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NON-LINEAR RESPONSE OF SOIL-STRUCTURE SYSTEMS

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ABSTRACT

Induced variations in effective structural dynamic properties due to soil-structure interaction can lead to higher or lower inertial forces depending on (1) the response spectra ordinates at the resonant periods of the structure, (2) the damping levels and (3) the ductility factors. In spite of the fact that unsafe side errors can be introduced in the process, design criteria usually do not take into account soil-structure interaction effects in the structural ductility. So far, their implications on the response are not well known.

The main interest in this study is to identify the key parameters that control the variations on the non-linear structural response due to its interaction with the soil. Different soil-foundation-structure scenarios are considered. Numerical results of ductility demands which were computed using an approximated formula and a rigorous solution are presented.

KEYWORDS

Replacement oscillator, flexibility time functions, ductility demand.

LINEAR SOIL-STRUCTURE SYSTEM

For multistory structures embedded in layered soil deposits, soil-structure systems can be idealized as shown in figure 1. Structures responding as single degree of freedom oscillators in their fixed-base condition and layered soil deposits essentially as single strata are required. The foundation is assumed to be axisymmetric and rigid with two degrees of freedom, lateral translation and rocking. This coupled system is suitable to consider interaction effects in the fundamental mode of vibration. The contribution of higher modes may be determined by standard procedures disregarding these effects. In the figure T_e and ζ_e are the period and damping of the fundamental mode, respectively, M_e is the effective mass participating in the fundamental mode and H_e is the effective height of the resultant of the corresponding inertial forces. The foundation is represented by means of their parameters the radius R, the mass M_c, the mass inertia moment J_c and the depth D. The real deposit is represented by means of the mean shear wave velocity β_s and the dominant period of the site $T_s = 4H_s / \beta_s$, where H_s is the depth of the soil. The degrees of freedom of the system are the strain of the structure x_e (that is, the structural displacement relative to the base of the foundation), the horizontal displacement of the foundation relative to the surface of the ground x_c and the rotation of the foundation ϕ_c.

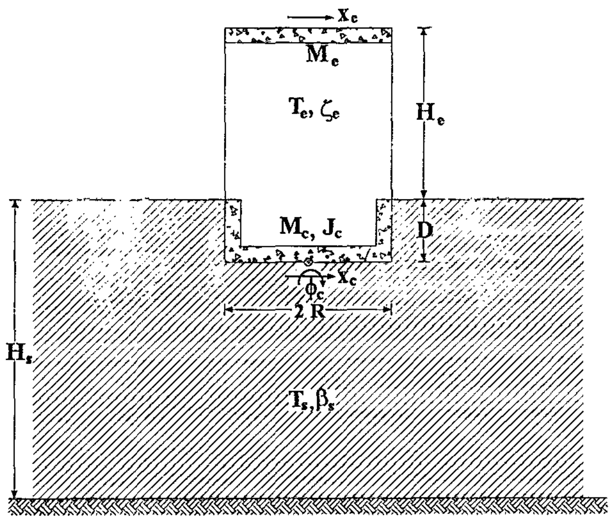

Fig 1 Soil-structure system for interaction effects in the fundamental mode

The dynamic equilibirum equation of this system is given by

$$\mathbf{M}_s\ddot{\mathbf{X}}_s + \mathbf{C}_s\dot{\mathbf{X}}_s + \mathbf{K}_s\mathbf{X}_s = -\ddot{x}_o\mathbf{M}_o \tag{1}$$

where $\mathbf{X}_s = \{x_e, x_c, \phi_c\}^\mathbf{T}$ is the degrees of freedom vector while $\dot{\mathbf{X}}_s$ and $\ddot{\mathbf{X}}_s$ are their first and second time derivatives. Besides $\mathbf{M}_s$, $\mathbf{C}_s$ and $\mathbf{K}_s$ are the mass, damping and stiffness matrices, respectively. $\mathbf{M}_o$ is the load vector and $\ddot{x}_o$is the outcrup acceleration. The matricial components have the following forms.

$$\mathbf{M}_s = \begin{bmatrix} M_e & M_e & M_e(H_e+D) \\ M_e & M_e+M_c & M_e(H_e+D)+M_cD/2 \\ M_e(H_e+D) & M_e(H_e+D)+M_cD/2 & M_e(H_e+D)^2+J_c \end{bmatrix} \tag{2}$$

$$\mathbf{C}_s = \begin{bmatrix} C_e & 0 & 0 \\ 0 & C_h & C_{hr} \\ 0 & C_{rh} & C_r \end{bmatrix}, \quad \mathbf{K}_s = \begin{bmatrix} K_e & 0 & 0 \\ 0 & K_h & K_{hr} \\ 0 & K_{rh} & K_r \end{bmatrix}, \quad \mathbf{M}_o = \begin{Bmatrix} M_e \\ M_e+M_c \\ M_e(H_e+D)+M_cD/2 \end{Bmatrix} \tag{3,4,5}$$

Let m be h, r or hr = rh in order to indicate any of the horizontal, rocking and coupled modes. Then, in these expresions K_m and C_m are the real and imaginary parts of the dynamic stiffnesses, also called dynamic impedances, which are complex functions of the frequency. This functions relate the force (or moment) of excitation with the displacement (or rotation) of the foundation in the steady state. K_m represents the stiffness and inertia of the soil as well as C_m accounts for the material (due to viscosity) and geometrical (due to wave radition) dampings. These parameters are interpreted as the equivalent springs and dashpots of the soil-foundation system and can be coveniently expressed as (Gazetas, 1983)

$$K_m(\omega) = K_m^o\left[k_m(\eta) - \zeta_s\eta c_m(\eta)\right] \tag{6}$$

$$\omega C_m(\omega) = K_m^o\left[\eta c_m(\eta) + 2\zeta_s k_m(\eta)\right] \tag{7}$$

where K_m^o is the static stiffness of the m mode, k_m and c_m are the corresponding stiffness and damping coefficients, respectively, and $\eta = \omega R / \beta_s$ is the normalized frequency being ω the circular frequency of the excitation.

A parametric analysis of linear soil-structure systems allows to identify the key parameters that control most of the interaction effects in the structural response. The relative stiffness between the structure and the soil, defined by $H_e T_s / H_s T_e$, controls the intensity of the interaction effects, which increase as the structure is founded in softer soils. On the other hand, the geometry of the soil-foundation-structure linkage, defined in terms of H_s/R the relative depth of the soil deposit, D/R the relative depth of foundation and H_e/R the slenderness ratio of the structure, increase or decrease the levels of interaction too. Bigger effects are found on higher structures with surficial foundations on shallow soil deposits. Under any soil-structure condition, interaction effects increase the period and shift the critical damping of the structure with respect to those corresponding to the fixed-base condition. The damping value increases or decreases dependig essentilly on both, $H_e T_s / H_s T_e$ the stiffness soil-structure ratio and ζ_s the damping factor of the soil. Structural transfer functions resemble the typical transfer function of the simplest dynamic damped system: a fixed-base single degree of freedom linear oscillator. It is suggested to use this system, which further on will be called replacement oscillator, in order to approximate the response of the idealized soil-structure system. Figure 2 shows the idealized soil-structure system an the replacement oscillator. In particular, Avilés and Pérez-Rocha, (1996) provide formulas to estimate the effective period ($\tilde{T}_e$) and damping ($\tilde{\zeta}_e$) that define the replacement oscillator.

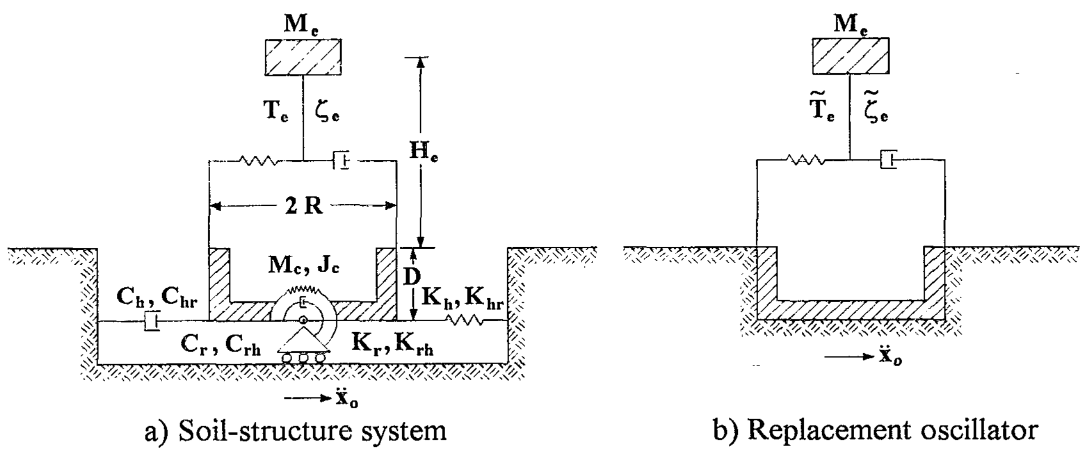

Fig 2 Soil-tructure system and the replacement oscillator subjected to the same control motion

NON-LINEAR ANALYSIS

The inertial interaction effects on the period and damping can be treated by assuming a linear behavior of the structure. In this case it is advisable to determine the soil-structure system response by means of typical analysis in the frequency domain and Fourier theorems. However, it is well known that the structures should be designed with suitable ductility to work adequately in the non-linear behavior range during intensive earthquakes. In order to achieve non-linear analysis, a scheme of solution of the equilibrium equations in the time domain is needed. It allows to account for non-linearity in an explicit way, however, in spite of impedance functions of the soil are dependent of the frequency, the use of convolutions integrals is required.

In non-linear real analysis, dynamic stiffness of the soil is usually approximated by means of invariant springs and dashpots. The values of stiffness for frequency zero and damping when frequency tends to infinite should be used. In this kind of approach, non-linear analysis of interaction systems is notably simplified since the procedure of calculus is completely similar to the one of systems without interaction.

Impedance and compliance functions

From the equilibrium equation of the elementary oscillator the following relation can be written

$$K(\omega)X(\omega) = P(\omega) \tag{8}$$

where $X(\omega)$ and $P(\omega)$ are the Fourier spectra of displacements and excitation forces, respectively, and $K(\omega)$ is the dynamic impedance of the oscillator which is expressed as

$$K(\omega) = k - \omega^2 + i\omega c \tag{9}$$

k and c are the spring and dashpot constants, respectively, and $i = \sqrt{-1}$. Differing from the elementary oscillator, springs and dashpots of the soil-foundation system are functions of the frequency. For this system, the dynamic stiffness matrix becomes

$$\tilde{\mathbf{K}}(\omega) = \mathbf{K}(\omega) + i\omega\mathbf{C}(\omega) \tag{10}$$

Tipically, it is not possible to make use of Fourier integrals to obtain the time representation of impedance functions because the amplitude spectrum of each one grows monotonically with the frequency. Instead, the inverse of dynamic stiffness matrix, that is, dynamic flexibilities or compliance functions, are quantities that can be synthetized by using Fourier synthesis. As well as dynamic stiffnesses can be interpreted as the set of forces that must be applied on the foundation to produce unitary displacements, dynamic flexibilities can be interpreted as the set of displacements of the foundation due to the action of unitary forces. The dynamic flexibility matrix in the frequency domain reads as

$$\tilde{\mathbf{F}}(\omega) = \tilde{\mathbf{K}}(\omega)^{-1} = \begin{bmatrix} F_h & F_{hr} \\ F_{hr} & F_r \end{bmatrix} \tag{11}$$

with

$$F_h = \frac{K_r}{K_h K_r - K_{hr}^2} \tag{12}$$

$$F_r = \frac{K_h}{K_h K_r - K_{hr}^2} \tag{13}$$

$$F_{hr} = \frac{K_{hr}}{K_h K_r^2 - K_h K_r} \tag{14}$$

Flexibility time functions are obtained by the inverse Fourier transform of (12-14). These time functions must be causal. They are requiered as convolution integrals in the scheme of solution of the dynamic equilibrium equations system.

Values of K^{o}_{m}, k_m and c_m for axisymmetric foundations embedded in a single homogeneous stratum with rigid base are obtained from a data base (Avilés and Pérez-Rocha, 1992). It was computed by performing an efficient numerical technique (Tassoulas and Kausel, 1983) that considers the effect of the rigid sidewall on the dynamic stiffness of axisymmetric embedded footings. Discrete impedance functions were reported for a wide range of normalized frequencies and for several soil-foundation conditions defined by ν_s the Poisson ratio of the soil, H_s/R the relative depth of the soil deposit and D/R the relative depth of foundation. In this formulation, hysteretic damping is accounted for.

Hysteretic damping factor is widely used in frequency computation since its simplicity, due to the non-dependency of frequency, makes it handy for many purposes. The assumtion of this kind of damping in frequency formulations leads to results which are compared with those obtained by using more realistic damping factors, such as the Voight and Kelvin damping models. In these models variations in the frequency domain are admitted to obtain causal time functions.

We propose a frequency dependent term which allows to obtain approximated causal flexibility functions from hysteretic impedance functions, that would be useful as convolution functions in a step-by-step integration scheme. In the linear case this functions lead to comparable results to those obtained in frequency domain. The form that we found for the dynamic stiffness of the m mode is

$$\tilde{K}_m(\omega) = K^{o}_{m}[k_m(\eta) + i\eta c_m(\eta)](1+i2\zeta)\frac{(1+i4\zeta\eta)}{(1+i\zeta\eta)} \tag{15}$$

Integration scheme of the equilibrium equation of soil-structure systems

To take into account the variations of the impedance functions controlled by frequency, it is necessary to solve the equilibrium equations in terms of flexibilities by means of convolution integrals (Wolf, 1988). The solution scheme for the i+1 step of integration is

$$\mathbf{M}_s\ddot{\mathbf{X}}_{s_{i+1}} + \mathbf{P}_{s_{i+1}} = -\ddot{\mathbf{X}}_{g_{i+1}}\mathbf{M}_o \tag{16}$$

where $\mathbf{P}_s$ is the vector of the forces that act on the springs and dashpots of the system, that is

$$\mathbf{P}_{s_{i+1}} = \begin{Bmatrix} \mathbf{P}_e \\ \mathbf{P}_c \end{Bmatrix}_{i+1} \tag{17}$$

Here $P_{c_{i+1}} = c_{e_{i+1}}\dot{x}_{e_{i+1}} + k_{e_{i+1}}x_{e_{i+1}}$ is the force that acts only on the stiffness of the structure. The forces that act on the foundation are

$$\mathbf{P}_{c_{i+1}} = \tilde{\mathbf{F}}^{-1}_{c_o}\left\{\mathbf{X}_{c_{i+1}} - \sum_{j=1}^{i}\tilde{\mathbf{F}}_{c_{i+1-j}}\mathbf{P}_{c_j}\right\} \tag{18}$$

where $\tilde{\mathbf{F}}_{c_o}^{-1}$ and $\tilde{\mathbf{F}}_{c_{i+1-j}}$ are integrals of the flexibility function given by

$$\tilde{\mathbf{F}}^{-1}_{c_o} = \int_0^{\Delta t}\frac{\tau}{\Delta t}\mathbf{F}_c(\Delta t - \tau)d\tau \tag{19}$$

and

$$\tilde{\mathbf{F}}_{c_{i+1-j}} = \int_0^{\Delta t}\frac{\tau}{\Delta t}\mathbf{F}_c((i+2-j)\Delta t - \tau)d\tau + \int_0^{\Delta t}\left(1 - \frac{\tau}{\Delta t}\right)\mathbf{F}_c((i+1-j)\Delta t - \tau)d\tau \tag{20}$$

Equations (19-20) can be evaluated by using linear variation of the matricial function $\mathbf{F}_c$ within the interval Δt. Regarding the relationship of velocity and displacement of the β–Newmark integration method for constant acceleration it is possible to write

$$\mathbf{P}_{\mathbf{s}_{i+1}} = \mathbf{P}^*_{\mathbf{s}_{i+1}} + \mathbf{P}_{\mathbf{s}_i} \tag{21}$$

where

$$\mathbf{P}^*_{\mathbf{s}_{i+1}} = \begin{bmatrix} c_e \frac{\Delta t}{2} + k_e \frac{\Delta t^2}{4} & 0 \quad 0 \\ 0 & \\ 0 & \frac{\Delta t^2}{4} \tilde{\mathbf{F}}^{-1}_{c_o} \end{bmatrix} \begin{Bmatrix} \ddot{x}_e \\ \ddot{\mathbf{X}}_c \end{Bmatrix}_{i+1} = \mathbf{R}_s \ddot{\mathbf{X}}_{s_{i+1}} \tag{22}$$

and

$$\mathbf{P}_{\mathbf{s}_i} = \begin{Bmatrix} (c_e \frac{\Delta t}{2} + k_e \frac{\Delta t^2}{4}) \ddot{x}_{e_i} + (c_e + k_e \Delta t) \dot{x}_{e_i} + k_e x_{e_i} \\ \mathbf{F}^{-1}_{c_o} \left\{ \mathbf{X}_{c_i} + \dot{\mathbf{X}}_{c_i} \Delta t + \ddot{\mathbf{X}}_{c_i} \frac{\Delta t^2}{4} - \sum_{j=1}^{i} \mathbf{F}_{c_{i+1-j}} \mathbf{P}_{c_j} \right\} \end{Bmatrix} \tag{23}$$

With these results, dynamic equilibrium equation (16) is written as

$$[\mathbf{M}_s + \mathbf{R}_s] \ddot{\mathbf{X}}_{s_{i+1}} = -\ddot{X}_{g_{i+1}} \mathbf{M}_o - \mathbf{P}_{s_i} \tag{24}$$

In particular, this formulation allows to account for non-linar variations of the matrix $\mathbf{R}_s$

INTERACTION EFFECTS ON THE STRUCTURAL DUCTILITY

The elastoplastic non-linear model was used to study the effect of soil-structure interaction on the structural ductility. The behavior laws of this model in both, fixed- and flexible- base conditions are shown in figure 3. The values of x_y and $\tilde{x}_y$ correspond to the displacements at the limit of yielding of the structure without and with interaction, respectively, while the values μx_y and $\tilde{\mu}\tilde{x}_y$ are the maximum relative displacements reached by the structure in those conditions. The parameters μ and $\tilde{\mu}$ are the corresponding ductility factors.

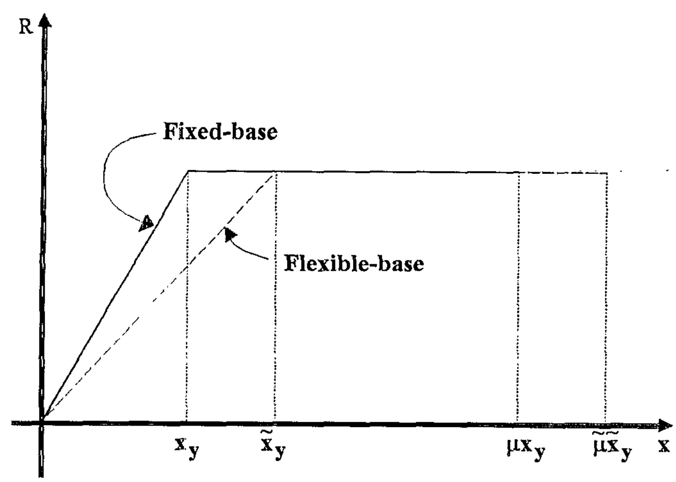

Fig 3 Load-displacement relations of an elastoplastic single degree of freedom system, wiht and without interaction

Rosenblueth and Reséndiz (1988) found the ductility demand of the equivalent replacement oscillator with interaction should be

$$\tilde{\mu} = \left(\frac{T_e}{\tilde{T}_e}\right)^2 (\mu - 1) + 1 \qquad (25)$$

where $\tilde{T}_e$ is the effective period of the system for a certain soil-structure condition. Since $0 < T_e / \tilde{T}_e \leq 1$ (25) yields $1 < \tilde{\mu} \leq \mu$. It implies that the ductility factor is reduced due to the soil-structure interaction.

Using the time integration procedure described, we determined X_e^{max} / X_y the ductility demands of soil-strucutre systems with 5% of critical damping for the same range $0 \leq T_s H_e / T_e H_s \leq 2$ of relative stiffness between the structure and the soil accounting for $\mu = 2$ y 4. SCT (Secretaria de Comunicaciones y Transportes) and CAO (Oficinas de Central de Abastos) sites at the lake bed zone of Mexico City valley were selected. The excitations were given by the EW component of the accelerations of the great September 19, 1985, Michoacan earthquake recorded at this sites. The stratigraphic profiles were idealized by means of single homogeneus strata. The properties for both, SCT and CAO sites are $H_s = 38\text{m}$, $\beta_s = 76$ m/s and $H_s = 56$ m, $\beta_s = 64$ m/s, respectively. According to the one-dimensional shear wave propagation model, the dominant periods are $T_s = 2$s and 3.5s for SCT and CAO sites, respectively.

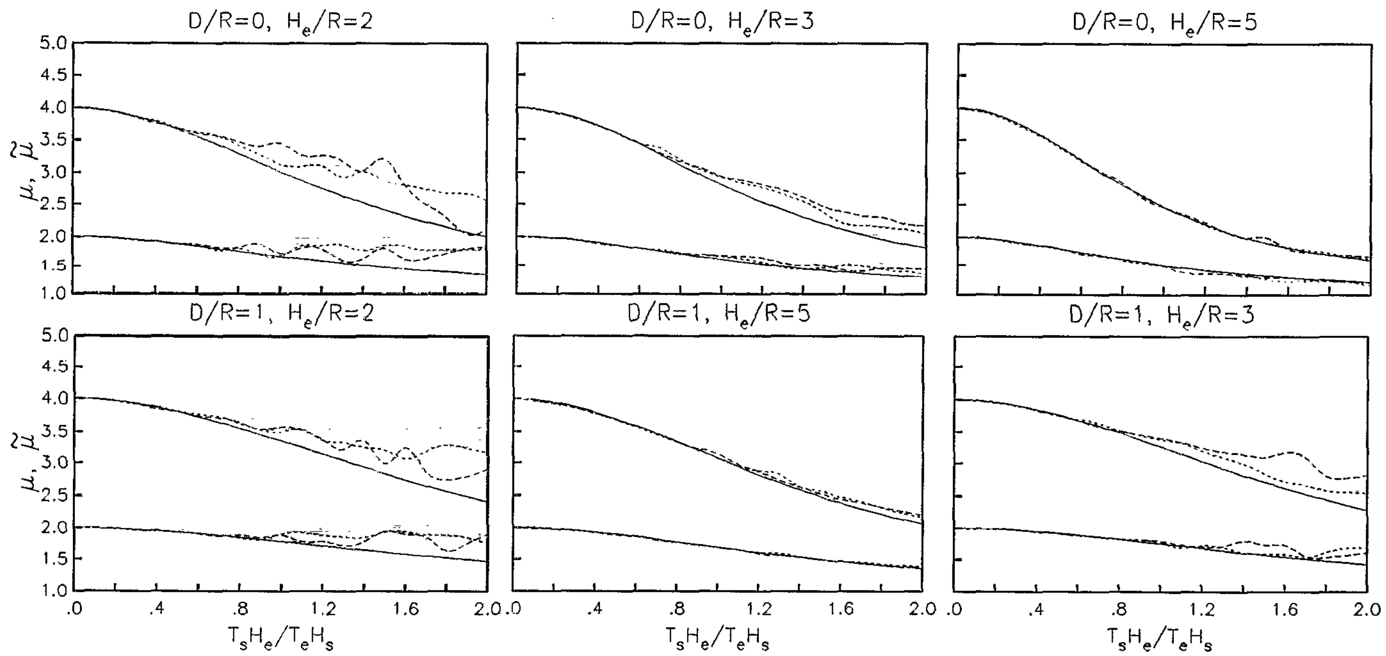

Fig 4 Ductility demands of soil-structure systems for $H_s / R = 5$ at SCT (dashed lines) and CAO (dotted lines) sites. Continuous line indicates the effective ductility of the replacement oscillator.

Results for different soil-structure scenarios defined by the relative depth of the soil deposit $H_s / R = 5$, the relative depth foundation $D / R = 0$, $1/2$ and 1 and the slenderness ratio of the structure $H_e / R = 2,3$ and 5 are displayed in figure 4. Continuous lines correspond to the ductility demands of the replacement oscillator whose strength is the same that the one required by the coupled non-linear system to reach the ductility value μ in the fixed-base condition. They were computed with (25). For the flexible base condition these values, that is, the ductility demands of the coupled systems for the SCT and CAO sites, are represented by dashed and dotted lines, respectively. Note that the ductility demand is independent of the site effects. As it seems, the most of the ductility demand reduction are controled by the relative stiffness between the structure and the soil.

CONCLUSIONS

A rigorous formulation was presented in order to compute the non-linear response of a soil-structure system. This method uses flexibility functions as convolution integrals in a time domain solution. Results show that the main effect in the structural behavior is a reduction in the ductility demand. The key parameter that controls the major part of this reduction is the effective period. In particular, it depends mostly on the relative stiffness between the structure and the soil. These results were compared with those obtained by using an approximate algorithm, based on the replacement oscillator equivalence, which does not depend either on the excitation or on the site effects. The agreement suggest that the approximation is useful for practical purposes.

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