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AN ENERGY APPROACH TO POUNDING OF STRUCTURES

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ABSTRACT

An energy approach to study pounding problems is presented based on the use of a Pseudo Energy Radius concept. The critical gap size to avoid pounding, estimates of the amplification effects if pounding occurs, and estimates of the effectiveness of some mitigation techniques are presented in terms of energy.

KEYWORDS

Pounding; Energy; Pounding Mitigation.

INTRODUCTION

In the last decades, major earthquakes affecting large metropolitan areas have induced severe pounding damage (Wada *et al.*, 1984; Kasai and Maison, 1991). During the 1985 Mexico City earthquake, about 40% of the damaged structures experienced some level of pounding, 15% of them leading to structural collapse (Rosenblueth and Meli, 1986). Pounding between two structures occurs when, due to their different dynamic characteristics, the structures oscillate out of phase, and the separation between them is not sufficient to accommodate the relative displacements. In general, building codes specify a minimum gap size to avoid pounding interactions. Nevertheless, at present, there is an important number of buildings in major metropolitan areas that do not have adequate separation, and therefore, are prone to pounding damage.

Pounding between buildings have been studied by a number of researchers. Some early studies were performed by the nuclear power industry (Wolf and Skikerud, 1980) to retrofit nuclear power plants due to increases in the seismic requirements. Studies have been made to model the response of several multistory buildings in series (Anagnostopoulos and Spiliopoulos, 1992). The double difference combination rule was introduced by Kasai and Jagiasi (1993) to calculate the minimum gap between inelastic structures to avoid pounding. Furthermore, some pounding mitigation techniques have been proposed, including the use of damper elements connecting adjacent buildings (Filiatrault and Folz, 1992; Sues *et al.*, 1991), and supplemental energy dissipation (Kasai et al., 1993).

In this paper, the concept of Pseudo Energy Radius (Valles, 1995) is introduced as a simple tool to study pounding problems using an energy approach. The Pseudo Energy Radius (PER) is then used to present simple formulations to: (*i*) calculate the minimum gap to avoid pounding between inelastic structures; (*ii*) estimate the amplifications in the response due to pounding interactions; and, (*iii*) estimate the effectiveness of different mitigation techniques.

ENERGY TRANSFER DURING POUNDING INTERACTIONS

When pounding occurs, an interaction force between the two structures is observed. Consider the equations of motion of two linear structures prone to pounding:

$$m_1\ddot{u}_1 + c_1\dot{u}_1 + k_1u_1 + f_c = -m_1\ddot{x}_g \tag{1}$$

$$m_2\ddot{u}_2 + c_2\dot{u}_2 + k_2u_2 - f_c = -m_2\ddot{x}_g \tag{2}$$

where u_i, $\dot{u}_i$, and $\ddot{u}_i$ correspond to the displacement, velocity, and acceleration of structure "*i*"; m_i, c_i, and k_i are the mass, damping, and stiffness parameters for structure "*i*"; $\ddot{x}_g$ is the earthquake acceleration; and f_c is the contact force between the structures.

The presence of the interaction force alters the energy balance in the structure, leading to amplifications or reductions in the response. The equations describing the energy balance can be obtained from (1) by multiplying both sides of the equations by the corresponding velocities, and integrating over time. Using a relative energy formulation:

$$E_{K1} + E_{\xi 1} + E_{P1} + E_{Tr12} = E_{I1} \tag{3}$$

$$E_{K2} + E_{\xi 2} + E_{P2} + E_{Tr21} = E_{I2} \tag{4}$$

where E_{Ii} is the energy input for structure "*i*"; and the kinetic, viscous, and deformation energies for structure "*i*" are given by:

$$E_{Ki} = \frac{m_i}{2}\left(\dot{u}_i\right)^2;\ E_{\xi i} = \int_0^t c_i\left(\dot{u}_i\right)^2 d\tau;\ E_{Pi} = \frac{k_i}{2}\left(u_i\right)^2 \tag{5}$$

The energy transfer from structure 1 to structure 2 is denoted by E_{Tr12}, while the energy transfer from structure 2 to structure 1 is E_{Tr21}. Note that in this formulation the energy transfer terms can be negative, since they are direction dependent. That is, a positive energy transfer indicates that part of the input energy is transferred to the other structure, while a negative energy transfer indicates that energy is received from the other structure.

For simplicity, a distinction can be made between the input energy (E_I), the transfer energy (E_{Tr}), and the structural energy (E_S), defined as:

$$E_S = E_K + E_\xi + E_P \tag{6}$$

Figure 1 presents the variation over time of the structural and input energies of two linear single-degree-of-freedom systems subjected to pounding interactions. Note that one structure increases its structural energy level, while the other reduces it. The energy transfer is the difference between the input and structural energies. The sudden variations in the structural energy are due to the significant changes in the velocity (kinetic energy) of the structures when pounding occurs.

The theory of stereomechanical impact can be used to estimate the sudden changes in velocity when pounding occurs. The post impact velocities ($\dot{u}_1^f$ and $\dot{u}_2^f$) can be determined from the approaching velocities ($\dot{u}_1^0$ and $\dot{u}_2^0$) prior to impact, according to:

$$\dot{u}_1^f = \dot{u}_1^0 - (1+e)\frac{m_2\left(\dot{u}_1^0 - \dot{u}_2^0\right)}{m_1 + m_2} \tag{7}$$

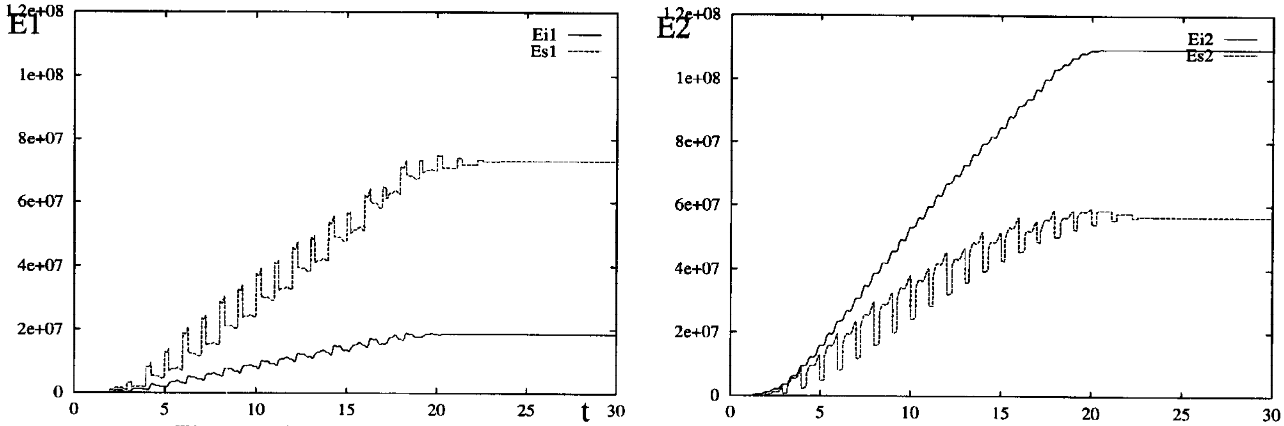

Fig. 1 History of input and structural energies during pounding interactions.

$$\dot{u}_2^f = \dot{u}_2^0 + (1+e)\frac{m_1(\dot{u}_1^0 - \dot{u}_2^0)}{m_1 + m_2} \tag{8}$$

where e is the coefficient of restitution, that takes into account nonlinearities and energy dissipation at the contact interface. The coefficient of restitution range from a value of 1 for elastic impact, to a value of 0 for perfectly plastic impacts. The stereomechanical formulation accounts for the influence of relative masses, initial velocities, and energy dissipation at the interface.

The formulas for stereomechanical pounding can be used to calculate the instantaneous energy transfer (Valles, 1995):

$$E_{Tr12} = \frac{1}{2}\frac{(1+e)m_1 m_2}{(m_1+m_2)^2}\left[\left(-2m_1 + (e-1)m_2\right)\dot{u}_1^0 - (1+e)m_2\dot{u}_2^0\right]\left(\dot{u}_1^0 - \dot{u}_2^0\right) \tag{9}$$

$$E_{Tr12} = -\frac{1}{2}\frac{(1+e)m_1 m_2}{(m_1+m_2)^2}\left[-(1+e)m_1\dot{u}_1^0 + \left(-2m_2 + (e-1)m_1\right)\dot{u}_2^0\right]\left(\dot{u}_1^0 - \dot{u}_2^0\right) \tag{10}$$

Note that only for elastic impacts ($e = 1$) the transfer energies have the same magnitude but different sign ($E_{Tr12} = -E_{Tr21}$). Therefore, energy is conserved only when elastic pounding occurs, otherwise some energy is dissipated at the interface.

PSEUDO ENERGY RADIUS (PER)

The Pseudo Energy Radius (PER) was introduced to calculate the critical gap to avoid pounding interactions, estimate amplification effects if pounding occurs, and estimate the effectiveness of various mitigation techniques (Valles, 1995). Consider a single-degree-of-freedom system, with frequency ω, subjected to an earthquake excitation. The response of the system can be visualized using the state space representation of the response (displacement versus velocity over frequency), as shown in Fig. 2a.

Using this graphical representation, the distance of any point along the response trace to the origin provides a measurement of the instantaneous structural energy E_e (kinetic plus potential):

$$\frac{E_e}{m} = \frac{1}{2}\omega^2\left(\frac{\dot{u}^2}{\omega^2} + u^2\right) = \frac{\omega^2}{2}r^2 \tag{11}$$

Therefore, the distance r can be interpreted as the radius of concentric circles defining constant energy

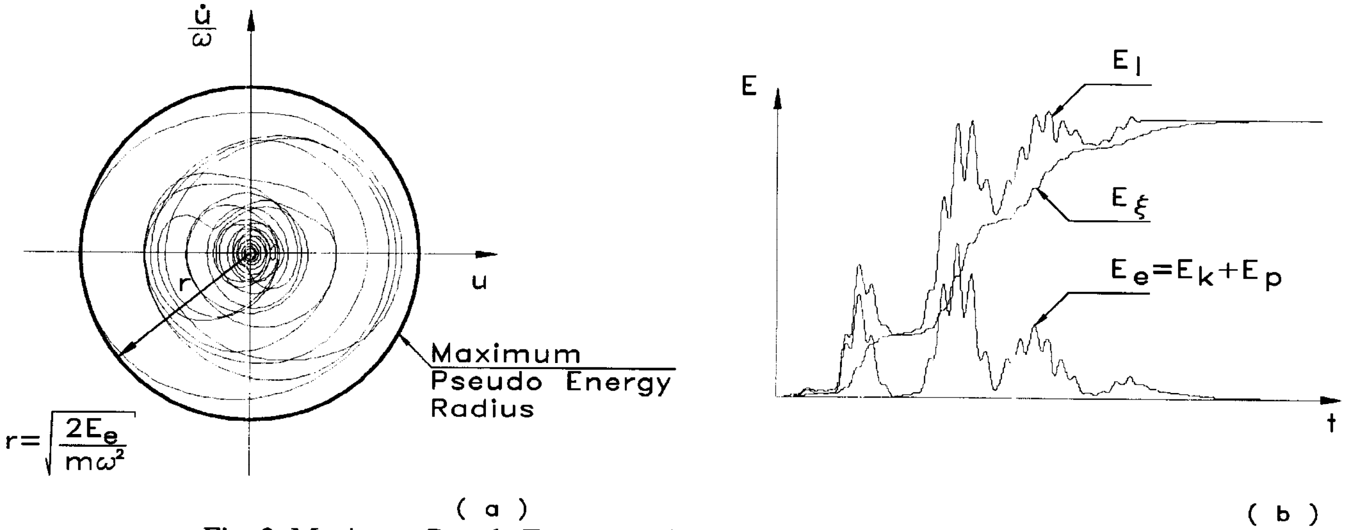

Fig. 2 Maximum Pseudo Energy Radius from a state space representation.

levels in the structure:

$$r = \frac{1}{\omega}\sqrt{\frac{2E_{\varepsilon}}{m}} \tag{12}$$

Therefore, changes in r correspond to changes in the energy level of the system.

The maximum experienced distance r (energy level), is referred to as the Pseudo Energy Radius (PER), and denoted as r_{PER}. Some differences between the PER and the commonly used input and viscous energies can be identified. First, the PER is expressed in units of displacement and not energy. This will prove to be useful to solve pounding problems since r_{PER} can be directly correlated to the critical gap g_{cr}, or the actual gap g_p between two adjacent structures. The second is that the PER is directly related to the maximum response quantities:

$$u_{\max} = r_{PER} \tag{13}$$

$$\dot{u}_{\max} = \omega r_{PER} \tag{14}$$

while the other energy measurements are not, since other parameters, such as the duration of the event considerably changes these quantities (see Fig. 2b). Figure 3 presents the PER spectrum for the 1985 Mexico city earthquake, recorded at SCT.

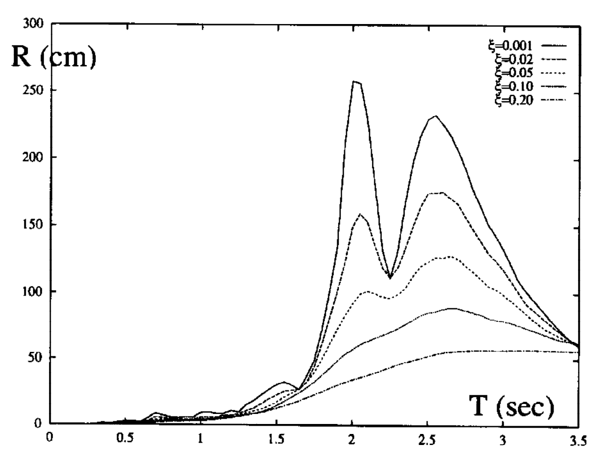

Fig. 3 Pseudo Energy Radius spectrum for 1985 Mexico City earthquake.

CRITICAL GAP COMPUTATION

The critical gap between two structures is defined as the minimum distance between the structures to avoid pounding. Under these conditions the structures may come into contact but with zero relative velocity. The critical gap (g_{cr}) is determined using the underlying principles of the Double Difference Combination (DDC) rule (Jeng *et al.*, 1992) combined with the PER formulation:

$$g_{cr} = \sqrt{r_{PER1}^2 + r_{PER2}^2 - 2\rho r_{PER1} r_{PER2}} \qquad (15)$$

where ρ is the correlation coefficient, and accounts for the phase difference in the response between the two structures. Note that, for a perfectly correlated response ($\rho = 1$), the critical gap is the absolute value of the difference in PER for the structures. For an uncorrelated response ($\rho = 0$), the critical gap corresponds to the SRSS combination of both PER; and for a negatively correlated response ($\rho = -1$), the critical gap corresponds to the absolute sum of the two PER.

Using the PER formulation, the critical gap size and the effect of the correlation coefficient can be easily visualized. Figure 4 presents the response traces for two linear structures, separated by the critical gap, for different combinations of periods. Some overlapping of the energy levels is possible without inducing pounding effects. Using the state space representation, u versus $\dot{u}/\omega$, yields two points that identify the onset of pounding: structures become in contact but with zero relative velocity. The points coincide in the horizontal axis, but have different vertical ordinates. This is due to the difference in the frequencies of the structures, that yield two different vertical scales for each energy circle. Note that the maximum overlapping is significantly influenced by the correlation coefficient ρ.

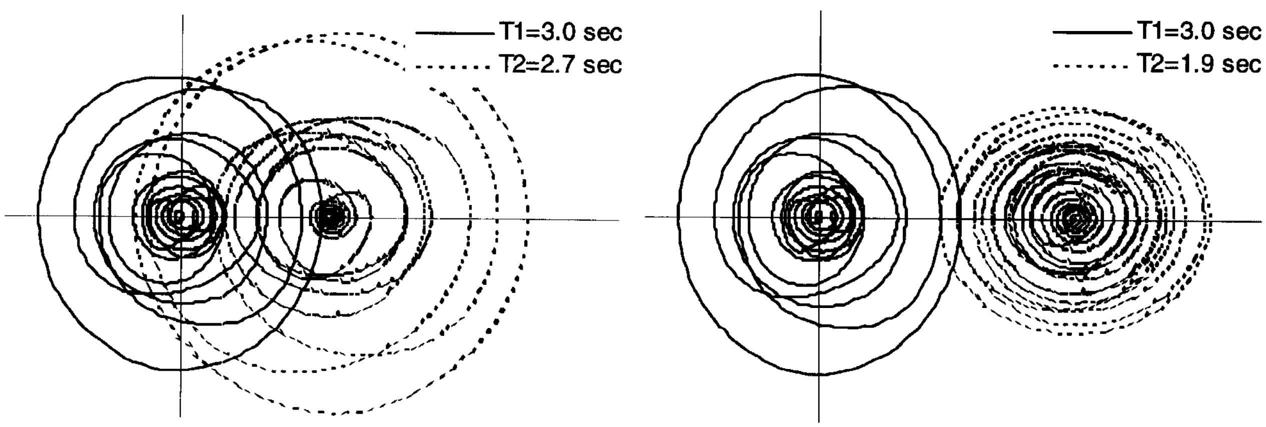

Fig. 4 Critical gap for various structural periods, Mexico City earthquake.

The correlation coefficient can be approximately calculated according to (Jeng *et al.*, 1992):

$$\rho = \frac{8\sqrt{\xi_1 \xi_2}\left(\xi_2 + \xi_1 T_2 / T_1\right)\left(T_2 / T_1\right)^{3/2}}{\left[1-\left(T_2/T_1\right)^2\right]^2 + 4\xi_1\xi_2\left[1+\left(T_2/T_1\right)^2\right]\left(T_2/T_1\right) + 4\left(\xi_1^2+\xi_2^2\right)\left(T_2/T_1\right)^2} \qquad (16)$$

derived as a simplification for linear oscillators subjected to a white noise input. Jeng *et al.* (1992) proposed simple formulas to calculate an effective period and an equivalent critical damping ratio for bilinear structures, that can be used with (16). A summary of other linearization methods is given by Iwan and Gates (1979). Using statistical linearization Valles and Reinhorn (1996) developed plots to calculate the correlation coefficient for bilinear structures subjected to broad band (i.e., Taft), or narrow band (i.e., Mexico City), earthquakes. The use of these plots is suggested, specially for nonlinear structures subjected to narrow band inputs, since (13) is not applicable, and may lead to insufficient gap sizes.

ESTIMATING POUNDING EFFECTS

Estimating the effects that pounding imposes in structures separated by a gap less than critical is a cumbersome task, but can be undertaken using a nonlinear analysis program. A simple method based on the PER formulation and the formulas for stereomechanical pounding is summarized below. The method is based on a single hit event to estimate pounding effects. The method conservatively assumes that both structures are at their maximum energy levels at the onset of pounding, and contact occurs only once. The method yields good results for gap sizes close to the critical gap, in which case a single hit takes place, or if subsequent interactions occur, the overall maximum amplification is still governed by the first hit.

The single hit procedure is carried out in three steps (see Fig. 5): (*i*) identify the state of the structures at the onset of critical pounding, assuming that they are separated with the critical gap; (*ii*) backtrack in time along the PER circle to the onset of actual pounding with the structures separated with the actual gap; and, (*iii*) using the state at the onset of actual pounding, determine the post-impact states and the post-impact energy levels. See Valles and Reinhorn (1996) for a more detailed presentation of the method, and the formulas used for each step.

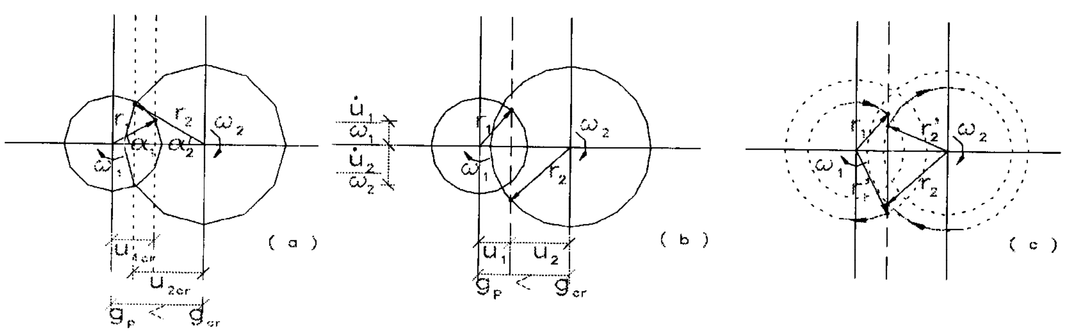

Fig. 5 Simplified single hit procedure to estimate pounding effects.

Note that while one structure experiences an increase in the energy level, the other will undergo a reduction. However, although a reduction in the PER takes place in one of the structures, the no pounding maximum was already observed, and must be used as minimum for design. A comparison of the predicted response amplification effects calculated using the simplified formulation discussed, and the exact results from time-history analyses is shown in Fig. 6. Note that a fairly good estimate is obtained with a considerably less computational effort.

EVALUATING THE EFFECTIVENESS OF MITIGATION TECHNIQUES

A number of pounding mitigation techniques using damper elements have been proposed, and can be broadly classified according to their installation: link elements, bumper damper elements, and supplemental energy dissipation elements. Link elements are used to connect adjacent structures. Forces in the links can be, for extreme cases, in the same order of magnitude as the base shears. Furthermore, the link may induce force distributions that differ considerably from the original design forces, and significant retrofitting may be necessary to withstand them. Retrofit solutions using link elements can only be

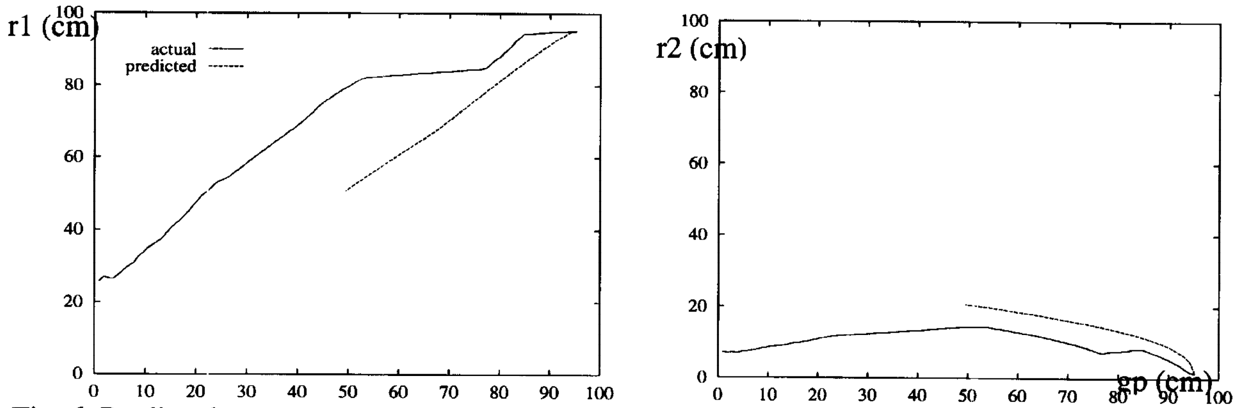

Fig. 6 Predicted versus actual maximum Pseudo Energy Radius considering pounding interactions.

analyzed using linear or nonlinear structural analysis programs, depending on the link behavior. A simple energy formulation to estimate the effectiveness of this technique is not available.

Bumper damper elements are link elements that are activated when a gap is closed. Such elements reduce the energy transfer during pounding and the high frequency acceleration pulses. Initial estimates on the effectiveness of this retrofit technique can be obtained using an equivalent coefficient of restitution for the damper. The damper will yield a smaller value for the coefficient of restitution, and therefore will lead to smaller PER amplifications (see Fig. 7). Supplemental energy dissipation devices can be used in the structures for pounding prevention or mitigation, depending on the amount of additional damping supplied. Using the PER spectrum, the minimum additional damping required to avoid pounding, or to reduce pounding effects to an acceptable level, can be calculated (see Fig. 8).

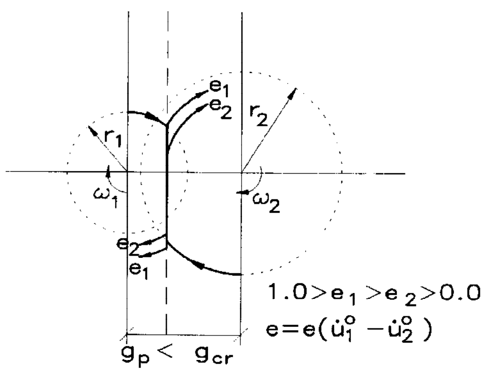

Fig. 7 Influence of the coefficient of restitution.

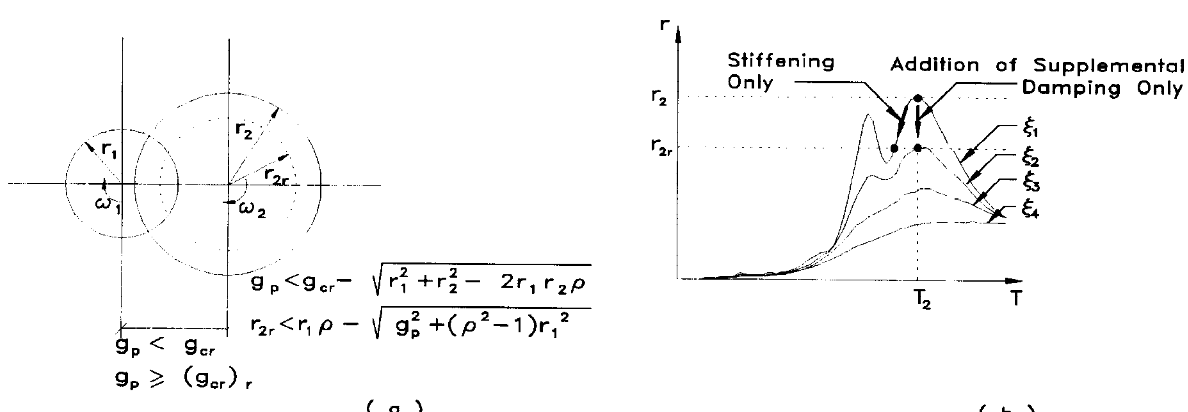

Fig. 8 Minimum supplemental damping for pounding prevention or mitigation.

CONCLUSIONS

The Pseudo Energy Radius (PER) was presented as a tool to study pounding problems using an energy approach. The PER provides an energy measurement that can be directly compared to the lateral deflections and gap sizes. The PER combined with the DDC rule can be used to estimate the critical gap size. Overlapping in the energy levels, governed by the correlation coefficient, is possible without inducing pounding effects. For bilinear structures subjected to Mexico City type of narrow banded earthquakes, the procedure suggested by Valles and Reinhorn (1996) is suggested, since the simple formula may lead to inadequate estimates. The single hit methodology, based on the PER, was presented to estimate pounding effects. The method yields a good approximation for gaps close to critical. The use of the PER formulation to estimate the effectiveness of bumper damper elements, and supplemental energy devices was discussed.

The use of the PER approach to pounding problems provides a simple and powerful method that can be easily understood and adopted by practicing engineers. The energy approach presented for buildings can be extended to study other types of structures that are prone to pounding damage, such as bridges and base isolated structures.

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