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BEARING CAPACITY OF STRIP FOOTINGS NEAR SLOPING GROUND DURING EARTHQUAKES

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ABSTRACT

Seismic bearing capacity factors are derived for strip footings situated near sloping ground using limit equilibrium technique. The failure mechanism is composed of an active and a passive wedge and a sheared transition zone is sandwiched between the two wedges. The most critical mechanism is found by trial and error. The acceptability of the solutions are checked. Results show that the bearing capacity factors are quadratic functions of the slope. The constants of the quadratic functions are given in tabular forms for a range of values of the friction angle and the seismic accelerations. The constants are derived for the footing at the edge of the slope and a technique is developed to use the same results when the footing is away from the edge. The effect of ground water table as well as the effect of different soil and structural accelerations are considered in a simple way.

KEY WORDS

Seismic Bearing Capacity, Shallow Strip Footing, Slope, Water Table, Factor of Safety.

INTRODUCTION

It is quite often necessary to site a structure near a sloping ground, particularly in hilly regions and obviously, the strength of the foundation material, the geometry of the slope and the proximity of the structure to the slope affect the load that the soil can carry. It is known that a slope exists with a factor of safety against failure without an external load and this safety factor is affected by any external load that is placed on it. The failure of the slope not only affects the particular structure but also has damaging consequences on the environment in general.

The bearing capacity defines the ultimate load that the foundation soil can sustain at the state of incipient failure and is usually expressed by the linear combination of the three bearing capacity factors, N_c, N_q, and N_γ as:

$$Q = q\,N_q + c\,N_c + 0.5\gamma B\,N_\gamma \tag{1}$$

where, Q is the ultimate bearing pressure of the footing, B is the base width of the footing, q is the surcharge pressure, c is the cohesion and γ is the unit weight of the soil. The bearing capacity factors are functions of the friction angle ϕ' of the soil, the seismic acceleration kg and also, they are functions of the slope angle β when the footings are situated near sloping ground.

Bearing capacity factors for non-seismic case on level ground are well defined and are available in literature. Most of these works also extend to the case of the sloping ground. Caquot and Kerisel (1956) provides tables of bearing capacity factors which are frequently used by the designers. Brinch Hansen (1961, 1970), Vesic (1975) provides empirical equations for these factors which take into account many design variables. For the seismic case on level ground, the usual practice is to consider inclined static load and use the tables available for static cases. These factors ignore the inertia of the soil mass. Sarma and Iossifelis (1990) derived the bearing capacity factors for the seismic case on level ground based on the limit equilibrium technique and provides design charts for these factors. These factors do take into account the inertia of the soil mass as well as that of the structure. For the seismic case near sloping ground, limited results are available from Shikhiev & Jakovlev (1977), Soubra & Reynolds (1992) and Sawada, Nomachi & Chen (1994). A detailed list of references and design values are available from Sarma & Chen (1995). Their results are also being presented here.

ANALYSIS

The method of analysis is presented in Sarma & Chen (1995). This is based on the limit equilibrium technique and is an extension of the method given by Sarma & Iossifelis (1990). The failure mechanism, as shown in figure 1, is composed of an active wedge and a passive wedge and a sheared transition zone is sandwiched between the two wedges. The curve linking the two wedges is composed of a log-spiral curve whose origin is not necessarily at the edge of the footing. The convenience for search is the reason for using a log-spiral and the analysis does not take into account the special property of the log-spiral. The shear zone radiates from the end of the footing and not from the centre of the spiral. The footing is placed at the edge of the slope on the flat ground. Mohr-Coulomb failure criterion defines the soil strength. On the internal radial surfaces, soil strength is fully mobilised which is an essential condition for incipient failure without which a kinematic slip mechanism cannot develop. If the kinematic slip mechanism condition is neglected, then it is possible to obtain smaller values of bearing capacity which implies that even though the stresses on the main rupture surface is at limiting equilibrium, a failure mechanism cannot develop. The analysis does not take into account the excess pore water pressure generated by the cyclic loading nor does it take into account any static pore water pressure. The analysis and the strength parameters therefore represent total stress conditions. The effect of ground water table at the surface level without any seepage pressure can be considered in the results as shown later. It is also assumed in the analysis that the seismic acceleration on the structure and on the soil is the same. This represents the average acceleration on the structure. In reality, the two accelerations may be different and a simple technique is presented here to take this difference into account. The effect of the footing being away from the edge of the slope is also considered.

EFFECT OF GROUND WATER ON BEARING CAPACITY

This assumes that the water table is static at the surface level and no seepage pressure is present. Only the N_γ parameter is affected. For the static case, I) use effective strength parameters; ii) use submerged unit weight of soil (γ') and forget water and determine N_γ; iii) $Q = 0.5\ \gamma'\ B\ N_\gamma$

For the Seismic case, we assume that there is no change in pore water pressure due to earthquake and I) again use effective strength parameters; ii) use submerged unit weight of soil γ' (and forget water); iii) modify seismic coefficient from k to $\bar{k}=k\gamma/\gamma'$; iv) determine $\bar{N}_\gamma$ from the charts corresponding to $\bar{k}$; v) modify $\bar{N}_\gamma$ to the following N_γ value:

$$N_\gamma = \mu\ \bar{N}_\gamma \qquad (2)$$

The reason for the technique is the following. The presence of the ground water (without any seepage pressure) is equivalent to using submerged weight of the soil in the static case but the inertia force of the soil mass corresponds to the total weight of the soil. Therefore, the seismic coefficient is changed to $\bar{k}$. The equilibrium of the system is maintained in the solution with $\bar{Q}$ and $\overline{kQ}$ as the load on the foundation whereas the corresponding load should be Q and kQ. Therefore, from the equilibrium of the active wedge (assuming that the geometry of the failure surface will be the same for both cases), we obtain $N_\gamma = \mu \bar{N}_\gamma$ where

$$\mu = \frac{\tan(\alpha_1 - \phi') + \bar{k}}{\tan(\alpha_1 - \phi') + k} \qquad (3)$$

We can accept

$$\alpha_1 = \frac{\pi}{4} + \frac{\phi'}{2} - \frac{1}{2}\left[\sin^{-1}\left(\frac{\sin i}{\sin \phi'}\right) + i\right] \qquad (4)$$

$$i = \tan^{-1}\bar{k} \quad or \quad \tan^{-1}k \qquad (5)$$

Use the value of i which gives smaller μ.

Example: $k=0.1$, $\phi' = 30°$, $\beta=0$; $\bar{k}=0.2$, $\bar{N}_\gamma= 9.53$, $\mu=1.2$, $N_\gamma=11.44$

EFFECT OF DIFFERENT ACCELERATION ON THE FOOTING AND IN THE SOIL

The solution is obtained by considering the equilibrium of the active wedge assuming that the failure geometry for the variable acceleration is the same as that for similar accelerations. Let us assume that the soil acceleration is $\bar{k}$g while the structure acceleration is kg. We obtain the bearing capacity factor $\bar{N}_\gamma$ corresponding to the acceleration $\bar{k}$g. The factor is then modified according to equation 2. If $k > \bar{k}$, then it is safer to determine N_γ directly for k without any modification.

EFFECT OF THE POSITION OF THE FOOTING AWAY FROM THE EDGE

When the footing is placed away from the edge, the safety is obviously increased. The geometry of the critical failure surface depends on the slope angle when the footing is on edge and it changes when the footing is placed away from the edge. Ultimately, when the footing is placed so far away that the failure geometry is contained in the level part of the ground, then the slope has no effect on the bearing capacity. We therefore try to find an equivalent slope angle for footing on edge which corresponds to the distance away from the edge. Figure 2 shows the equivalent geometry. For the N_q case, the angles $\alpha_{1,2,3,4}$ are clearly defined and the centre of the spiral is at the corner. Therefore L_p is known and given d and β', β can be calculated. Therefore β' is a function of d and β. For the N_c case, the angles α_1 and α_4 are undefined while for the N_γ case the centre of the spiral is not necessarily at the corner even though the angles $\alpha_{1,2,3,4}$ are defined and therefore the relationship between β', β and d is to be determined numerically. Figure 3 shows such a relationship. The weight of the soil above the β' line is distributed over the slope as a surcharge load and its effect is considered through the N_q parameter.

RESULTS

Figure 4 shows the effect of the cohesion and the weight on the bearing capacity for a specific slope and seismic acceleration. These curves are obtained by considering the two factors simultaneously in the analysis and therefore gives slightly higher values of bearing capacity compared to that obtained by the combination of the N_c and the N_γ values. The figure shows that the contours of equal bearing capacity can be defined by curved lines but the parameters defining these lines are functions of the bearing capacity itself along with the slope angle and the seismic accelerations. Therefore, this is not a convenient way to present the result. However, it should be noticed that if we consider a point X on this graph which corresponds to the available soil properties (ϕ=35°, $c/\gamma B$=0.5), then we get a bearing capacity ($Q/\gamma B$) of this soil as 18. If we apply a factor of safety on bearing capacity equal to 2, then this corresponds to a point X_d as shown in the figure. This point represents a factor of safety on available strength of 1.25 which is obtained by the ratio OX/OX_d. This shows clearly that there is no correspondence between the factor of safety on bearing capacity and the factor of safety on strength of material.

Bearing capacity factors, which are reproduced here from Sarma & Chen(1995) are represented as quadratic functions of the angle β° (for N_c) and of tan(β) (for N_γ and N_q). The constants of the quadratic functions are presented in Tables 1,2,3. The errors associated with using the quadratic functions are less than 4% compared to the calculated values.

CONCLUSIONS

Limit equilibrium technique provides a convenient and useful tool in determining the bearing capacity factors for shallow strip foundations. The factors are quadratic functions of the slope angle and the constants of the quadratic functions are dependent on the friction angle and the seismic coefficient. The results for the footing on edge can be easily transformed for the case of footing away from the edge. The presence of ground water can be accommodated in determining the bearing capacity factors. Also the factors for the variable seismic accelerations on the soil and the structure can be obtained from the results of the similar accelerations. The factor of safety on bearing capacity has no relationship with the factor of safety on the strength of the material.

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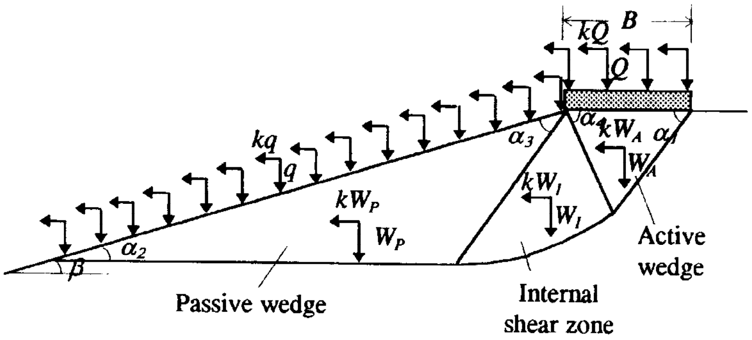

Fig.1. The failure mechanism and the applied forces of the foundation-soil system.

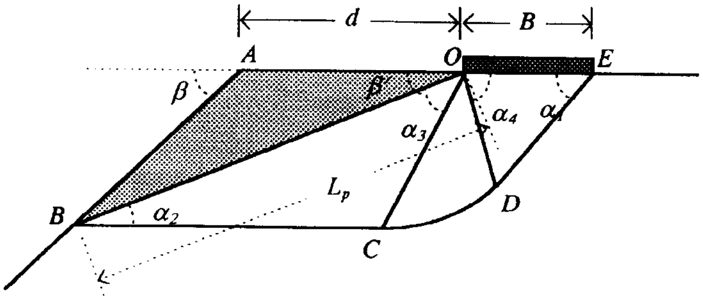

Fig. 2. Geometry of the failure surface for footings away from the edge.

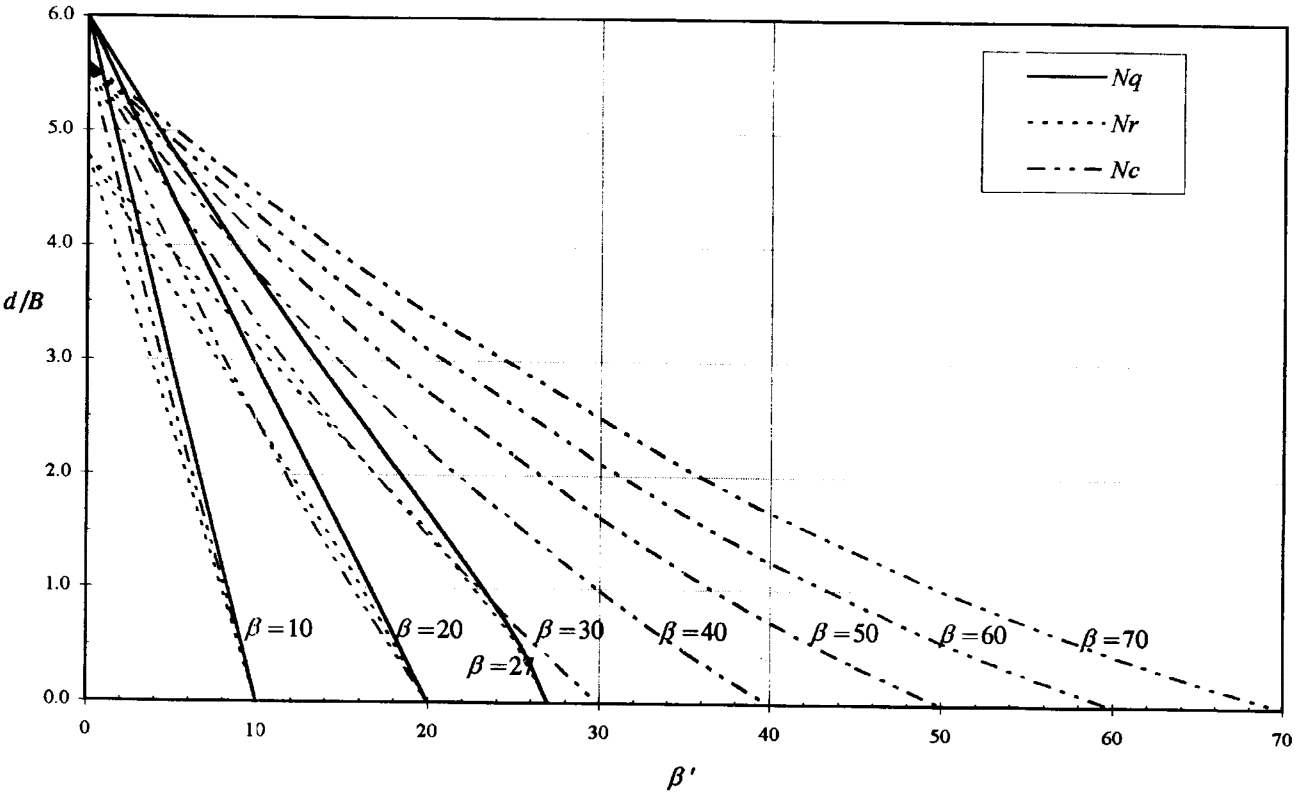

Fig.3. β curves (with respect to d & β') determined from N_c, N_q & N_γ conditions, when ϕ =40, k =0.2 (ϕ, β, β' are in degrees)

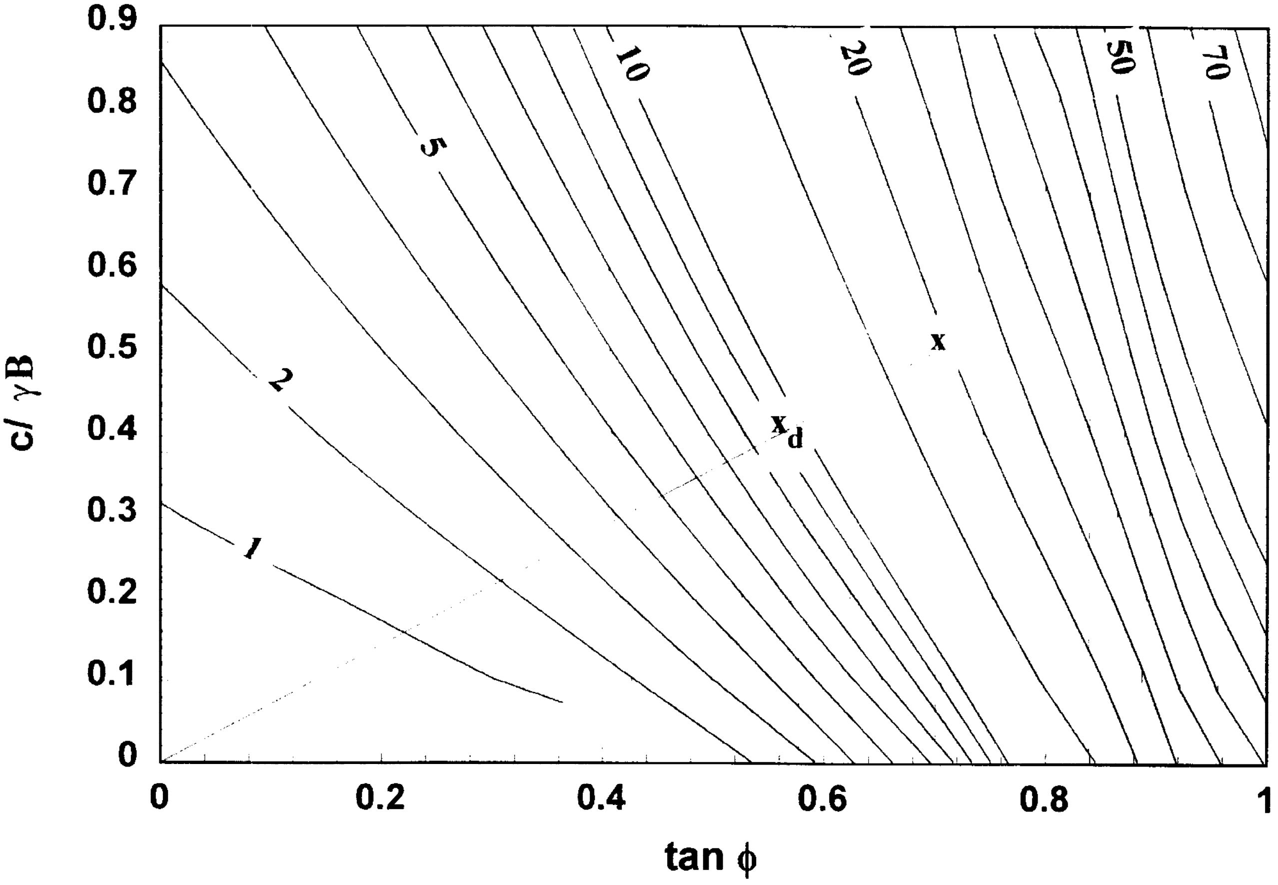

Fig. 4. Contours of Equal Bearing Capacity Q/γB

β=10° & k = 0.2

Table 1. N_r **expressed in terms of slope angle, i.e.** $\log(N_r)=a\tan^2\beta+b\tan\beta+c$**, with respect to friction angle (**ϕ **in degrees) and ground acceleration (**kg**).**

k \ ϕ	10	15	20	25	30	35	40	45	50
0	-6.492 -0.595 0.085	-3.923 -0.483 0.435	-2.479 -0.550 0.759	-1.755 -0.634 1.075	-1.205 -0.780 1.402	-0.830 -0.940 1.750	-0.545 -1.130 2.130	-0.252 -1.380 2.564	-0.006 -1.680 3.068
0.1		-10.440 -0.680 0.166	-4.234 -0.740 0.535	-2.600 -0.790 0.875	-1.724 -0.894 1.212	-1.177 -1.035 1.561	-0.796 -1.210 1.940	-0.436 -1.440 2.364	-0.157 -1.720 2.853
0.2			-12.415 -0.880 0.215	-4.895 -0.930 0.614	-2.742 -1.020 0.979	-1.786 -1.130 1.342	-1.139 -1.290 1.723	-0.699 -1.500 2.141	-0.369 -1.770 2.625
0.3				-14.210 -1.060 0.262	-5.370 -1.130 0.692	-2.960 -1.230 1.087	-1.796 -1.370 1.482	-1.131 -1.570 1.907	-0.693 -1.810 2.379
0.4					-14.880 -1.240 0.326	-5.638 -1.330 0.786	-3.030 -1.450 1.212	-1.776 -1.635 1.646	-1.138 -1.860 2.120
0.5						-15.115 -1.420 0.422	-5.544 -1.530 0.907	-3.008 -1.700 1.368	-1.789 -1.895 1.845
0.6							-13.830 -1.610 0.557	-5.331 -1.760 1.066	-2.970 -1.935 1.565
0.7								-12.394 -1 820 0.747	-5.530 -1.980 1.282
0.8									-9.560 -2.020 0.964

* Note:

1. The values listed above, from top to bottom, are the coefficients a, b & c respectively.
2. The coefficients are valid for $(i+\beta)<\phi$ only, outside this region, N_r drops to zero.

Table 2. N_q **expressed in terms of slope angle, i.e.** $\log(N_q)=a\tan^2\beta+b\tan\beta+c$**, with respect to friction angle (**ϕ **in degrees) and ground acceleration (**kg**).**

k \ ϕ	10	15	20	25	30	35	40	45	50
0	-3.530 -0.363 0.393	-2.825 -0.328 0.594	-2.119 -0.370 0.802	-1.638 -0.431 1.020	-1.282 -0.519 1.255	-0.899 -0.677 1.513	-0.638 -0.828 1.799	-0.382 -1.030 2.125	-0.187 -1.243 2.502
0.1		-4.383 -0.586 0.486	-2.819 -0.573 0.699	-2.069 -0.595 0.918	-1.575 -0.652 1.151	-1.200 -0.742 1.402	-0.892 -0.867 1.680	-0.624 -1.027 1.994	-0.384 -1.230 2.359
0.2			-5.105 -0.802 0.559	-2.932 -0.788 0.789	-2.130 -0.788 1.025	-1.616 -0.834 1.276	-1.227 -0.924 1.549	-0.905 -1.058 1.855	-0.622 -1.239 2.208
0.3				-6.694 -0.925 0.625	-3.583 -0.910 0.876	-2.514 -0.905 1.132	-1.868 -0.957 1.404	-1.397 -1.062 1.705	-1.015 -1.223 2.048
0.4					-7.756 -1.060 0.696	-4.012 -1.049 0.970	-2.769 -1.046 1.248	-2.033 -1.110 1.549	-1.503 -1.239 1.885
0.5						-7.562 -1.259 0.781	-3.998 -1.232 1.079	-2.771 -1.235 1.385	-2.032 -1.318 1.720
0.6							-7.968 -1.359 0.889	-4.602 -1.294 1.211	-3.143 -1.335 1.550
0.7							-59.079 1.142 0.663	-12.959 -0.839 1.023	-7.959 -0.900 1.371
0.8									-5.779 -1.766 1.199

* Note:

1. The values listed above, from top to bottom, are the coefficients a, b & c respectively.
2. The coefficients are valid for $(i+\beta)<\phi$ only, outside this region, N_q drops to zero.

Table 3. N_c **expressed in terms of slope angle (in degrees), i.e.** $\log(N_c)=a\beta^2+b\beta+c$, **with respect to friction angle (**ϕ **in degrees) and ground acceleration (**kg**).**

ϕ k	5	10	15	20	25	30	35	40	45	50
0 ($\beta<=90$)	-2.275E-05 -3.099E-03 0.8080	-2.237E-05 -3.922E-03 0.9173	-2.173E-05 -4.870E-03 1.0363	-2.083E-05 -5.962E-03 1.1670	-1.968E-05 -7.221E-03 1.3121	-1.829E-05 -8.677E-03 1.4749	-1.670E-05 -1.037E-02 1.6597	-1.492E-05 -1.237E-02 1.8728	-1.298E-05 -1.475E-02 2.1228	-1.093E-05 -1.765E-02 2.4227
0.1 ($\beta<=80$)	-2.010E-05 -3.207E-03 0.7543	-1.961E-05 -4.047E-03 0.8598	-1.879E-05 -5.015E-03 0.9746	-1.780E-05 -6.119E-03 1.1004	-1.655E-05 -7.390E-03 1.2398	-1.509E-05 -8.858E-03 1.3959	-1.344E-05 -1.056E-02 1.5728	-1.164E-05 -1.256E-02 1.7765	-9.723E-06 -1.495E-02 2.0150	-7.749E-06 -1.784E-02 2.3006
0.2 ($\beta<=75$)	-1.943E-05 -3.012E-03 0.6837	-1.850E-05 -3.909E-03 0.7869	-1.741E-05 -4.920E-03 0.8985	-1.606E-05 -6.069E-03 1.0205	-1.456E-05 -7.376E-03 1.1551	-1.292E-05 -8.872E-03 1.3053	-1.112E-05 -1.060E-02 1.4751	-9.212E-06 -1.263E-02 1.6700	-7.213E-06 -1.503E-02 1.8978	-5.259E-06 -1.793E-02 2.1699
0.3 ($\beta<=70$)	-2.076E-05 -2.484E-03 0.5980	-1.875E-05 -3.500E-03 0.7002	-1.673E-05 -4.615E-03 0.8101	-1.474E-05 -5.845E-03 0.9293	-1.269E-05 -7.220E-03 1.0601	-1.050E-05 -8.782E-03 1.2055	-8.329E-06 -1.056E-02 1.3692	-6.111E-06 -1.263E-02 1.5564	-3.941E-06 -1.507E-02 1.7743	-1.861E-06 -1.801E-02 2.0339
0.4 ($\beta<=65$)	-2.622E-05 -1.473E-03 0.4977	-2.199E-05 -2.701E-03 0.6008	-1.837E-05 -3.977E-03 0.7103	-1.474E-05 -5.366E-03 0.8282	-1.147E-05 -6.867E-03 0.9565	-8.325E-06 -8.533E-03 1.0982	-5.374E-06 -1.040E-02 1.2568	-2.584E-06 -1.255E-02 1.4373	6.716E-08 -1.505E-02 1.6466	2.418E-06 -1.804E-02 1.8948
0.5 ($\beta<=60$)	-3.502E-05 -1.076E-04 0.3892	-3.131E-05 -1.318E-03 0.4892	-2.406E-05 -2.898E-03 0.6003	-1.786E-05 -4.512E-03 0.7183	-1.221E-05 -6.223E-03 0.8457	-7.307E-06 -8.053E-03 0.9849	-2.891E-06 -1.007E-02 1.1398	1.037E-06 -1.233E-02 1.3148	4.480E-06 -1.493E-02 1.5167	7.382E-06 -1.800E-02 1.7549
0.6 ($\beta<=55$)	-4.259E-05 1.166E-03 0.2843	-3.916E-05 -6.580E-06 0.3797	-3.503E-05 -1.362E-03 0.4837	-2.622E-05 -3.178E-03 0.6011	-1.693E-05 -5.174E-03 0.7287	-9.037E-06 -7.252E-03 0.8671	-1.840E-06 -9.490E-03 1.0194	4.022E-06 -1.193E-02 1.1904	9.012E-06 -1.468E-02 1.3862	1.293E-05 -1.787E-02 1.6159
0.7 ($\beta<=55$)	-4.670E-05 2.118E-03 0.1905	-4.836E-05 1.411E-03 0.2724	-4.317E-05 3.578E-06 0.3728	-3.860E-05 -1.505E-03 0.4813	-2.790E-05 -3.621E-03 0.6067	-1.530E-05 -6.045E-03 0.7455	-4.209E-06 -8.581E-03 0.8969	5.644E-06 -1.130E-02 1.0653	1.258E-05 -1.424E-02 1.2563	1.879E-05 -1.762E-02 1.4790
0.8 ($\beta<=50$)	-5.563E-05 3.442E-03 0.0941	-5.465E-05 2.520E-03 0.1770	-4.903E-05 1.109E-03 0.2733	-4.867E-05 -3.935E-05 0.3703	-4.282E-05 -1.787E-03 0.4852	-2.797E-05 -4.357E-03 0.6213	-1.101E-05 -7.288E-03 0.7731	3.701E-06 -1.035E-02 0.9403	1.479E-05 -1.358E-02 1.1279	2.392E-05 -1.720E-02 1.3450
0.9 ($\beta<=45$)		-5.321E-05 2.913E-03 0.1025	-6.136E-05 2.637E-03 0.1726	-6.016E-05 1.475E-03 0.2650	-5.334E-05 -3.119E-04 0.3757	-5.314E-05 -1.954E-03 0.4942	-2.503E-05 -5.518E-03 0.6485	-2.484E-06 -9.064E-03 0.8162	1.423E-05 -1.265E-02 1.0017	2.771E-05 -1.657E-02 1.2146
1 ($\beta<=45$)			-6.215E-05 3.241E-03 0.0950	-6.413E-05 2.407E-03 0.1762	-6.098E-05 9.672E-04 0.2755	-5.879E-05 -6.684E-04 0.3869	-4.751E-05 -3.243E-03 0.5240	-1.680E-05 -7.266E-03 0.6920	8.605E-06 -1.138E-02 0.8776	2.872E-05 -1.570E-02 1.0876
1.1 ($\beta<=40$)				-7.432E-05 3.669E-03 0.0888	-6.936E-05 2.181E-03 0.1842	-7.528E-05 1.104E-03 0.2821	-7.174E-05 -9.768E-04 0.4068	-4.187E-05 -4.929E-03 0.5689	-3.117E-06 -9.717E-03 0.7557	2.643E-05 -1.457E-02 0.9646
1.2 ($\beta<=35$)						-9.020E-05 2.641E-03 0.1883	-8.167E-05 3.715E-04 0.3095	-6.600E-05 -2.754E-03 0.4552	-2.601E-05 -7.543E-03 0.6358	1.937E-05 -1.313E-02 0.8453
1.3 ($\beta<=35$)						-8.886E-05 3.231E-03 0.1137	-8.499E-05 1.337E-03 0.2237	-8.110E-05 -1.001E-03 0.3527	-5.965E-05 -4.864E-03 0.5174	1.698E-06 -1.120E-02 0.7282
1.4 ($\beta<=35$)							-9.332E-05 2.495E-03 0.1422	-1.084E-04 1.312E-03 0.2500	-7.865E-05 -2.884E-03 0.4127	-2.324E-05 -8.926E-03 0.6142
1.5 ($\beta<=30$)								-1.343E-04 3.228E-03 0.1615	-9.904E-05 -1.067E-03 0.3178	-6.637E-05 -6.065E-03 0.5033

* Note:

1. The values listed above , from top to bottom, are the coefficients a, b & c respectively.
2. β and ϕ are in degrees.