×10^6	142	13	7	3.9

Table 1. Values of $\exp(\frac{1}{2}k_1^2\sigma_{lnR}^2)$ the "Correction Factor" on $H(\hat{R})$ in Eq. 6 for Different Values of the δ_{lnR} , the COV of R , and the Hazard Curve Slope.

One immediate advantage of this explicit failure probability form is the ease with which it can be inverted for *design* interpretation. From Eq. 6, for a reliability specification $p_f \leq p_o$, the median "capacity" must be

$$\hat{R} \geq H^{-1}(p_o) e^{1/2 k_1 \sigma_{lnR}^2} = S_a^o e^{1/2 k_1 \sigma_{lnR}^2} \quad (7)$$

in which $S_a^o = H^{-1}(p_o) = (k_o/p_o)^{1/k_1}$ is the demand spectral acceleration value with an exceedance probability equal to p_o . This is our basic probability-based design equation. It says that the median "capacity" must *exceed* this ("first order") design value S_a^o by an exponential "load factor" $\exp(1/2 k_1 \sigma_{lnR}^2)$ which is tabulated in Table 2 as a function of hazard curve "slope" k_1 (or a_R) and COV of R . These values are *smaller* than the probability correction factors in Table 1. These "load increases" necessary to accommodate "capacity" variability are only about 10 to 20% even for moderate capacity variability, provided the hazard curves are shallow enough. In different words, if one specifies that for a particular performance level the mean frequency of exceedance of the critical ductility level should be, say, once per N years, then the appropriate design spectral acceleration is typically about 10 to 20% larger than the spectral acceleration with a mean return period of N years. But in general this multiplier depends on the product of the hazard curve slope and the square of the capacity COV (recall $\delta_R \approx \sigma_{lnR}$).

COV of R	σ_{lnR}	Hazard Curve Slope					
		k_1	10.32	5.68	4.11	3.32	2.51
		a_p	1.25	1.5	1.75	2.0	2.5
0	0		1.0	1.0	1.0	1.0	1.0
0.2	0.197		1.22	1.12	1.08	1.07	1.05
0.4	0.385		2.15	1.52	1.35	1.28	1.20
0.6	0.55		4.89	2.39	1.87	1.66	1.46

Table 2. Values of $\exp(1/2 k_1 \sigma_{lnR}^2)$ the Multiplier on S_a° in Eq. 7b

The right hand side of the inequality in Equation 3 (or R) contains the factor $F_R(\hat{\mu}_c)$ that reflects the ability of a ductile structure to accommodate dynamic loads beyond those causing first yield. For SDOF systems this ductility effectiveness factor has been extensively studied for seismic problems (e.g., Cornell and Sewell, 1987, Sewell, 1988, and Miranda and Bertero, 1994). The factor is a record property, varying from record to record. For example, its median value $\hat{F}_R(\hat{\mu}_c)$ is approximately equal to $\hat{\mu}_c$ for moderate period SDOF elastoplastic (or "better"⁴) systems; this is equivalent to saying that for such systems the displacement of the nonlinear system is approximately equal to that of a linear elastic system with the same period and damping. The empirically confirmed insensitivity of the median of $F_R(\mu)$ to magnitude and distance (Sewell, 1988) has been used implicitly in the development of Equation 6 and hence 7. If, in the future, post-yield measures of performance other than ductility (such as hysteretic energy) are adopted and they prove sensitive to magnitude (e.g., because, implicitly, of duration effects), then the procedure here must be modified. This is a subject of current research.

Recall that the median capacity, \hat{R} , is the product of (1) the median spectral acceleration that causes yield, which is itself the median *yield force capacity* \hat{R}_{yield} divided by the mass, times (2) the median *nonlinear capacity factor* $\hat{F}_R(\hat{\mu}_c)$ evaluated at the median *ductility capacity*. In some recent code developments the factor $F_R(\mu_c)$ called here the nonlinear capacity factor $\hat{F}_R(\hat{\mu}_c)$ is replaced by the product of a *static overstrength factor* Ω and a *dynamic nonlinear or ductility effectiveness factor*, which we denote here $D_R(\mu)$. This factor is simply $F_R(\mu)$ divided by Ω , where Ω is the ratio of the static ultimate force capacity to the yield force capacity. Note that ductility is measured with respect to the displacement at yield not that at ultimate capacity. As suggested above the nonlinear capacity factor is not sensitive to the force-deformation curve and therefore to the static ultimate strength per se. In contrast, the dynamic factor $D_R(\mu)$ is. If further one attempts to reference it with respect to the displacement at ultimate capacity it becomes sensitive to the shape of the force-deformation curve, e.g., to the second slope of a tri-linear force deformation law. In this author's view it is preferable therefore to avoid the introduction of the static overstrength factor in the formulation. Its attraction apparently lies in the use of static pushover analyses to provide an estimate of the factor.

The capacity uncertainty (Eq. 4) contains contributions (1) from F_R (e.g., record-to-record variability in the nonlinear capacity factor reflecting factors such as the duration of response and the precise time variation of the accelerogram); (2) from yield force capacity ($\hat{R}_{yield} = mS_{a,ref}$) as reflected here in $S_{a,ref}$; and (3) from ductility capacity (μ_c as reflected in ϵ_c). The first COV in Eq. 4 is surprisingly only 20 to 30% and it increases for increasing ductility levels (Sewell, 1988); the second may be 10 to 15%; and the last, less studied, may be 30%

⁴ "Better" means a force deformation behavior that retains some stiffness after first yield. Many investigators have noted the insensitivity of $F_R(\mu)$ to the precise form of the force deformation curve, provided there is no softening, as with a buckling brace.

or more, depending on the performance limit state of interest, for a net value of about 40% for limit states associated with large (6 to 8) median ductility capacities.

As numerical examples consider two performance levels. The onset of damage might be associated with incipient yield, implying $\hat{\mu}_c$ and $\hat{F}_R(\hat{\mu})$ equal unity and have no uncertainty. Then $\hat{R} = \hat{S}_{a_{ref}} = \hat{R}^{yield}/m$ and the net COV is only that in the yield force capacity, perhaps 10 to 15%. A typical value of $H(\hat{R})$ might be 0.01. For a typical value (3.0) of k_1 the probability of exceeding yield would be only about 5% larger than 0.01 (Table 1). In another case, if uneconomical repair were associated with an median ductility level of 4 and the structure had a one second natural period, the median value of the nonlinear capacity factor might be about 3.8. The median yield force capacity necessary to limit the annual probability of such an event to a target value of, say, 0.001 would be the mass times the 1000-year 1-second spectral acceleration (say, 0.4g) divided by 3.8 and multiplied the factor from Table 2 to account for the variability in the several capacity factors. For the net COV value deduced in the previous paragraph, 40%, and an a_R of 1.75 (based on the 100-year spectral acceleration being found to be about 0.4/1.75 or 0.23g), the necessary "load factor" is 1.35 (Table 2). Therefore the design yield force must be the mass times 1.35 times (0.4/3.8) or 0.14g to meet the probabilistic performance goal. Note that if the two cases are presumed to be the same site, then median yield force in the former, onset-of-damage case would be the mass times 0.23g. If 0.01 and 0.001 were considered the two limit state specifications, then it is clear that in this problem the onset of damage is the performance goal that governs design. This conclusion could change with the change of any of several elements, even the period of the structure, $\hat{F}_R(\hat{\mu})$ being smaller for smaller period structures.

NONLINEAR MULTI-DOF SYSTEMS

Recent work (with stick models, as well as 2D and 3D frames, including large offshore steel jacket platforms; e.g., Bazzurro and Cornell, 1994a, b) demonstrates that MDOF problems can be put into the same format. The most difficult step is to estimate the statistics of the factor $F_R(\hat{\mu}_c)$ by subjecting a non-linear model of the MDOF structural system to a sample of 4 or 5 input histories (either recorded or simulated). The sample size can be small, making the procedure practical, because the COV of this factor is found to be comparatively small (especially compared to that of the demand ground motion, as reflected in typical attenuation law COV values of more than 50% and other factors that produce flat hazard curve slopes, i.e., small values of k_1). The median of $F_R(\hat{\mu}_c)$ can be estimated with a standard error factor of only δ_{F_R}/\sqrt{n} in which n is the sample size; this statistical uncertainty can be added to the analysis if necessary (see below). The value of δ_{F_R} normally needs to be estimated only crudely, e.g., from past experience, because it plays a limited role in the net COV of the capacity and because the correction factor and load factor (Tables 1 and 2) are insensitive to this COV over much of the range of interest.

In the simplest case, in each simulation the accelerogram is first scaled up⁵ until it causes incipient yield at some critical point in the structure, the structure and the analysis being linear elastic. For the natural frequency of some meaningful "reference" SDOF linear oscillator, e.g., one with a frequency equal to that of the first mode of the MDOF structure, the $S_{a_{ref}}$ for this record is defined as the spectral acceleration at this initial level.⁶ The record is then scaled to several higher levels, a non-linear dynamic analysis is conducted at each level. Interesting local

⁵ Scaling up the record, which some object to, is equivalent numerically to scaling down the capacity of the system, but for complex MDOF structural systems involving buckling and/or multiple materials this may not be a practical approach. The properties of the structure, e.g., yield level, etc., should be set to their median ("best estimate") values in the model.

⁶ For SDOF oscillators, this value is identical for all records. For realistic MDOF systems it displays a small variability due to variability in higher mode response contributions; see, e.g., Inoue and Cornell (1992) and Bazzurro and Cornell (1994a,b). This variability can be "added" to the others in Eq. 4.

and global response measure values, such as maximum story drift, are recorded for each ground motion level. From a plot of the scale factors versus response measure values, simple statistics of the scale factor (i.e., of $F_R(\mu)$) for a given response level, (e.g., global ductility, μ , equal 4 or maximum story drift equals 1%) can be estimated. These statistics are then used in the analysis scheme outlined above. (Practical implementation details and empirical justification for implicit assumptions, e.g., independence between $F_R(\mu)$ and magnitude and distance, are discussed in the references cited above). As some current practice suggests, it may be possible to use an MDOF nonlinear static "pushover" analysis together with SDOF $F_R(\mu)$ results to obtain an estimate of the $F_R(\mu)$ versus μ curve of the MDOF system, but one should supplement this estimate with additional uncertainty to reflect the incompleteness of the information it represents relative to dynamic analyses.

Figure 1 shows the static pushover for an actual four-leg steel jacket structure and pile-soil foundation in the North Sea, and the plot of median and 16th and 84th percentile values of $F_R(\mu)$ versus μ ; the COV's are quite small in this "softening" case. The subscripts indicate that the ductility is a global ductility and that it is a composite (maximum) of X and Y direction values. Figure 1 suggests that the global ductility capacity with respect to a collapse limit state, $\hat{\mu}_c$, is about 3, and that the median of $F_R(\hat{\mu}_c)$ is about 1.75 with a COV of 10% or less (Bazzurro and Cornell, 1994b). The value of \hat{S}_1 for this 1.8 second (first mode) structure was about 0.36g; it displayed a record-to-record COV of about 3%. If the COV of the ductility capacity were 30%, that of ϵ_c (in Eq. 3) would be about 15% because $F_R(\mu)$ is approximately $\mu^{0.5}$ (Footnote 2). With these and a few additional numbers similar to those used in the SDOF examples plus a site hazard curve for spectral acceleration one can use the scheme developed here to easily determine the annual likelihood of collapse or other less severe deformation states, as shown in the SDOF examples above.

Simple static additive loads can be incorporated within this context by treating them as reductions from the right hand side of Eq. 3 and hence part of a new right hand side (capacity) with smaller median and larger COV. The lognormal distribution assumption must be adopted for the new right hand side to maintain the explicit results here and their advantages. For cases of smaller k_1 and smaller δ_R the result should be insensitive to the precise distribution form of the right hand side.

Epistemic Uncertainty. Uncertainty in the distributions and parameters of the model due to limited statistical data is referred to here as "epistemic" (as opposed to "aleatory") uncertainty. If it can be adequately modeled here (1) by assigning a lognormal (epistemic) distribution to $H(x)$ (with $COV = \delta_H$), and (2) by an epistemic lognormal distribution on the median, \hat{R} , of the right hand side (of the inequality in Eq. 3) with $COV = \delta_{\hat{R}}$, then the a mean probability of failure is

$$\bar{p}_f = \bar{H}(\hat{R}^o) e^{1/2 k_1^2 (\sigma_{HR}^2 + \sigma_{\hat{R}}^2)} \tag{8}$$

in which $\bar{H}(\hat{R}^o)$ is the mean (estimate of the) hazard curve evaluated at the median estimate of the median of R , denoted here \hat{R}^o . Note the simple combination rule for aleatory and epistemic uncertainty in R and the apparent independence of the result from δ_H . The independence is illusory; the mean hazard is larger than the median estimate by a factor $\exp 1/2 \delta_H^2$. The assumptions made here are those commonly made in nuclear power plant probabilistic risk assessment practice.

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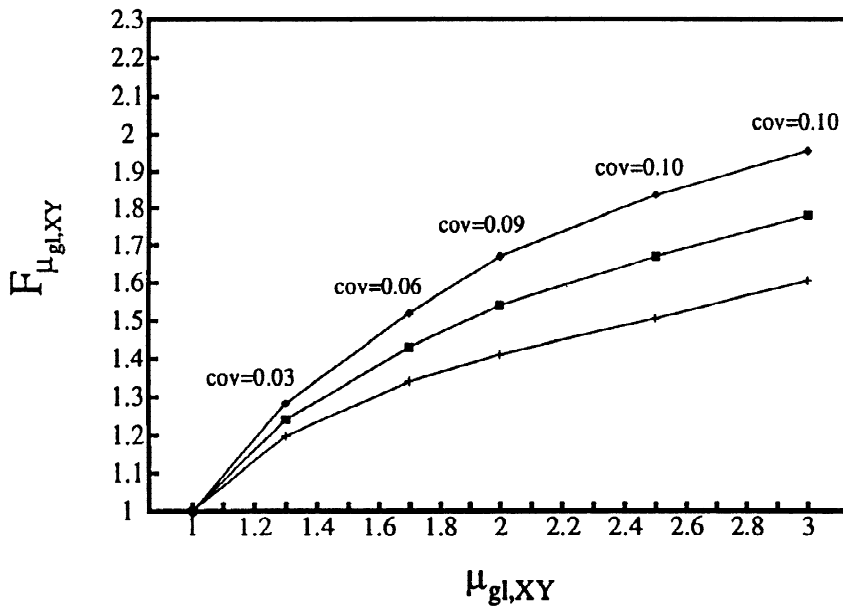
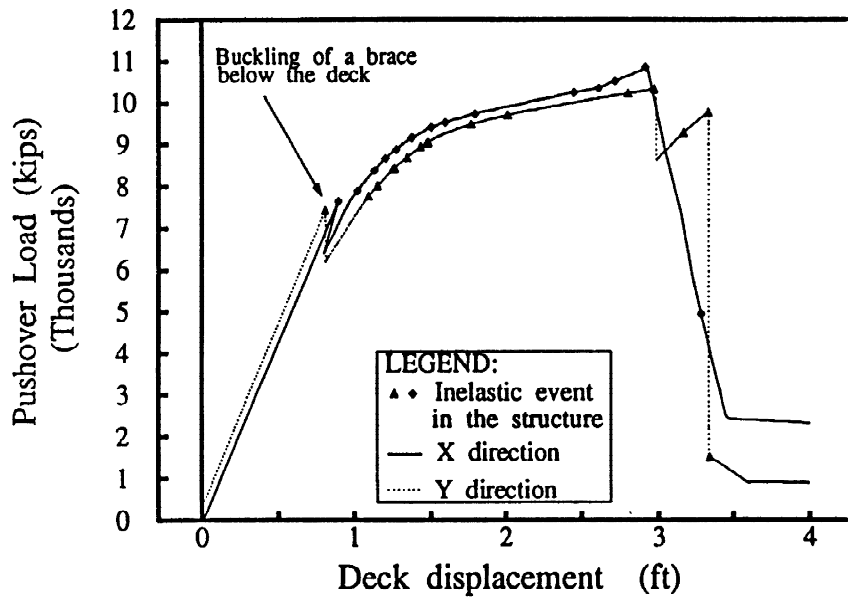


Figure 1: Pushover results; F_{μ} vs. μ statistics.