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ACCIDENTAL AND NATURAL TORSION IN EARTHQUAKE RESPONSE AND DESIGN OF BUILDINGS

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ABSTRACT

Considering that a comprehensive review of past work in this field is available, this paper focuses on the description of two recently developed procedures to incorporate the effects of accidental and natural torsion in earthquake analysis and design of asymmetric buildings. The new procedure for accidental torsion consists in specifying an increase in edge displacements due to the most important sources of elastic accidental torsion, such as uncertainty in stiffness, mass, and base rotation, as a function of two building parameters, the ratio between the plan dimension b and the radius of gyration r, and the frequency ratio Ω. This procedure has several advantages over the current code accidental torsion provisions, such as the elimination of the two extra cumbersome analyses in each lateral direction specified by current codes. On the other hand, the new procedure for natural torsion provides a globally applicable framework to understand and evaluate the inelastic behavior of asymmetric structures even prior to any elaborate inelastic analysis. This framework is based on using the story shear and torque surfaces in conjunction with a simplified structural model of a single element per building story. Such a model is tested in this paper using a building with a setback and a strong asymmetry.

KEYWORDS

accidental torsion, accidental eccentricity, design envelope, frequency ratio, natural torsion, single element model, story shear and torque surface, asymmetric structures, inelastic behavior

INTRODUCTION

Buildings subjected to ground shaking undergo lateral as well as torsional motions simultaneously. Such motions are due to *natural torsion* in buildings with asymmetric plan; and *accidental torsion* in all buildings, even those with symmetric plan. Symmetrical buildings undergo torsional motions because no building as built is *perfectly* symmetric, and unrecorded rotational motion at the building base, among other factors, causes torsional motion of the structure.

The coupling between lateral and torsional motions in a building with plan asymmetry inevitably leads to non-uniform displacement demands on the lateral resisting planes of the system. Such displacement demands are of principal interest in the sizing and detailing of structural elements for earthquake resistance. Consequently, many studies have attempted to understand the change in building displacements that arise from building asymmetry; see list of over seventy references in a recent review paper (Rutenberg, 1992). Most of these investigations involve interpretation of parametric studies of the earthquake response of simple idealizations of asymmetric one-story structures. Although these results are important for understanding the inelastic response of asymmetric systems, they have two major limitations. First, because of the inherent complexity of the problem, it has not been possible to glean general trends that apply to structures other than those analyzed. Second, these conclusions do not seem extendable to multistory buildings. The results available for the response of code-designed asymmetric multi-story buildings are even more problem-specific and difficult to generalize. These limitations of past parametric studies of inelastic earthquake response of asymmetric structures are discussed in a review paper (Rutenberg, 1992) which states:

> "The picture emerging from the foregoing review is somewhat confusing. It appears that most of the problems and disagreements among researchers are the result of using different models. This is not surprising considering the large number of parameters involved. . . the lack of a unique system definition by an accepted set of parameters is perhaps the main obstacle to progress in this field."

Considering that a comprehensive review of past work is available, we have adopted an unconventional approach in this state-of-the-art paper. To overcome the limitations of past parametric studies and their sometimes inconsistent conclusions, we will present a simple conceptual framework to enable the designer to understand the performance of different structural configurations prior to any elaborate inelastic analysis. Also included is a simplified method for the analysis and design of asymmetric-plan structures which is suitable for trying alternative structural configurations and seeking the most seismically efficient and cost-effective configuration.

The subject of accidental torsion is not amenable to investigation by traditional analytical approaches because standard dynamic analyses do not predict torsion in symmetric-plan buildings. Perhaps for this reason the work on this subject is meager compared to the substantial body of literature on natural torsion. Summarized herein is a recently developed procedure to estimate accidental torsion in the design of buildings.

PART I: ACCIDENTAL TORSION

Code Predictions of Accidental Torsion

Building codes require consideration of accidental torsion in one of two ways: (1) apply the equivalent static lateral forces at a distance e_d from the center of stiffness (CS), which includes the accidental eccentricity $e_a = \pm\beta b$; and (2) perform dynamic analyses with the center of mass (CM) of each floor shifted a distance equal to the accidental eccentricity $e_a = \pm\beta b$ from its nominal position, where b is the plan dimension of the building perpendicular to the direction of ground motion. For each structural element the e_a value leading to the larger design force is to be used. Implementation of these code provisions requires two three-dimensional (3-D) static or dynamic analyses of the building for each lateral direction. The two types of analyses--static and dynamic--predict a significantly different increase in design forces resulting from accidental eccentricity; the code-static analyses are not consistent with the analytical results (Fig. 2) (De la Llera and Chopra, 1994d).

Analytical Estimates of Accidental Torsion

The increase in building displacements due to individual sources of accidental torsion, such as stiffness and mass uncertainty, base rotational excitation, and other less important sources, is a function of the system parameters p, which are random variables describing the stiffness and mass matrices of the system, and a random base rotational excitation $a_{g\theta}(t)$. The normalized displacement of the building plan at a distance x from the CM is defined as the ratio $\hat{u}_x = u_x^* / u_x$ between the displacement u_x^* considering the effect of all sources of accidental torsion and the displacement u_x neglecting accidental torsion; then, the increase in response is $\hat{u}_x - 1$. Based on the assumption of statistical independence among the system parameters, first-order approximations for the mean and standard deviation of the normalized displacement are given by (De la Llera and Chopra, 1994d):

$$\mu_{\hat{u}_{b/2}} = 1 + \mu_{\hat{u}_{g\theta}} \qquad \sigma_{\hat{u}_{b/2}} = \sqrt{\sum_{i=1}^{N_p} \left[\frac{\partial \hat{u}_{b/2}}{\partial p_i} \right]^2 \sigma_{p_i}^2 + \sigma_{\hat{u}_{g\theta}}^2} \tag{1}$$

where $\mu_{(\cdot)}$ and $\sigma_{(\cdot)}$ denote the mean and standard deviation of $(\cdot)$, $\mu_{\hat{u}_{g\theta}}$ and $\sigma_{\hat{u}_{g\theta}}$ are the mean and standard deviation of $\hat{u}_{b/2}$ resulting from base rotational excitation, N_p is the number of system parameters, σ_{p_i} is the standard deviation of the parameter p_i, and $\partial \hat{u}_{b/2} / \partial p_i$ is the sensitivity of $\hat{u}_{b/2}$ with respect to the parameter p_i.

Determined in earlier investigations (De la Llera and Chopra, 1994b,e,f) are the normalized response and increase in response due to the following sources of accidental torsion: (1) rotational motion of the building foundation, (2) uncertainty in the stiffness of structural elements in both principal directions of analysis, (3) uncertainty in the location of the CM, and (4) uncertainty in stiffness and mass distributions in stories of a building other than the story analyzed. The variation of normalized displacements with $\Omega = \omega_\theta / \omega_y$ (the ratio between the natural frequencies of uncoupled torsional and lateral motions of the building) in Fig. 1 leads to the following observations:

1. The largest increase in building displacements results from uncertainty in the location of the CM orthogonal to the direction of ground motion (Fig. 1a). Somewhat smaller are the effects of uncertainty in the stiffness of structural elements in the direction of ground motion (Fig. 1b). These two sources combined account for over 70% of the total increase in response due to accidental torsion (De la Llera and Chopra, 1994b,f).

2. Stiffness uncertainty and uncertainty in the location of the CM can be modeled as a perturbation of the static eccentricity of the system, an observation that partially justifies the code dynamic analysis procedure.

3. The increase in edge displacement due to base rotational motion, derived from true base rotations of thirty buildings during recent California earthquakes (De la Llera and Chopra, 1994e), is generally less than 8% for systems with uncoupled lateral vibration period T_y over, say, 1/2 sec. and a wide range of Ω (Fig. 1c); it may reach values as large as 40% for short period systems (T_y < 1/2 sec.) that are torsionally flexible (Ω < 2/3).

4. The increase in edge displacement resulting from uncertainty in the stiffness of structural elements in the direction orthogonal to the direction of ground motion and in the location of the CM along the direction of ground motion are in general less than 5% (Fig. 2d) (also, De la Llera and Chopra, 1994f). This increase is zero for nominally symmetric systems and increases with increasing stiffness eccentricity in the system.

5. The increase in edge displacements due to uncertainty in the stiffness and mass distributions in stories other than the one analyzed (Fig. 2e) is between one-third to one-half of the increase due to the uncertainty of these properties in the story considered (Fig. 2b) (also, De la Llera and Chopra, 1994f).

6. Most sources of accidental torsion increase the response of nominally symmetric systems more than they do for unsymmetric systems (De la Llera and Chopra, 1994b,e,f).

7. The increase in edge displacements due to stiffness and mass uncertainty is essentially insensitive to changes in T_y (De la Llera and Chopra, 1994b,f).

8. Buildings with the plan dimension perpendicular to the direction of ground motion much larger than the other dimension (large b/r) show the largest increase in response due to accidental torsion.

9. The increase in response of single-story systems due to accidental torsion is also the exact result for a special class of multistory systems with the following properties: (a) the CM of all floors lie on a vertical line; (b) the resisting planes are arranged such that their principal axes form an orthogonal grid in plan and are connected at each floor by a rigid diaphragm; and (c) the lateral stiffness matrices of all resisting planes are proportional.

Shown in Fig. 2 are the mean and mean-plus-one standard deviation of $\hat{u}_{b/2}$ computed from Eq. (1) considering all sources of accidental torsion (De la Llera and Chopra, 1995a). The mean value of the increase in response, $\hat{u}_{b/2} - 1$, is usually less than 3%. Furthermore, with the exception of systems with $T_y < 0.5$ sec. and $\Omega < 1$, this mean increase in response is insensitive to Ω. The mean-plus-one standard deviation value of the response increase reaches a peak value of 45% for systems with $\Omega \simeq 0.85$ or $\Omega \simeq 1.1$; it decreases steadily for values of Ω larger and smaller than these two values; and it varies rapidly between its peaks at $\Omega \simeq 0.85$ and 1.1 to a minimum at $\Omega \simeq 1$. As shown in the figure this increase in response is larger for nominally symmetric systems ($e_s/b = 0$) than for unsymmetric systems (De la Llera and Chopra, 1994c).

Design Considerations

Compared in Fig. 3 is $\hat{u}_{b/2}$ predicted by code-dynamic analysis with $e_a = \pm 0.05b$ and the "true" value (Fig. 2). The code increase in edge displacements is much larger than the mean value of the "true" increase. However, it is about one-half of the mean-plus-one standard deviation of the "true" value; it corresponds to an exceedance probability of about 30%.

The discrepancy in the design forces due to accidental torsion, as predicted by code-specified static and dynamic analysis procedures, can be overcome by defining a unique design envelope for the edge displacements:

$$\hat{u}_{b/2} = \begin{cases} A & 0 \le \Omega \le 1 \\ A - \dfrac{A-1}{\Omega_c - 1}(\Omega - 1) & 1 < \Omega \le \Omega_c \\ 0 & \Omega > \Omega_c \end{cases} \tag{2}$$

where $\Omega_c = 1.8$ and

$$A = 1 + 0.0475(b/r)^2 \qquad (3)$$

Equation (3) is a good approximation to the maximum value of $\hat{u}_{b/2}$ over all Ω (Fig. 4a) determined by code-specified dynamic analysis (Fig. 4b). Furthermore, Eqs. (2) and (3) have been intentionally calibrated to produce values that are conservative, especially for the range $0.9 \leq \Omega \leq 1.1$. There are three reasons for this. First, the estimation of Ω is obviously subject to error; therefore, taking advantage of the dip in the response curve near $\Omega = 1$ is not appropriate for design. Second, this conservatism proves to be useful in preventing resisting planes in the interior of the building plan to be underdesigned by the procedure developed. Third, the "recorded" increase in response for a system with $\Omega \simeq 1$ can be larger than predicted by code-specified dynamic analysis for accidental torsion (De la Llera and Chopra, 1995a).

Simplified Procedure

In order to overcome the limitations of the present code procedures, a procedure has been developed to include accidental torsion in the seismic design of buildings. This procedure is implemented in four steps:

1. Determine $\Omega = \omega_\theta/\omega_y$; ω_θ and ω_y can be estimated by Rayleigh's Method:

$$\omega_y \simeq \sqrt{\frac{\sum_i F_i\delta_i}{\sum_i m_i\delta_i^2}} \qquad \omega_\theta \simeq \sqrt{\frac{\sum_i T_i\theta_i}{\sum_i I_{pi}\theta_i^2}} \qquad (4)$$

   where F_i and T_i are any reasonable heightwise distribution of equivalent static forces and torques (e.g., $T_i = F_i e$, where e is an arbitrary eccentricity value), δ_i and θ_i are floor displacements and rotations associated with these forces, and m_i and I_{pi} are the floor masses and moments of inertia.

2. Calculate from Eqs. (2) and (3) the normalized edge displacement $\hat{u}_{b/2}$.

3. Compute the normalized displacement $\hat{u}_x$ at distance x from the CM by

$$\hat{u}(x) = 1 + (\hat{u}_{b/2} - 1)\left|\frac{x}{b/2}\right| \qquad (5)$$

   which states that the displacement varies linearly from the CM to the edge.

4. Compute the forces in the structural members of each resisting plane by amplifying the forces without accidental torsion by the normalized displacement determined in Step 3.

This procedure is "exact" for single-story systems and for multistory buildings belonging to the special class defined earlier; it is also a good approximation for other multistory systems. It has several important advantages over the current seismic code procedures. First, it avoids the two additional 3-D static or dynamic analyses of the building in each lateral direction. Second, it includes the effects of all sources of accidental torsion whereas codes include only those that can be represented by a constant accidental eccentricity. Third, it gives a unique value for the increase in a design force due to accidental torsion, whereas current codes give very different results depending on whether the analysis is static or dynamic. Fourth, the procedure defines explicitly the expected increase in design forces due to accidental torsion, in contrast to the use of accidental eccentricity in codes implying an indirect increase in member forces. Fifth, the increase in design forces specified by the new procedure has a well-established probability of exceedance.

Evaluation of Procedure

The key element of the simplified procedure described above are the design displacement envelopes of Fig. 4. These envelopes are evaluated against motions of seven nominally symmetric buildings recorded during the Loma Prieta and Northridge earthquakes (Table 1). At each instrumented floor of these buildings, three channels of acceleration were recorded. First, the recorded accelerations, after they have been appropriately processed, are integrated to determine the corresponding displacements. From these "recorded" displacements the "true" value of the normalized edge displacement is computed assuming the floor diaphragm to be rigid; this is the ratio between the "recorded" value of displacement and the value excluding accidental floor rotation. The ratio between two values of the maximum edge displacement, one including floor rotation and the other excluding it, are presented in Fig. 5 for each building corresponding to its frequency ratio Ω determined from its recorded motions and the b/r value (Table 1). The vertical line for each building shows the range of values for $\hat{u}_{b/2}$ for various floors. Superimposed on these results are the design displacement envelopes for $b/r=1,2$, and 3.

Table 1. Nominally Symmetric Buildings Considered

Buildings	CSMIP	PGA	Material	$(b/r)_x$[1]	$(b/r)_y$	Ω_x	Ω_y
A:Richmond	58506	0.11g	Steel	3.12	1.49	1.36	1.52
B:Pomona	23511	0.13g	RC	2.22	2.67	1.42	1.34
C:San Jose	57562	0.20g	Steel	3.22	1.28	1.00	1.03
D:Sylmar	24514	0.67g	Steel	2.44	2.45	1.10	1.10
E:Burbank	24385	0.29g	RC	2.45	2.45	1.64	1.53
F:Burbank	24370	0.29g	RC	3.28	1.10	0.97	0.80
G:Warehouse	24463	0.26g	RC	2.74	2.12	1.37	1.42

It is apparent from Fig. 5 that buildings with larger frequency ratio Ω present a smaller increase in edge displacements, which is in agreement with theoretical predictions (Fig. 2). Besides, the increase in response for buildings with frequency ratio close to one varies from essentially zero for building F to about 40% for building D. This is precisely the main reason why the proposed envelopes have ignored the dip in the theoretical curves at $\Omega=1$ (Fig. 3). However, Figure 5 must be interpreted with caution since for each frequency ratio Ω, the increase in displacements is a random variable. Thus, the points presented in the figure are just few "true" outcomes of this set of random variables.

[1]In computing (b/r), the subscripts x and y denote selection of the plan dimension b perpendicular to the direction of analysis; in computing Ω, x and y denote the direction of vibration of the structure.

PART II: NATURAL TORSION

Story Shear-Torque (SST) Yield Surface

To develop a conceptual and engineering-practice-oriented strategy to understand the seismic response of asymmetric buildings, the story shear and torque response histories are presented in the force space spanned by the story shears in the two principal directions of the building and by the story torque. At each instant of the response the story shears and torque define one point in this space. These combinations of shears and torques are bounded by the SST yield surface, a polygonal surface defined by a set of story shear and torque combinations corresponding to different collapse mechanisms that can be developed in the story. The parameters that define the SST yield surface are (1) strength of resisting planes; (2) strength of resisting planes in the orthogonal direction; (3) asymmetry in stiffness; (4) asymmetry in strength; (5) planwise distribution of strength; and (6) number of resisting planes. How these parameters affect the earthquake response of a structure can be predicted based on their influence on the yield surface (De la Llera and Chopra, 1995b).

In order to avoid detailed and "exact" calculation of the SST surface for the specific system to be analyzed, a new superelement model (SEM) has been developed to represent the elastic and inelastic properties of each story of the building (Fig. 6). The eight-vertex polygon presented in Fig. 7 is the exact model of a section of the SST yield surface at a constant story shear V_x for a story with three resisting planes along the y-axis, the direction of asymmetry and of ground motion, and two resisting planes in the orthogonal direction. This model can also be used to predict with sufficient accuracy the responses of systems with an arbitrary number of resisting planes. The coordinates (x_j, y_j) of the vertices of this surface are given by the following expressions:

$$
\begin{aligned}
x_1 &= V_{yo} & y_1 &= V_{yo}x_p + T_\perp(1-\hat{V}_x) \\
x_2 &= V_{yu} + V_{yc} & y_2 &= T_o - T_\perp \hat{V}_x \\
x_3 &= V_{yu} - V_{yc} & y_3 &= T_o - T_\perp \hat{V}_x \\
x_4 &= -V_{yo} & y_4 &= -V_{yo}x_p + T_\perp(1-\hat{V}_x)
\end{aligned}
\qquad (6)
$$

$$x_5=-x_1,\ x_6=-x_2,\ x_7=-x_3,\ x_8=-x_4; \quad y_5=-y_1,\ y_6=-y_2,\ y_7=-y_3,\ y_8=-y_4$$

where,

(1) $\hat{V}_x = V_x / V_{xo}$ is the normalized story shear in the x-direction, $V_{xo}=\sum_{i=1}^{M} f_x^{(i)}$ is the lateral capacity of the story in the x-direction, $f_x^{(i)}$ is the capacity of the i^{th} resisting plane in the x-direction, and M is the number of resisting planes in the x-direction;

(2) $V_{yo} = \sum_{i=1}^{N} f_y^{(i)}$ is the lateral capacity of the story in the y-direction, $f_y^{(i)}$ is the capacity of the i^{th} resisting plane in the y-direction, and N is the number of resisting planes in the y-direction;

(3) V_{yc} is the capacity of the resisting planes in the y-direction passing through the CM of the system (in practical terms it will represent the capacity of all resisting planes "close" to the CM);

(4) $T_o = \sum_{i=1}^{N} |f_y^{(i)}x^{(i)}| + \sum_{i=1}^{M} |f_x^{(i)}y^{(i)}|$ is the torsional capacity of the system;

(5) $T_\perp = \sum_{i=1}^{M} f_x^{(i)}y^{(i)}$ is the torque provided by the resisting planes in the orthogonal direction;

(6) $x_p = \sum_{i=1}^{N} f_y^{(i)}x^{(i)}/V_{yo}$ is the strength eccentricity, or first moment of strength; and

(7) $V_{yu} = \sum_{i=1, i\neq 2}^{N} f_y^{(i)}x^{(i)}/|x^{(i)}|$ is the "strength unbalance" in the story.

In order to test the accuracy of the SEM, the inelastic response--of a four-story steel frame building (Fig. 8a-b) to the N-S component of 1940 El Centro ground motion amplified by a factor of 2--was computed using the SEM and the exact multi-plane models. This building exhibits coupling of lateral and torsional motions because of asymmetry due to the setback at the second floor level. The SEM for the building is shown in Fig. 8c. Shown in Fig. 9 is the SST surface for the first story and a comparison between the story shears and torque histories computed from the SEM and the exact multiplane models for each building story. Both responses show remarkable similarity in spite of the significant lateral-torsional coupling of the structure. The fact that yielding occurs on branches of the SST surface with negative slope implies that yielding in the first story occurs mainly on the planes to the right of the CM (1995b,c). Figure 10 indicates that the results predicted by the SEM are satisfactory for practical design purposes, although the SEM uses only four superelements instead of the 54-column elements in the "exact" analysis. In the example considered, the peak displacement ductilities for the resisting planes are about 2. This small ductility demand is typical of steel-frame buildings where sizing of the elements is usually controlled by serviceability (stiffness) requirements. More important, this intermediate ductility case constitutes a tough test for the SEM since the model is based on the assumption of an elastic-perfectly-plastic behavior, which ignores the transition between the elastic and plastic states.

Design Aspects

A comprehensive investigation was conducted to understand the effect of the following six characteristics controlling the behavior of multistory asymmetric structures: (1) strength of resisting planes in the orthogonal direction; (2) stiffness asymmetry; (3) strength asymmetry; (4) planwise distribution of strength; (5) number of resisting planes; and (6) bidirectional ground motion (De la Llera and Chopra, 1995b,1996). Computed for five-story buildings with several different plans were the story shear and torque history superimposed on the SST surface, building displacements, and member forces. Interpretation of these results led to the following conceptual guidelines for improving the design of asymmetric structures:

1. The responses of asymmetric-plan single and multistory buildings belonging to the special class considered, i.e., with regular asymmetry in height, show trends that are similar (1995b,1996). This suggests that, at least conceptually, earlier results for single-story systems may be applicable to this class of multistory systems.

2. Stiffness asymmetry of a system influences the story shear and torque combinations inside the SST surface and hence the elastic response of the system. However, changes in the stiffness eccentricity will affect the inelastic response of the system only if they lead to a shift of the inelastic action to a different region (or branch) of the SST surface.

3. Strength asymmetry produces concentration of deformation demand in resisting planes that are farther from the strongest plane in the plan. Furthermore, buildings with strength asymmetry are prone to develop torsional mechanisms and, hence, uneven displacement demands among resisting planes.

4. The two observations above may be combined into an interesting point. Since stiffness asymmetry controls the behavior inside the surface and strength asymmetry controls the shape of the surface, we can, theoretically speaking, adjust both to direct the inelastic behavior to any desired region of the surface.

5. A reduction in the torsional capacity of stiffness-asymmetric systems may produce, at the expense of larger displacements, more uniform displacement demands among resisting planes, implying a dominantly translational response.

6. Equivalent three-plane models of buildings with multiple resisting planes lead to sufficiently accurate estimates of the building displacements, story shears, and story torques. This is the basis for the SEM developed in the preceding section for preliminary analysis and design of buildings.

7. Increased strength in the resisting planes in the direction orthogonal to the ground motion reduces the torsion in an asymmetric structure. Indeed, substantial yielding of the resisting planes in the direction orthogonal to the ground motion implies that desirable lateral mechanisms with uniform displacement demand for all resisting planes are essentially impossible to generate, even in nominally symmetric structures.

Retrofit Design Example

The conceptual guidelines developed in the previous section are applied next to a hypothetical retrofit solution of a building. We seek a retrofit solution for the five-story building in Fig. 11 for the design earthquake equal to twice the E-W and N-S components of the El Centro earthquake in the x- and y-directions, respectively. The stiffnesses of the resisting planes are as shown, and their yield deformations are all assumed to be equal to v_y = 0.02 ft. Observe that the system is asymmetric in both directions as a result of the setback in the second story and because, in the second and upper stories, resisting plane 5 is stiffer and stronger than resisting plane 2. Besides, the building has resisting planes with lateral stiffness matrices which are not proportional.

Before proposing different retrofit solutions for this structure, it is necessary to understand its inelastic dynamic behavior (Fig. 12). Response results are presented for the second story of the building; columns (a), (b), and (c) present the floor displacements, force-deformation relations, and story shear and story torque histories, respectively. The floor displacements at the left and right edges of the plan are substantially different; the ratio of the two ranges between 3 and 5 for different floors (e.g. Fig. 12, part a). The peak deformation ductility demand for the most critical resisting plane in the second story is approximately 15 (Fig. 12, part b). The ductility demands on the various resisting planes differ by close to 100% in the first story and 60% in the second story. All the above observations are verified by the base shear and torque response histories superimposed on the SST surfaces. Figure 12, part c shows that yielding in the second story is quite extensive and spreads to the large-torque regions of the constant story shear branches where mechanisms become increasingly torsional. Therefore, there is clear evidence that the system is torsionally unbalanced, and any proposed retrofit solution should aim to correct this unbalance in order to lead to more uniform deformation demands among resisting planes.

First Retrofit Solution. The strength of resisting planes A and C is increased by factors of 2.5 and 2 in the first story and all upper stories, respectively, such that the lateral capacity of each story in the x-direction has been doubled (Fig. 11b). It is apparent by comparing the earthquake response of the modified building (Fig. 13) with the response of the original building (Fig. 12) that, as expected, the torsional unbalance in the system has been partially corrected, i.e., the displacements of different resisting planes become less different (Fig. 13, part a), their deformation ductility demands also become more similar (Fig. 13, part b), and the mechanisms developed at different stories are less torsional (Fig. 13, part c).

Although the benefit of increasing the strength in the orthogonal planes is apparent, some aspects of the behavior of the new system are not completely satisfactory. First, given the large increase in strength of the orthogonal planes, we would have preferred a better agreement between the left and right edge displacement histories. Second, the force-deformation histories in the second story (Fig. 13, part b) show that yielding in the resisting planes occurs asymmetrically about the force axis, indicating a residual drift in the structure. Third, the increase in strength in the orthogonal planes leads to an increase in the story torques developed in the system (Fig. 13, part c). Despite these deficiencies, the retrofit solution proposed accomplishes our goal of reducing differences in demands among resisting planes. However, as shown next, it is possible to achieve a much better performance by adjusting the stiffness and strength of the resisting planes.

Second Retrofit Solution. We first recall from points 2, 3, and 4 in the previous section that, by changing the stiffness and strength distribution in the system, we may concentrate yielding in specific resisting planes of the structure. Thus, the strategy for this solution is to decrease the capacity of those planes that remain essentially elastic in the original system and increase the capacity of planes that yield excessively in that system; however, the overall lateral capacity is kept the same as that of the original system. For this purpose, in the first story the capacity of plane 5 is increased and that of plane 1 is decreased, both by 25%; in the second story the strength of resisting plane 3 is increased by 50% and that of plane 5 is reduced by 25%.

The dynamic response of the modified system (Fig. 14) is remarkable in many respects. First, the displacement histories at different locations of the building plan (Fig. 14, part a) are very similar, especially for the second and upper stories. Second, the force-deformation histories (Fig. 14, part b) show essentially identical peak deformation demands on the different resisting planes, as well as symmetric behavior about the force axis. Further, since all resisting planes are used more effectively, the peak ductility demands have been reduced from 15 in the original system (Fig. 12, part b) to about 8 in the new system (Fig. 14, part b). Third, the second story shear and torque combinations now lie close to the zero torque axis (Fig. 14, part b), implying that the mechanisms developed are mainly translational.

As demonstrated by this example, the conceptual guidelines presented in the previous section concerning the inelastic behavior of asymmetric buildings provide a basis to develop practical solutions to improve the torsional behavior of an existing structure, even if that structure is highly asymmetric. Thereafter, the retrofit solution can be tested further by inelastic dynamic analyses of the system. Such analyses, which are costly if standard methods are used, can be greatly simplified using the SEM presented earlier.

CONCLUSIONS

This investigation has led to two proposals to include the effects of accidental and natural torsion in building design.

First, a new procedure for accidental torsion has been developed. This procedure consists in specifying an increase in edge displacements due to all sources of accidental torsion in the elastic range as a function of two building parameters, the ratio b/r and the frequency ratio Ω.
This procedure has several advantages over the current code accidental torsion provisions. First, it avoids the important discrepancies between the increase in response obtained by static and dynamic analysis procedures to account for accidental torsion. Second, it is simple to use and eliminates the two extra 3D-static or dynamic analyses of the building in each lateral direction. Third, it includes the effect of all sources of accidental torsion whereas codes include only those that can be represented by an accidental eccentricity. Fourth, it defines explicitly the expected increase in design forces due to accidental torsion, in contrast to the use of an accidental eccentricity which implies an indirect increase in member forces. Fifth, the envelopes for the increase in response are associated with a well established probability of exceedance.

This investigation has also led to a new procedure for integrated analysis and design of asymmetric plan structures. The procedure differs from previous work in the sense that it does not develop simple rules based on extensive parametric studies for the effects of natural torsion, but provides a model which enables the engineer to understand, prior to any elaborate inelastic analysis if the planwise earthquake performance of the building is adequate or not. Based on this framework, we developed an explanation for the parameters that control the inelastic behavior of torsionally coupled systems, a tool for simplified inelastic analysis of asymmetric multistory buildings (SEM model), and a set of guidelines to improve their inelastic performance.

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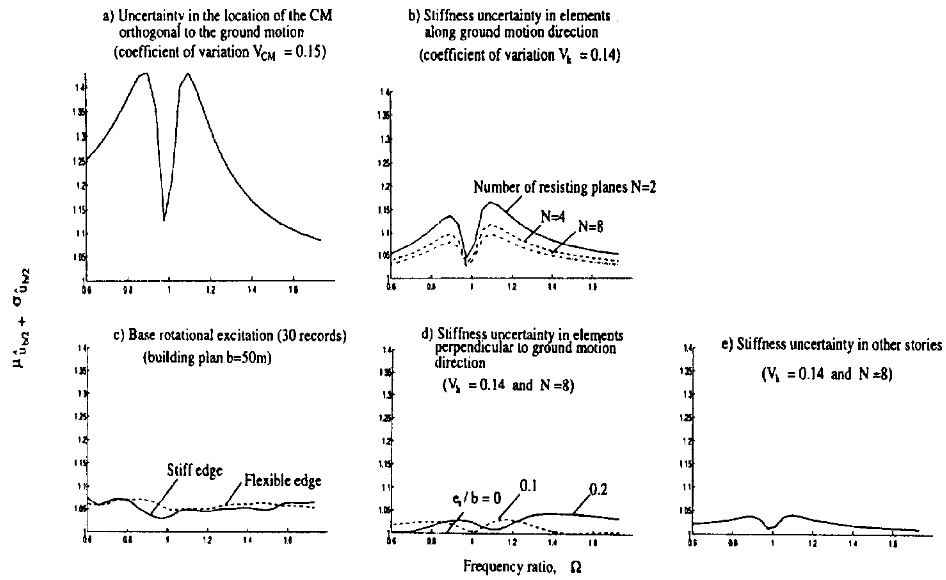

Fig.1 Mean-Plus-One Standard Deviation $\mu_{\hat{u}_{b/2}} + \sigma_{\hat{u}_{b/2}}$ of Normalized Edge Displacement $\hat{u}_{b/2}$ due to different sources of accidental torsion.

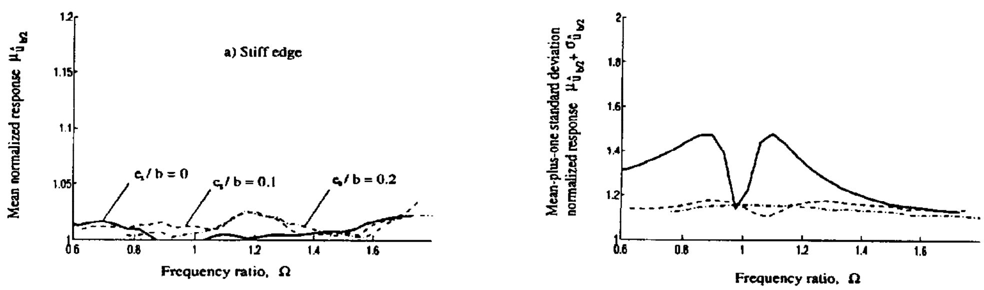

Fig.2 Mean and Mean-Plus-One Standard Deviation of Normalized Edge Displacement $\hat{u}_{b/2}$-Systems with $T_y = 1$ and square plan $b/r = \sqrt{6}$

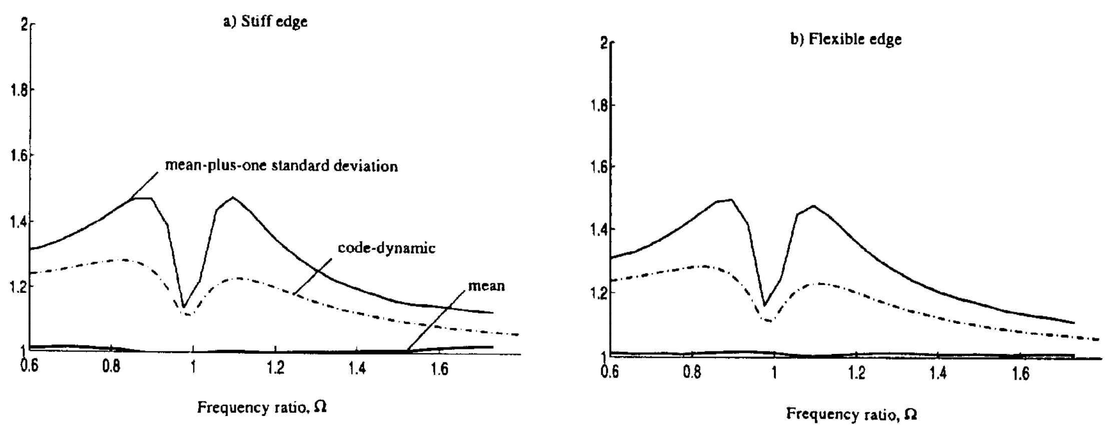

Fig.3 Comparison between $\hat{u}_{b/2}$ Computed from Code-Dynamic Analysis and from Statistical Analysis of Different Sources of Accidental Torsion-Systems with $T_y = 1$ sec and Square Plan $b/r = \sqrt{(6)}$

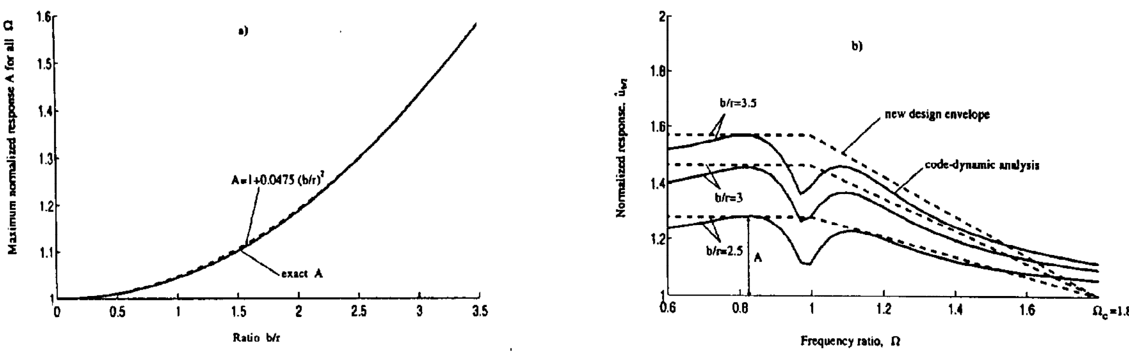

Fig.4 Design Envelopes for Normalized Edge Displacement $\hat{u}_{b/2}$: (a) Variation of A as Function of b/r; and (b) Design Envelopes for $b/r = 1.5, 3$, and 3.5 in Systems with $T_y = 1$ sec

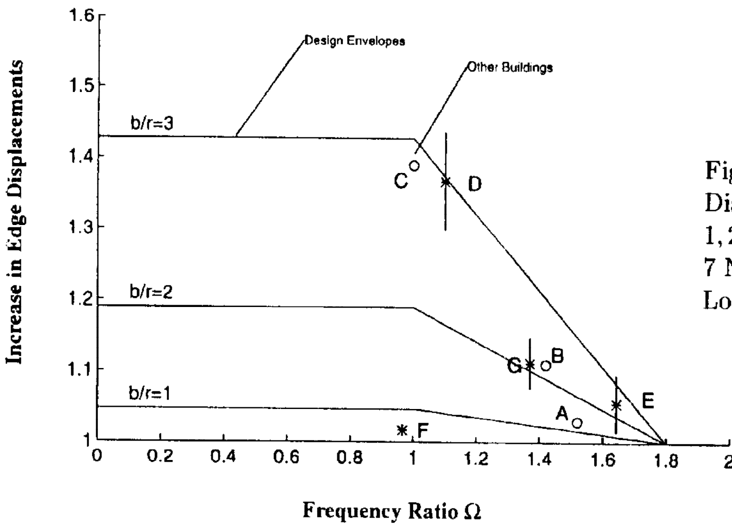

Fig.5 Design Envelopes for Normalized Edge Displacement $\hat{u}_{b/2}$ Corresponding to $b/r = 1, 2$ and 3, and "True" Results Obtained in 7 Nominally Symmetric Buildings During the Loma Prieta and Northridge Earthquakes

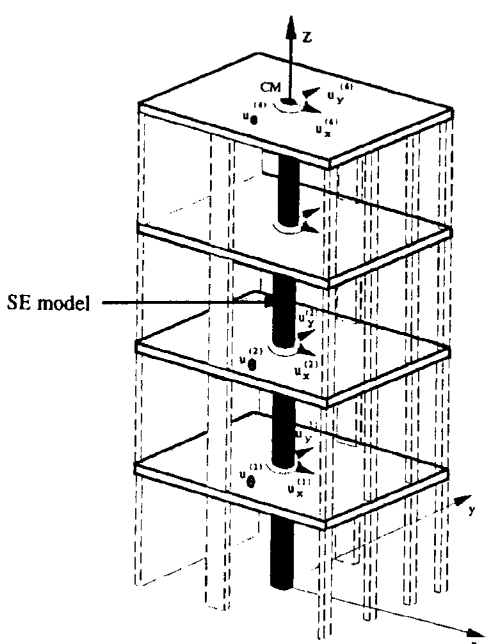

Fig.6 Single Element Model of a Four-Story Building

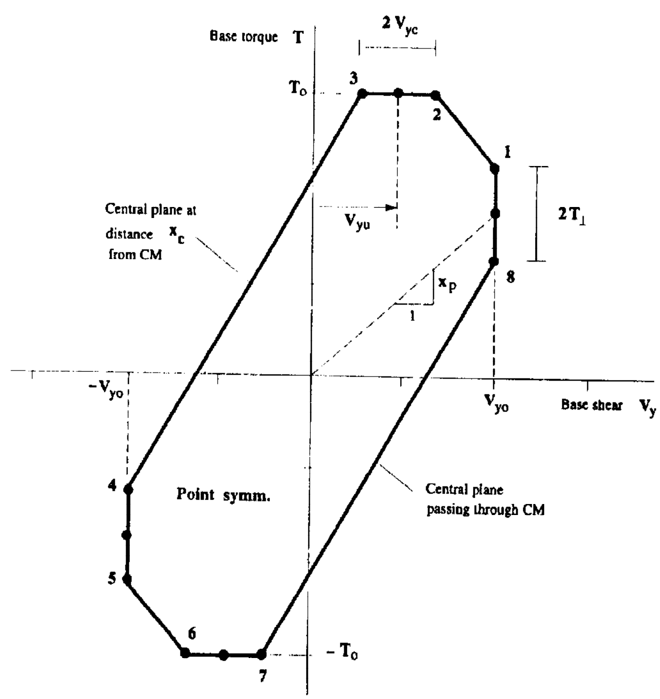

Fig.7 Parametric Representation of the Story-Shear and Torque Surface

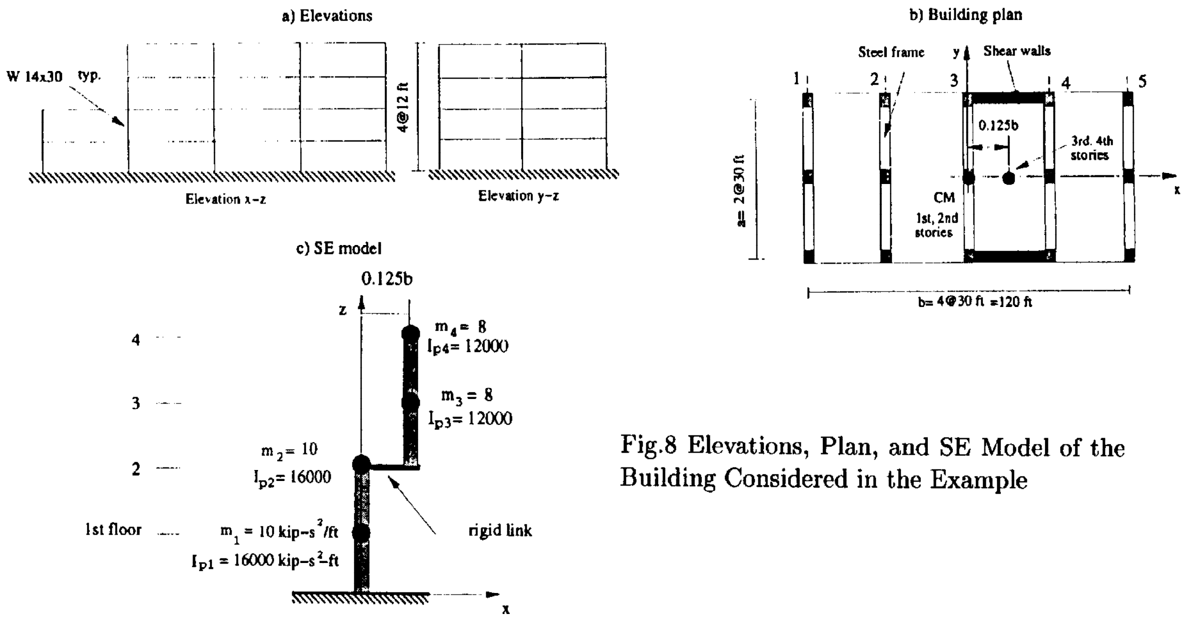

Fig.8 Elevations, Plan, and SE Model of the Building Considered in the Example

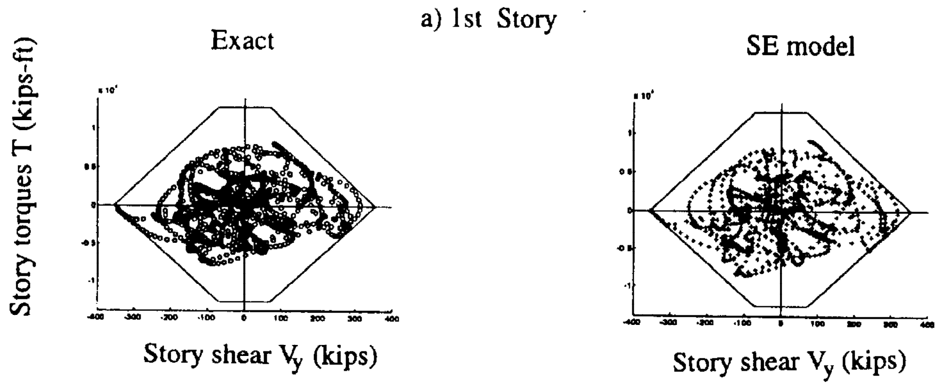

Fig.9 Comparison of Base-Shear and Torque Diagrams Computed from the Exact and SE models

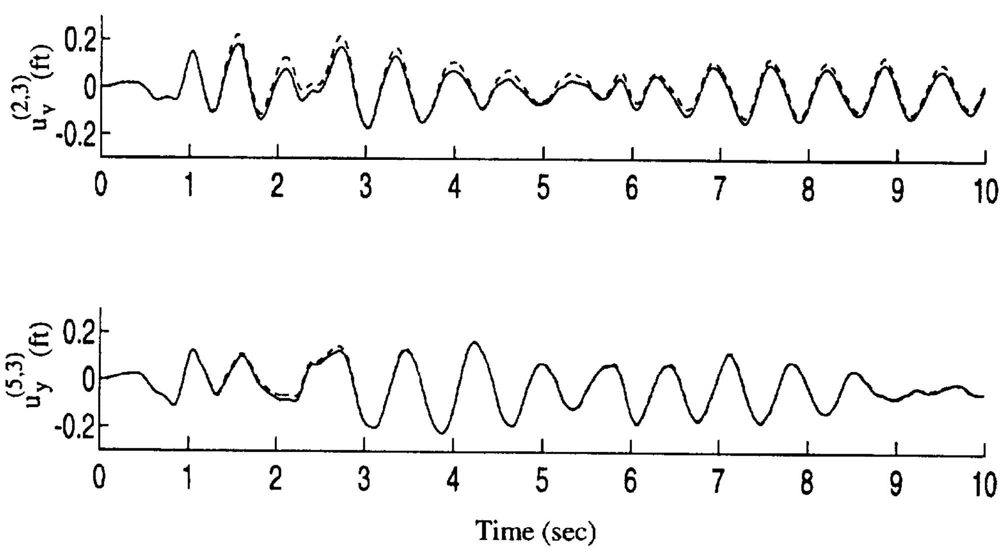

Fig.10 Displacements at Location of Resisting Planes 2 and 5 on the Third Floor

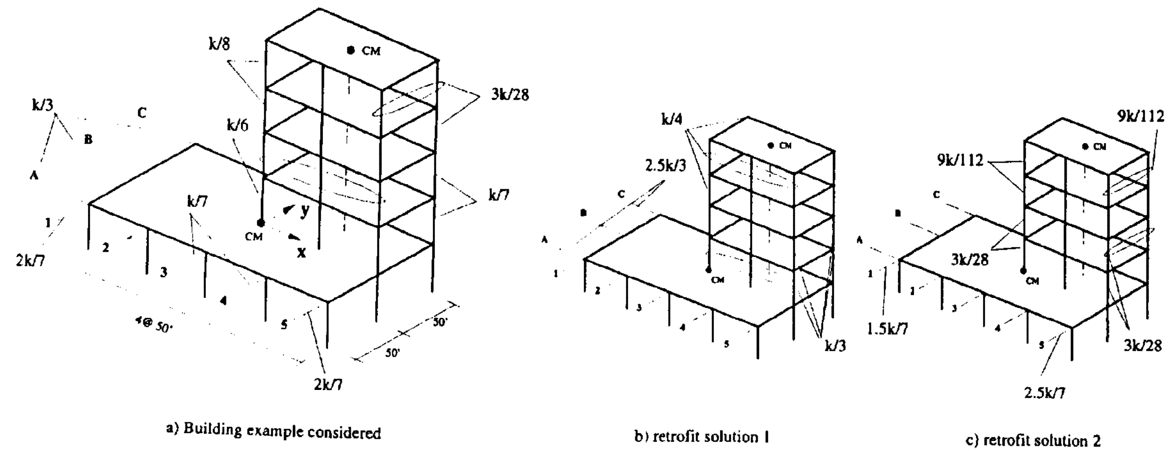

Fig.11 Building with Setback Considered in the Example and Two Possible Retrofit Solutions

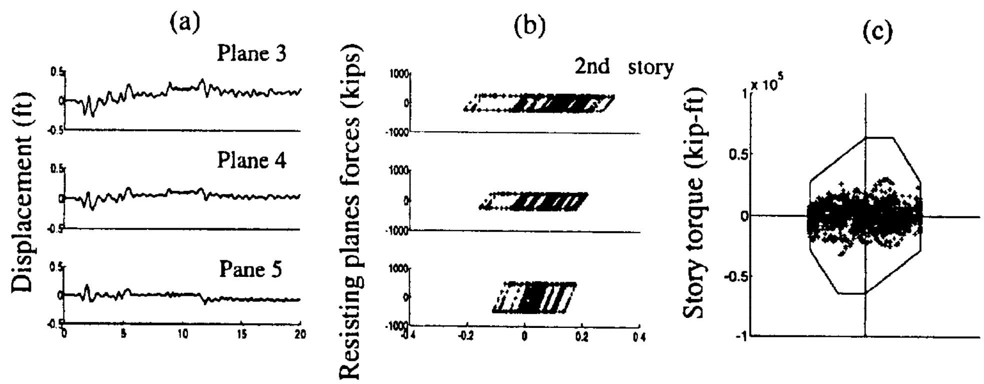

Fig.12 Behavior of the Second Story of the Original Building

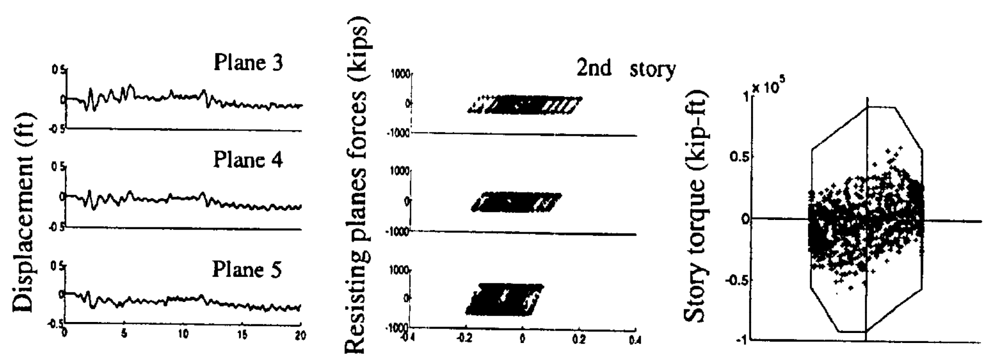

Fig.13 Behavior of Building with Increased Strength in the Orthogonal Resisting Planes

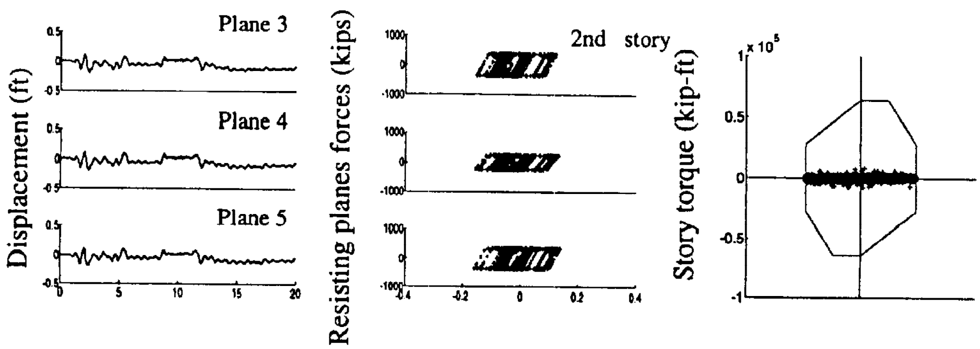

Fig.14 Behavior of Building with Modified Stiffness and Strength in Edge Resisting Planes