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SEISMIC BEHAVIOUR OF SHALLOW FOUNDATIONS BY SIMPLE ELASTO-PLASTIC MODELS

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ABSTRACT

The earthquake response of shallow foundations, including the nonlinear interaction effects with the superstructure, is studied by a simple method that, unlike other simplified approaches, directly takes into account the coupling between the soil-foundation system and the superstructure. The proposed model consists of a 4 degree-of-freedom (dof) oscillator, namely 1 dof for the superstructure and 3 for the foundation, and introduces nonlinear effects in the calculation of soil reactions through a failure criterion and plastic potential taken from previous experimental work. Comparing the results of an application to granular soils with those of more sophisticated finite element analyses, a satisfactory agreement is found, suggesting that the main features of this complicated nonlinear problem can be captured without resorting to an accurate description of the soil behaviour.

KEYWORDS

Shallow foundations, dynamic nonlinear interaction, failure surface, plastic potential, 4 dof model.

INTRODUCTION

Recent research on the seismic bearing capacity of shallow foundations (Sarma and Iossifelis, 1990; Pecker and Salençon, 1991; Budhu and Al-Karni, 1993) has lead to a better understanding of the governing factors. Using yield design theory, Pecker and Salençon (cit.) and Paolucci and Pecker (1996) have shown that both for cohesive and granular soils, the detrimental effects of soil inertia become significant only when the foundation is designed with a low factor of safety (indicatively $F_s < 2$). Otherwise, they can be neglected with respect to the inertial actions transmitted by the superstructure.

On the other hand, truly dynamic approaches to the evaluation of permanent foundation displacements during earthquakes have been relatively few, and mostly limited to the well-known method of Newmark (1965), under the basic assumption that such displacements develop only upon the attainment of failure loads. Sarma and Iossifelis (cit.) and Richards et al. (1993) treat the soil as an assemblage of rigid blocks under the foundation , whereas Pecker and Salençon (cit.) and Pecker (1994) relaxed this assumption by introducing soil deformability. Also crucial to the Newmark class of models is the decoupling between the superstructure response, generally calculated by standard structural methods, and the foundation response. This "substructures" approach has the important drawback of neglecting the effects of plastic flow in the soil on the inertial loads that the superstructure transfers to the foundation.

A more complete and rigorous approach would be the global modelling of the soil-foundation-superstructure system by finite elements. In this case, realistic prediction of the permanent displacements would require a sophisticated constitutive description for the soil, with substantial increase of the computational load.

To overcome some of the previous problems, a simple method is presented here for nonlinear foundation interaction analyses and the results obtained are compared to those yielded by a more rigorous approach.

OUTLINE OF THE METHOD

The simple 4 degrees of freedom (dof) system illustrated in Fig. 1 is considered, with 1 dof describing the movement of the superstructure and 3 dof that of the foundation. The dynamic equilibrium is described by the following set of equations:

$$\underline{\underline{M}}\,\underline{\ddot{x}}_{n+1} + \underline{\underline{C}}\,\underline{\dot{x}}_{n+1} + \underline{\underline{K}}^s \underline{x}_{n+1} + \underline{F}_{n+1} = \underline{p}_{n+1} \qquad (1)$$

where:

$$\underline{x} = \begin{bmatrix} x_1 & x_0 & \phi & x_v \end{bmatrix}^T;\ \underline{F} = \begin{bmatrix} 0 & H & M & V \end{bmatrix}^T;\ \underline{p} = \begin{bmatrix} -m_1\ddot{x}_g & -m_0\ddot{x}_g & 0 & -(m_1+m_0)\ddot{y}_g \end{bmatrix}^T$$

$$\underline{\underline{M}} = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_0 & 0 & 0 \\ 0 & 0 & J & 0 \\ 0 & 0 & 0 & m_1+m_0 \end{bmatrix};\ \underline{\underline{C}} = \begin{bmatrix} c_1 & -c_1 & -c_1\bar{h} & 0 \\ -c_1 & c_1+c_0 & c_1\bar{h} & 0 \\ -c_1\bar{h} & c_1\bar{h} & c_1\bar{h}^2+c_r & 0 \\ 0 & 0 & 0 & c_v \end{bmatrix};\ \underline{\underline{K}}^s = \begin{bmatrix} k_1 & -k_1 & -k_1\bar{h} & 0 \\ -k_1 & k_1 & k_1\bar{h} & 0 \\ -k_1\bar{h} & k_1\bar{h} & k_1\bar{h}^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The following notations are used:

x_1, x_0=displacement of the structure and of the basement, respectively, relative to the soil;

ϕ = rocking motion of the basement;

x_v = vertical displacement of the structure;

$\ddot{x}_g, \ddot{y}_g$= horizontal and vertical component, respectively, of ground acceleration;

m_1 = effective mass of the superstructure;

m_0 = mass of the foundation

J = sum of the centroidal moments of inertia of the building and the foundation;

$\bar{h}$ = effective height of the superstructure;

k_1, c_1 = elastic stiffness and damping of the superstructure;

c_0, c_r, c_v = equivalent elastic spring and dashpot coefficients of the soil-foundation system corresponding to the translational, rocking and vertical modes of vibration, respectively;

H, M, V = soil reactions, horizontal, rotational and vertical, respectively.

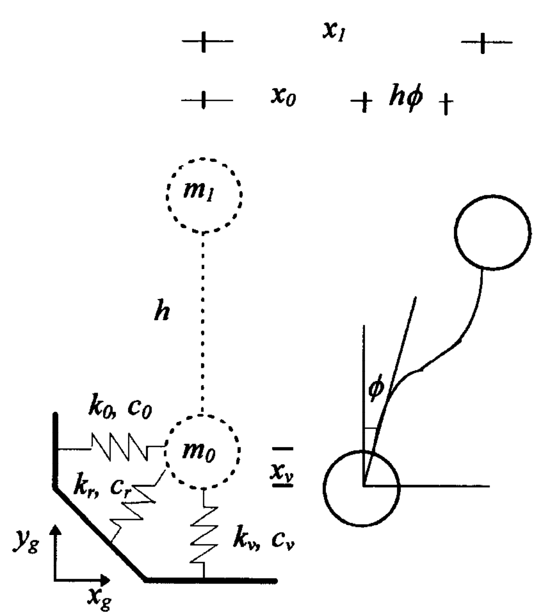


Fig. 1. The 4 dof model for the analyses of non-linear dynamic soil-structure interaction.

Note that the structural stiffness matrix $\underline{\underline{K}}^s$ is separated from the vector $\underline{F}$ of the soil reactions, where the non-linearity of the problem is concentrated. The non-linearity is introduced in the calculation of the soil reactions, whereas the superstructure is assumed to behave elastically. A failure surface in the H-M-V space is introduced and the soil behaviour is assumed to be linear visco-elastic until the failure surface is reached. This means that the plastic deformations induced by an earthquake for load conditions inside the failure surface are negligible with respect to those occurring at failure. For simplicity, no hardening is considered, although it could significantly affect the development of permanent deformations. Thus, the overall soil behaviour is assumed to be elastic-perfectly plastic. The radiation damping in the terms c_0, c_r, c_v is considered also during the plastic calculations.

If we call $f(\underline{F})$ the yield function which defines the failure surface (with $f(\underline{F}) \leq 0$), and $g(\underline{F})$ the plastic potential, the vector of the soil reactions can be calculated in one of the following two ways:

- if $f(\underline{F}) < 0$, or ($f(\underline{F}) = 0$ and $df(\underline{F}) < 0$), the response is elastic, so that:

$$\underline{F}_{n+1} = \underline{F}_n + \underline{\underline{K}}^i (\underline{x}_{n+1} - \underline{x}_n), \text{ where: } \underline{\underline{K}}^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k_0 & 0 & 0 \\ 0 & 0 & k_r & 0 \\ 0 & 0 & 0 & k_v \end{bmatrix} \tag{2}$$

and k_0, k_r, k_v are the equivalent elastic spring and dashpot coefficients of the soil-foundation system corresponding to the translational, rocking and vertical modes of vibration, respectively;

- if $f(\underline{F}) = 0$ and $df(\underline{F}) = 0$, plastic flow occurs, with

$$\underline{F}_{n+1} = \underline{F}_n + \underline{\underline{K}}^{iep} (\underline{x}_{n+1} - \underline{x}_n) \tag{3}$$

where $\underline{\underline{K}}^{iep}$ is the elastoplastic stiffness matrix of the soil foundation system. This can be calculated in terms of the elastic stiffness matrix $\underline{\underline{K}}^i$ and the derivatives of the yield and plastic potential functions (see e.g. Zienkiewicz and Taylor, 1991):

$$\underline{\underline{K}}^{iep} = \underline{\underline{K}}^i - \underline{\underline{K}}^i \left(\frac{\partial g}{\partial \underline{F}}\right) \left(\frac{\partial f}{\partial \underline{F}}\right)^T \underline{\underline{K}}^i \left[\left(\frac{\partial f}{\partial \underline{F}}\right)^T \underline{\underline{K}}^i \left(\frac{\partial g}{\partial \underline{F}}\right)\right]^{-1} \tag{4}$$

The time integration is carried out by the explicit Newmark scheme, whereas the solution of the non-linear system (1) is obtained by the modified Newton-Raphson method.

CHOICE OF THE YIELD FUNCTION AND FLOW RULE

The previous method applies both for cohesive and granular soils, provided that an adequate yield function and flow rule are chosen. For cohesive soils, the assumption of associated flow rule makes things easier, in that the yield function and plastic potential coincide. In this paper, however, we will concentrate on the case of granular soils in drained conditions. The failure of model strip footings on dry granular soils under general planar loading conditions, as well as the development of permanent displacements, has been thoroughly studied in recent years, particularly by Butterfield and Gottardi (1994), Gottardi and Butterfield (1995) and by Nova and Montrasio (1991). Due to the simplicity of the analytical expressions and the good agreement with the experimental results, the yield function and the plastic potential proposed by Nova and Montrasio (cit.) have been considered, namely:

$$f(\underline{F}) = h^2 + m^2 - v^2(1-v)^{2\beta} \tag{5}$$

$$g(\underline{F}) = \lambda^2 h^2 + \chi^2 m^2 - v^2(1-v)^{2\beta} \tag{6}$$

where $h=H/\mu V_{max}$, $m=M/\psi BV_{max}$, $v=V/V_{max}$, are the normalized soil reactions and μ, ψ, β, λ, χ are model parameters, whose values are discussed in the quoted paper. V_{max} is the static bearing capacity under vertical loads. Note that a non-associative flow rule has been considered in this case.

COMPARISON WITH 2D FINITE ELEMENT SIMULATIONS

This simplified method has been tested against a dynamic finite element simulation of the seismic response of a shear beam (superstructure) supported by a shallow strip foundation, 4 m wide, resting on cohesionless soil in drained conditions. The superstructure is 16.2 m high and has a fundamental period of vibration $T_0 = 0.5$ s. In static conditions, it transmits to the soil a vertical load V_d = 600kN. The finite element mesh is shown in Fig. 2. The soil behaviour has been described by the elastoplastic model of Hujeux (1985), which is a generalization of the well known Cam Clay model, with mobilization of both deviatoric and isotropic plastic mechanisms. The parameters of the model were chosen by fitting static and cyclic drained triaxial tests data for the medium dense Hostun sand, with relative density $D_r \cong 65\%$ (Faccioli et al., 1995). The elastic shear modulus G varies with depth as:

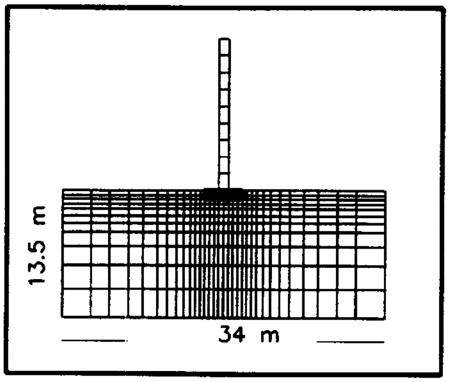
$$G = G_0\left(\frac{p}{p_0}\right)^n \tag{7}$$

where p = effective confining pressure, G_0 = 250Mpa, p_0 = 1Mpa, n = 0.5. The dynamic analyses were carried out by the finite element code GEFDYN (Aubry et al., 1986). A base excitation denoted as Gemona 13 has been considered, consisting of a real accelerogram from the September 1976 Friuli (Italy) strong aftershocks, with peak ground acceleration 0.63g.

The elastic spring and dashpot coefficients of the interaction problem have been calculated by the DYNA2 code (Novak et al., 1983), and the following parameters were used for the analyses with the simplified method:
$m_1 = 5\cdot10^4$ Kg, $k_1 = 7.9\cdot10^6$ N/m, c_1/c_{cr}= 10%, $m_0=1\cdot10^4$ Kg, $k_0 = 0.5\cdot10^8$ N/m, $c_0 = 1\cdot10^6$ Kgs/m, $J = 4.8\cdot10^4$ Kgm^2, $k_r = 1.5\cdot10^9$Nm/rad, $k_v = 5.5\cdot10^8$ N/m, $c_v = 3\cdot10^6$ Kgs/m, $\bar{h}$ = 10.8 m.

The model parameters $\mu = 0.46$, $\psi = 0.5$, $\beta = 0.95$, $\lambda = 0.4$, $\chi = 0.4$, close to the values suggested by Nova and Montrasio (cit.), have been chosen for the expressions (5) and (6) of the yield function and plastic potential. Note that μ is the tangent of the friction angle at the soil-foundation interface and has been calculated as $2/3\tan\phi$, where $\phi = 35°$ is the soil friction angle; the value of ψ indicates the maximum eccentricity ratio e/B that can be sustained by the foundation under vertical eccentric load. $V_{max}=9.1\cdot10^6$ was calculated by the standard superposition formula for the bearing capacity of shallow strip foundations on granular soils.
In Fig. 4 the horizontal, vertical and rocking components of the base motion obtained by the proposed method are compared with the relative motion between the foundation and the free-field calculated by finite element simulations. A satisfactory agreement is evident for the displacement components, whereas for the rocking motion the solutions agree only in the first 3 sec of the excitation. After the strong acceleration pulse occurring at ~3.2s, the behaviour predicted by the two solutions diverges, probably because of soil hardening effects that are not considered in this present version of the simplified method. Also, the latter does not take into account the changes of the center of rotation of the foundation during possible phases of uplift.

The plots of the base horizontal force vs. displacement and the base moment vs. rocking (Fig. 5) obtained by the simplified method show that the combination of shear force and overturning moment lead to reaching the failure surface during many phases of the excitation, although the final permanent displacements are not very large. In this respect, it should be noted that the non-linear response of the foundation system causes a significant reduction of the structural response, predicted both by the present simplified method and by the finite element simulation (Fig. 6).



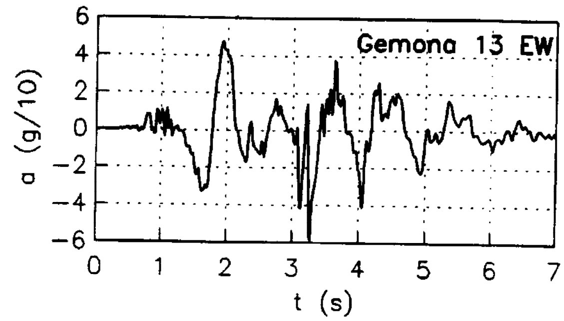


Fig. 2. Finite element mesh for dynamic analyses on non-linear soil-structure interaction.

Fig. 3. Base excitation.

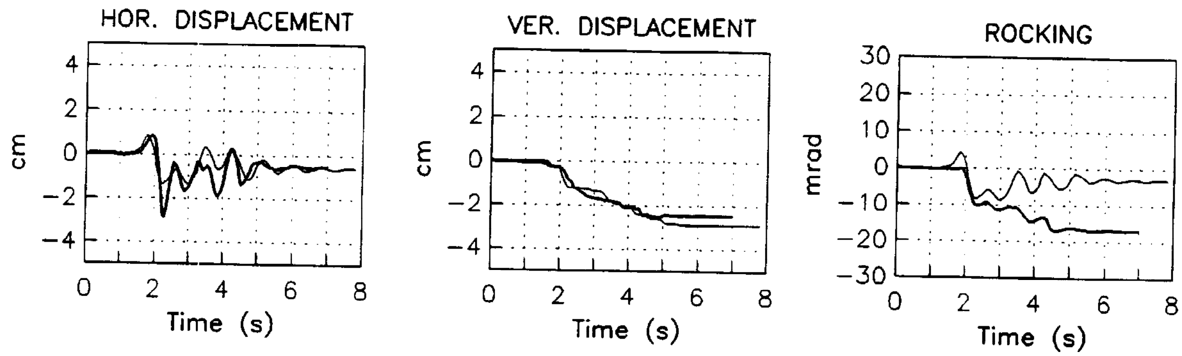


Fig. 4. Displacement and rocking components of base motion Thick line: finite element method. Thin line: present simplified method.

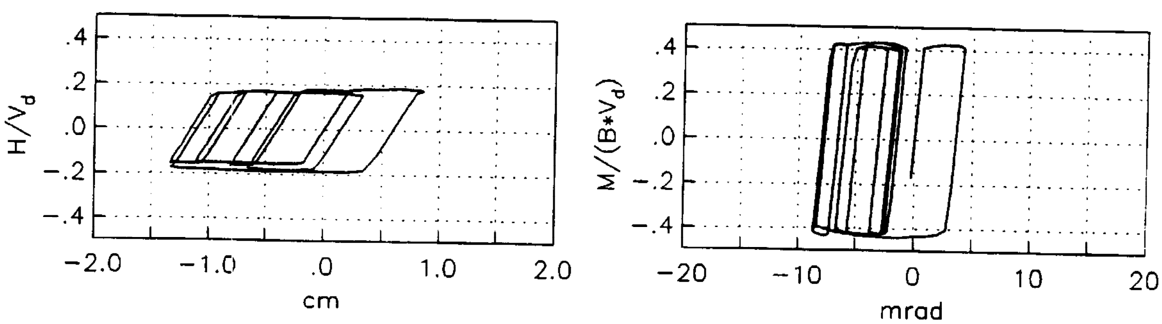


Fig. 5. Left: Horizontal soil reaction vs. displacement; right: moment vs. rocking. The values are normalized by the static vertical load V_d.

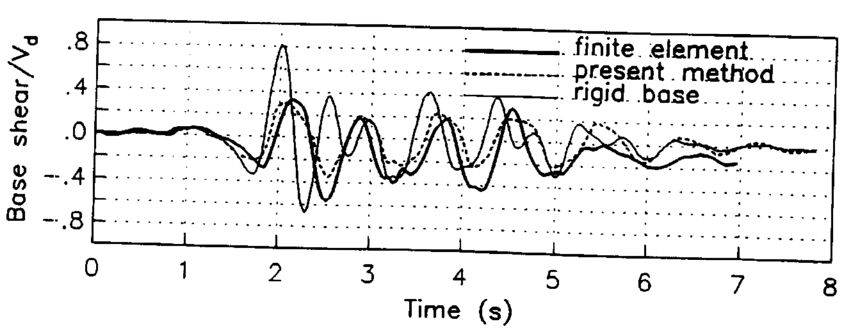


Fig. 6. Time history of the base shear normalized by V_d . Thick line: finite elements; Dashed line: present method; Thin line: rigid base assumption without non-linear effects.

CONCLUSIONS

In spite of the present limitations, the simple elasto-plastic model presented in this paper has been shown to describe in a satisfactory way most of the salient features of the seismic response of soil-foundation systems dynamically coupled with the superstructure. Additional work is needed for improving the non-linear description of the soil-foundation behaviour, mainly by the introduction of soil hardening, and for testing the method against experimental observations during earthquakes. Considering the heavy computational burden of sophisticated finite element analyses, this method seems to be a promising tool for simplified parametric analyses of non-linear soil structure interaction during earthquakes.

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