Paper No. 2016. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

11 WCEE

EMILIO ROSENBLUETH'S SELECTED RESULTS IN STRUCTURAL DYNAMICS

Anil K. Chopra

Department of Civil and Environmental Engineering
707 Davis Hall, University of California, Berkeley, CA 94720, U.S.A.

ABSTRACT

A historical perspective is presented for three important contributions of Emilio Rosenblueth to structural dynamics: (1) two modal combination rules; (2) a rule to combine effects of ground motion components; and (3) a response analysis procedure for multiple-support excitation.

KEY WORDS

Dynamics; earthquakes; linear analysis; modal combination rules; multiple excitations; response spectrum; Rosenblueth.

INTRODUCTION

Emilio Rosenblueth, through his contributions to research in earthquake engineering, had a profound influence on earthquake analysis and design of structures. His many seminal contributions have covered an amazingly wide variety of subjects, and many have become an integral part of textbooks and engineering practice.

They are so well integrated into earthquake engineering as we know it today that many researchers and engineers using these procedures are unaware of their origins. This paper provides a historical perspective of three of the perhaps most important contributions of Emilio Rosenblueth to the analysis of the dynamic response of structures. The contributions selected are (1) two rules for combining the peak modal responses to estimate the peak response of multi-degree-of-freedom (MDF) systems; (2) a rule to combine the peak responses due to various components of ground motion; and (3) a response analysis procedure for multiple support excitation.

MODAL COMBINATION RULES

Modal Analysis: A Review (Chopra, 1995)

The response of a viscously damped system described by the vector $\mathbf{u}$ of nodal displacements relative to the moving base is governed by the N (= number of degrees of freedom) differential equations

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{p}_{\text{eff}}(t) \tag{1}$$

where the effective earthquake forces are

$$\mathbf{p}_{\text{eff}}(t) = -\mathbf{m}\boldsymbol{\iota}\ddot{u}_g(t) \tag{2}$$

In Eqs. (1) and (2) $\mathbf{m}$, $\mathbf{c}$, and $\mathbf{k}$ are the mass, damping, and stiffness matrices, $\ddot{u}_g(t)$ is the ground acceleration, and $\boldsymbol{\iota}$ is the influence vector representing the displacements of the masses resulting from static application of a unit-ground displacement, $u_g = 1$.

The spatial distribution $\mathbf{s} = \mathbf{m}\boldsymbol{\iota}$ of the effective earthquake forces is expanded as

$$\mathbf{m}\boldsymbol{\iota} = \sum_{n=1}^{N} \mathbf{s}_n = \sum_{n=1}^{N} \Gamma_n \mathbf{m}\boldsymbol{\phi}_n \tag{3}$$

where $\boldsymbol{\phi}_n$ are the natural vibration modes of the system and

$$\Gamma_n = \frac{L_n}{M_n} \qquad L_n = \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\iota} \qquad M_n = \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n \tag{4}$$

The contribution of the nth mode to $\mathbf{s}$ is

$$\mathbf{s}_n = \Gamma_n \mathbf{m}\boldsymbol{\phi}_n \tag{5}$$

which is independent of how the modes are normalized. Equation (3) may be viewed as an expansion of $\mathbf{m}\boldsymbol{\iota}$ in terms of inertia force distributions $\mathbf{s}_n$ associated with natural vibration modes.

The nth-mode contribution $r_n(t)$ to any response quantity $r(t)$ is

$$r_n(t) = r_n^{\text{st}} A_n(t) \tag{6}$$

where r_n^{st} denotes the *modal static response*, the static value of r due to external forces $\mathbf{s}_n$. The algebraic sign of r_n^{st} is a property of the structure and the selected response quantity r. In Eq. (6),

$$A_n(t) = \omega_n^2 D_n(t) \tag{7}$$

$D_n(t)$ and $A_n(t)$ are the displacement and pseudo-acceleration response of the nth-mode single-degree-of-freedom (SDF) system, an SDF system with vibration properties—natural period T_n (natural frequency ω_n) and damping ratio ζ_n—of the nth mode of the MDF system excited by ground acceleration $\ddot{u}_g(t)$.

Combining the response contributions of all the modes gives the total response:

$$r(t) = \sum_{n=1}^{N} r_n(t) = \sum_{n=1}^{N} r_n^{\mathrm{st}} A_n(t) \tag{8}$$

Peak Modal Responses

The *peak* value of the nth-mode contribution $r_n(t)$ to response $r(t)$ can be obtained from the earthquake response spectrum or design spectrum:

$$r_{no} = r_n^{\mathrm{st}} A_n \tag{9}$$

where the subscript "o" denotes the peak value, defined as the maximum of the absolute value, and $A_n \equiv A(T_n, \zeta_n)$ is the pseudo-acceleration response spectrum ordinate corresponding to natural period T_n and damping ratio ζ_n. The algebraic sign of r_{no} is the same as that of r_n^{st} because A_n is positive by definition. Although it has an algebraic sign, r_{no} will be referred to as the peak modal response because it corresponds to the peak value of $A_n(t)$.

The peak value r_o of the total response $r(t)$ is estimated by combining the peak modal responses r_{no} ($n = 1, 2, \ldots, N$) according to the well known modal combination rules: *square-root-of-sum-of-squares (SRSS)* rule or *complete quadratic combination (CQC)* rule, as appropriate. The algebraic sign of r_{no} is relevant in the CQC rule, but inconsequential in the SRSS rule.

SRSS Rule

The SRSS rule for modal combination is

$$r_o \simeq \left(\sum_{n=1}^{N} r_{no}^2 \right)^{1/2} \tag{10}$$

The algebraic signs of r_{no} do not affect the value of r_o given by the SRSS rule.

The basis for Eq. (10) was developed by Rosenblueth in his Ph.D. thesis (1951); contained on page 97 is the following equation (in different notation):

$$E(r_o) = \left(\sum_{n=1}^{N} [E(r_{no})]^2 \right)^{1/2} \tag{11}$$

where $E(x)$ denotes the expectation of x.

Based on this thesis, an ASCE paper (Goodman, Rosenblueth, and Newmark, 1953) contains the result:

$$r_d = \left(\sum_{n=1}^{N} r_{nd}^2 \right)^{1/2} \tag{12}$$

where the subscript "d" denotes the design response.

Considering that, in the early 1950's, the subject of earthquake dynamics of structures was in its infancy and random vibration theory was unknown to structural engineers, Rosenblueth's results are especially impressive. Equations (11) and (12) were derived by expressing structural response as a convolution integral of ground acceleration and the unit impulse response function, idealizing the ground acceleration as white noise, and using probability theory. Although the "white noise" terminology was apparently not used, it was implied by the assumptions (Goodman, Rosenblueth, and Newmark, 1953):

> ". . . the writers consider motions consisting of random arrays of concentrated acceleration pulses. . . . The pulses are assumed to be distributed in time in a random order, either at small, uniformly spaced intervals or at randomly spaced instants of time."

Now contained in textbooks and taken for granted in engineering practice, the SRSS rule was first applied in the design of the Latino Americana tower in Mexico City in 1950. In a letter to me dated August 3, 1993, Rosenblueth stated:

> "If I'm not mistaken, when the Holy Ghost descended on me one cold night in early 1950 in Urbana and told me about SRSS combination of modal responses, He hadn't told anyone else. This allowed me to devote the sultry summer to computing the combined responses (which I had computed in pre-Illiac [computer] days) for the Latino Americana tower, of which Nate [Newmark] was consultant, and so SRSS was applied in design for the first time. SRSS was proposed in my Ph.D. thesis, presented in 1951."

This modal combination rule usually provides excellent response estimates for structures with well-separated natural frequencies. This limitation has not always been recognized in applying this rule to practical problems, and at times it has been misapplied to systems with closely spaced natural frequencies, such as piping systems in nuclear power plants and multistory buildings with asymmetric plan.

CQC Rule

The CQC rule for modal combination is applicable to a wider class of structures as it overcomes the limitations of the SRSS rule. According to the CQC rule,

$$r_o \simeq \left(\sum_{i=1}^{N} \sum_{n=1}^{N} \rho_{in} r_{io} r_{no} \right)^{1/2} \tag{13}$$

Each of the N^2 terms on the right side of this equation is the product of the peak responses in the ith and nth modes and the correlation coefficient ρ_{in} for these two modes; ρ_{in} varies between 0 and 1 and $\rho_{in} = 1$ for $i = n$. Thus, Eq. (13) can be written as

$$r_o \simeq \left(\sum_{n=1}^{N} r_{no}^2 + \underbrace{\sum_{i=1}^{N} \sum_{n=1}^{N}}_{i \neq n} \rho_{in} r_{io} r_{no} \right)^{1/2} \tag{14}$$

to show that the first summation on the right side is identical to the SRSS combination rule of Eq. (10); each term in this summation is obviously positive. The double summation includes all the cross ($i \neq n$) terms; each of these terms may be positive or negative. A cross term is negative when the modal static responses r_i^{st} and r_n^{st} assume opposite signs. Thus, the estimate for r_o obtained by the CQC rule may be larger or smaller than the estimate provided by the SRSS rule.

The preceding modal combination rule was first derived by E. Rosenblueth and J. Elorduy (1969), although they did not give it the CQC name, which is due to A. Der Kiureghian (1981). This result was based on an idealization of earthquakes as stationary Gaussian processes, but modified to recognize the transient character of actual ground motions. The methods that led to this result were extensions of Rosenblueth's early work underlying the SRSS rule.

The 1971 textbook *Fundamentals of Earthquake Engineering* by N. M. Newmark and E. Rosenblueth gives the Rosenblueth-Elorduy equations for the correlation coefficient:

$$\rho_{in} = \frac{1}{1 + \epsilon_{in}^2} \tag{15}$$

where

$$\epsilon_{in} = \frac{\omega_i \sqrt{1 - \zeta_i^2} - \omega_n \sqrt{1 - \zeta_n^2}}{\zeta_i' \omega_i + \zeta_n' \omega_n} \qquad \zeta_n' = \zeta_n + \frac{2}{\omega_n s} \tag{16}$$

s is the duration of the segment of white noise. Equations (15) and (16) show that $\rho_{in} = \rho_{ni}$; $0 \leq \rho_{in} \leq 1$; and $\rho_{in} = 1$ for $i = n$ or for two modes with equal frequencies and equal damping ratios.

Although used in some research studies (e.g. Kan and Chopra, 1977), this important result was not used very widely in engineering practice for many years perhaps for two reasons. First, the statement defining the algebraic sign of r_{no} may have been too subtle. This definition (with appropriate change in notation) appeared in Newmark and Rosenblueth, 1971:

> ". . . r_{no} is to be taken with the sign that ψ_{rn} [defined by the authors as the transfer function for r_n, but generally known as the unit impulse response function for r_n] has when it attains its maximum numerical value."

Second, it was not obvious how to define the duration s for an actual ground motion or a given design spectrum (Villaverde, 1984), and the recommendations for s (e.g., 12.5 sec. for earthquakes on the west coast of the United States) were not widely recognized (Rosenblueth and Esteva, 1964; Newmark and Rosenblueth, 1971, page 275).

Later, using a different approach based on random vibration theory in the frequency domain, Eq. (13) was derived together with the following equation for the correlation coefficient (Der Kiureghian, 1981):

$$\rho_{in} = \frac{8\sqrt{\zeta_i \zeta_n}(\zeta_i + \beta_{in}\zeta_n)\beta_{in}^{3/2}}{(1-\beta_{in}^2)^2 + 4\zeta_i\zeta_n\beta_{in}(1+\beta_{in}^2) + 4(\zeta_i^2+\zeta_n^2)\beta_{in}^2} \tag{17}$$

where $\beta_{in} = \omega_i/\omega_n$. This equation also implies that $\rho_{in} = \rho_{ni}$, $\rho_{in} = 1$ for $i = n$ or for two modes with equal frequencies and equal damping ratios. For equal modal damping $\zeta_i = \zeta_n = \zeta$ this equation simplifies to

$$\rho_{in} = \frac{8\zeta^2(1+\beta_{in})\beta_{in}^{3/2}}{(1-\beta_{in}^2)^2 + 4\zeta^2\beta_{in}(1+\beta_{in})^2} \tag{18}$$

This form of the correlation coefficient was easier to use relative to Eqs. (15) and (16), and it gained widespread popularity.

In order to compare the two equations for the correlation coefficient, Eq. (15) is specialized for systems with same damping ratio in all modes subjected to earthquake excitation with duration s long enough to replace Eq. (16b) by $\zeta_n' = \zeta_n$. We substitute $\zeta_i = \zeta_n = \zeta$ in Eq. (16a), introduce $\beta_{in} = \omega_i/\omega_n$, and insert Eq. (16a) into Eq. (15) to obtain

$$\rho_{in} = \frac{\zeta^2(1+\beta_{in})^2}{(1-\beta_{in})^2 + 4\zeta^2\beta_{in}} \tag{19}$$

Figure 1 shows Eqs. (18) and (19) for the correlation coefficient ρ_{in} plotted as a function of $\beta_{in} = \omega_i/\omega_n$ for four damping values: $\zeta = 0.02$, 0.05, 0.10, and 0.20. Observe that the two expressions give essentially identical values for ρ_{in}.

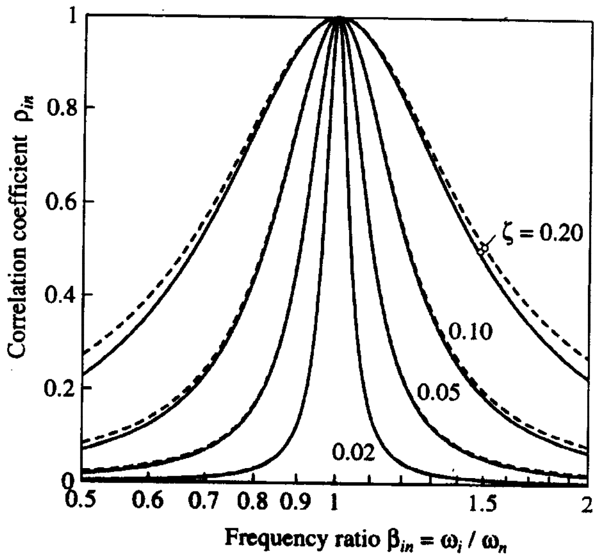

Fig. 1. Variation of correlation coefficient ρ_{in} with modal frequency ratio $\beta_{in} = \omega_i/\omega_n$ for four damping values; abscissa scale is logarithmic; Eqs. (18) and (19) are plotted in solid and dashed lines, respectively.

However, ρ_{in} should depend on the effective duration s of the ground motion, which is neglected in Eqs. (17) and (18). In this sense, Eqs. (15) and (16) have a wider range of applicability.

Figure 1 also provides an understanding of the correlation coefficient. Observe that this coefficient rapidly diminishes as the two natural frequencies ω_i and ω_n move farther apart. This is especially the case at small damping values that are typical of structures. In other words, it is only in a narrow range of β_{in} around $\beta_{in} = 1$ that ρ_{in} has significant values; and this range depends on damping. For structures with well-separated natural frequencies the coefficients ρ_{in} vanish; as a result all cross ($i \neq n$) terms in the CQC rule, Eq. (14), can be neglected and it reduces to the SRSS rule, Eq. (10).

COMBINED EFFECTS OF GROUND MOTION COMPONENTS

Most building codes specify how to combine the effects of various ground motion components. For example, the *Uniform Building Code* states:

> "The requirement. . . may be satisfied by designing such elements for 100 percent of prescribed seismic forces in one direction plus 30 percent of the prescribed forces in the perpendicular direction. The combination requiring the greater component strength shall be used for design."

While such combination rules had been proposed and used in the early 1970's, it appears that the first published work that provides a rational basis for these rules is due to Rosenblueth and Contreras (1977). Their approach assumes that such effects are uncorrelated Gaussian processes or that correlations are taken into account in computing modal responses. A simple approximation was derived under the additional assumption that the failure surfaces are convex.

The authors state that:

> "This approximate procedure is applied as follows:
>
> "1. Compute the responses to gravity loads and to the J components of ground motion regarded as potentially significant. Let these responses be arranged into vectors $\mathbf{r} = \mathbf{r}_g$ and $\mathbf{r}_j$, respectively, with $j = 1, 2, \ldots, J$.
>
> "2. Obtain the vectors
>
> $$\mathbf{r} = \mathbf{r}_g + \sum_{j=1}^{J} \alpha_j \mathbf{r}_j \tag{20}$$
>
> assigning plus and minus signs to $\alpha_j \mathbf{r}_j$, ordering the $\mathbf{r}_j$ values in all possible combinations, and giving the α_j terms the values in Table 1.
>
> "3. If the problem is one of analysis, find whether all points fall within the failure surface. If the problem is one of design, assign the design parameters such values that the safe domain will contain all the points."

It was shown that $\alpha_1 = 1$ and $\alpha_j = 0.3$ for $j \geq 2$.

The Acknowledgements section includes the following statement:

"The simplified procedure was proposed early in 1975 (with $\alpha_j = 1/3$ for $j \geq 2$) by A. S. Veletsos. Even earlier N. M. Newmark proposed this procedure with $\alpha_j = 0.4$ for $j \geq 2$. The main ideas in the present paper were developed in Ref. 8 [Rosenblueth, 1975]."

RESPONSE ANALYSIS FOR MULTIPLE SUPPORT EXCITATION

The governing equations for MDF systems excited by prescribed motions $\ddot{u}_{gl}(t)$ at the various supports ($l = 1, 2, \ldots, N_g$) of the structure are the same as Eq. (1) with the effective earthquake forces (Chopra, 1995)

$$\mathbf{p}_{\text{eff}}(t) = -\sum_{l=1}^{N_g} \mathbf{m}\boldsymbol{\iota}_l \ddot{u}_{gl}(t) \tag{21}$$

instead of Eq. (2) where $\boldsymbol{\iota}_l$ is the vector of static displacements in the structural DOF due to $u_{gl} = 1$.

The displacement response of the structure contains two parts (Chopra, 1995):

1. The dynamic displacements:

$$\mathbf{u}(t) = \sum_{l=1}^{N_g} \sum_{n=1}^{N} \Gamma_{nl} \boldsymbol{\phi}_n D_{nl}(t) \tag{22}$$

where

$$\Gamma_{nl} = \frac{L_{nl}}{M_n} \qquad L_{nl} = \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\iota}_l \qquad M_n = \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n \tag{23}$$

and $D_{nl}(t)$ is the deformation response of the nth-mode SDF system to support acceleration $\ddot{u}_{gl}(t)$.

2. The quasistatic displacements $\mathbf{u}^s$ are given by

$$\mathbf{u}^s = \sum_{l=1}^{N_g} \boldsymbol{\iota}_l u_{gl}(t) \tag{24}$$

Combining the two parts gives the total displacements in the structural DOF's:

$$\mathbf{u}^t(t) = \sum_{l=1}^{N_g} \boldsymbol{\iota}_l u_{gl}(t) + \sum_{l=1}^{N_g} \sum_{n=1}^{N} \Gamma_{nl} \boldsymbol{\phi}_n D_{nl}(t) \tag{25}$$

This well known procedure is now a part of textbooks and engineering practice. This author was first exposed to this treatment in the fall of 1964 when Prof. Rosenblueth presented a graduate course at the University of California, Berkeley and distributed to the class a draft of the early chapters of his book with N. M. Newmark; this procedure appeared in Section 2.4 of the published book. Soon thereafter, we at Berkeley used the procedure to analyze the earthquake response of earth dams (Chopra et al., 1969). Later, it was used in many research publications. However, for many years its practical application was limited for lack of analytical methods and instrumental records to define the spatial variation of ground motion. With advances in these subjects Rosenblueth's analysis procedure is being rediscovered and applied to practical problems, e.g., the seismic retrofit of the Golden Gate Bridge and the San Francisco-Oakland Bay Bridge.

ACKNOWLEDGEMENTS

A draft of this paper was reviewed by Profs. J. Bielak, A. Der Kiureghian, L. Esteva, W. J. Hall, J. Penzien, and R. Villaverde. The author is grateful to them for their comments.

REFERENCES

Chopra, A.K., M. Dibaj, R.W. Clough, J. Penzien, and H.B. Seed (1969). Earthquake analysis of earth dams. *Proceedings*, Fourth World Conference on Earthquake Engineering, Santiago, Chile.

Chopra, A.K. (1995). *Dynamics of Structures: Theory and Applications to Earthquake Engineering*, Prentice Hall, Englewood Cliffs, New Jersey.

Der Kiureghian, A. (1981). A Response Spectrum Method for Random Vibration Analysis of MDF Systems. *Earthquake Engineering and Structural Dynamics*, **9**, 419-435.

Esteva, L. and E. Rosenblueth (1964). Espectros de temblores a distancias moderades y grandes. *Boletin Sociedad Mexicana de Ingenieria Sismica, A.C.*, **2**, 1-18.

Goodman, L.E., E. Rosenblueth, and N.M. Newmark (1953). Aseismic design of firmly founded elastic structures. *Proceedings, Separate No. 349, ASCE*, **79**, 27 pages. Also *Transactions, ASCE*, 120, 782-802.

Kan, C.L. and A.K. Chopra (1977). Effects of Torsional Coupling on Earthquake Forces in Buildings. *Journal of the Structural Division*. ASCE, **103**, 805-819.

Newmark, N.M. and E. Rosenblueth (1971). *Fundamentals of Earthquake Engineering*, Prentice Hall, Englewood Cliffs, New Jersey.

Rosenblueth, E. (1951). *A Basis for aseismic design*, doctoral thesis, University of Illinois, Urbana, Illinois.

Rosenblueth, E. and J. Elorduy (1969). Response of linear systems to certain transient disturbances. *Proceedings, Fourth World Conference on Earthquake Engineering*, Santiago, Chile, **I**, A-1, 185-196.

Rosenblueth, E. (1975). Effectos del Componente Vertical en Edificios Altos. Presented at the Fourth National Conference on Earthquake Engineering, Oaxaca, Mexico, 29.

Rosenblueth, E. and H. Contreras (1977). Approximate Design for Multicomponent Earthquakes. *Journal of the Engineering Mechanics Division*, ASCE, **103**, 881-893.

Villaverde, R. (1984). On Rosenblueth's Rule to Combine the Modes of Systems with Closely Spaced Natural Frequencies. *Bulletin of the Seismological Society of America*, **74**, 325-338.