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DYNAMIC ANALYSIS OF STIFFENED COUPLED SHEAR WALLS

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ABSTRACT

This paper presents the free vibration analysis of stiffened coupled shear walls based on a discrete-continuous approach. The effect of the stiffening beam on the free vibration characteristics of the structure is investigated. The optimal position for the stiffening beam to increase as far as possible the first natural frequency of vibration is also presented.

KEYWORDS

Stiffened coupled shear walls; stiffening beam; free vibrations.

INTRODUCTION

Coupled shear walls are widely used in tall building systems to provide lateral resistance against external horizontal loads arising from wind or earthquakes. In the earlier papers (Choo and Coull, 1984, Chan and Kuang, 1989), it has been shown that the efficiency of coupled structural walls could under certain circumstances be increased significantly by the addition of a stiffer beam or a rigid truss at the top or some level of the structure. This induces additional axial forces, and thus reduces the bending moments in the walls as well as helping to reduce the lateral deflection of the structures.

In this paper, free vibration analysis of stiffened coupled shear walls is presented using a discrete-continuous approach (Li and Choo, 1995). The effect of the stiffening beam on the natural frequency characteristics of the structure is investigated. The optimal position for the stiffening beam, to increase as far as possible the first natural frequency of vibration, is also presented.

ANALYSIS

Free vibration analysis of coupled shear walls can be carried out by various methods, such as discrete finite element method and continuous approaches. FEM analysis may involve considerable data preparation effort and computing time; continuous approaches could require solving a high-order differential equation, to which a closed form solution is not obtainable. The mathematical difficulty involved in the continuous approaches can be overcome by employing techniques based on the Galerkin method of weighted residuals (Mukherjee and Coull, 1973) for approximate solutions. However, it is very hard to estimate the inherent errors of the continuous methods arising from the approximation process of the natural frequency calculations. The discrete-continuous (Li and Choo, 1995) approach overcomes the above-mentioned shortcomings and combines the advantages of both discrete and continuous approaches.

Consider a coupled shear wall system with a stiffening beam at level x_1 as shown in Fig. 1a. To obtain the mass matrix, the structure is considered as a discrete lumped mass system (Fig. 1b). It is assumed that the mass of the stiffened coupled shear walls may be replaced by a number of lumped masses evenly located along the structural height. The lumped-mass matrix $\mathbf{M}$ of this equivalent system is

$$\mathbf{M} = \mathrm{diag}[m_1,\ m_2,\ \ldots,\ m_n] \tag{1}$$

and the values of m_i can be determined by

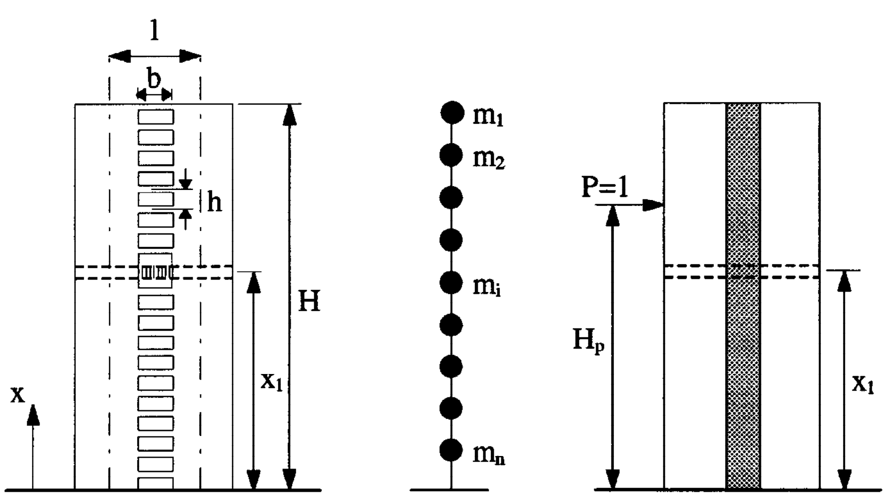
$$m_1 = \frac{1}{2n} M_T \qquad \text{and} \qquad m_i = \frac{1}{n} M_T \qquad \text{for } i > 1 \tag{2}$$

where m_i ($i = 1,\ 2,\ \ldots,\ n$) are the lumped masses at different location throughout the structural height; n is the number of the lumped masses, and M_T the total mass of the structure.

The stiffness matrix $\mathbf{K}$ of the system can be obtained by inverting the flexibility matrix,

$$\mathbf{K} = \mathbf{F}^{-1} \tag{3}$$

In the flexibility matrix $\mathbf{F}$ for the stiffened coupled shear walls, the values of the ith column elements represent the lateral deflections of the walls at all levels where lumped masses are located, induced by a unit load applied horizontally at the location of the ith lumped mass. Consider the stiffened coupled shear walls as a continuous system as shown in Fig. 1c, based on which the lateral deflections can be conveniently analyzed. The entire flexibility matrix of the structure may be formed by repeating the static analysis for a



a. Structural elevation b. Discrete system c. Continuous system
Fig. 1. Stiffened Coupled Shear Walls and Analytical Systems

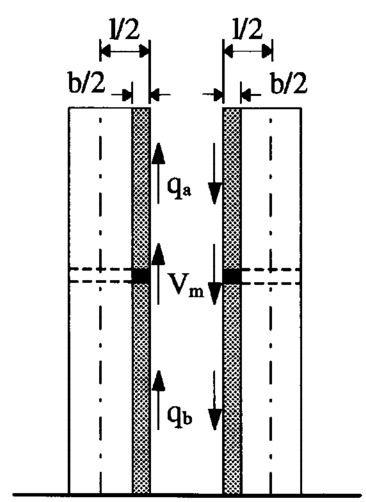


Fig. 2. Substitute Structure

unit lateral load on the walls at each and every level for which there is an assumed lumped mass.

After obtaining both the mass and stiffness matrices, the free vibration analysis of the stiffened coupled shear walls can be conducted by solving the following standard eigenvalue equation,

$$\{\mathbf{K}-\omega^2\mathbf{M}\}\{u\}=\{0\} \tag{4}$$

where ω and $\{u\}$ are circular frequency and deflection vector of vibration, respectively, of the structure.

It is obvious that the key aspect of the method is the derivation of the lateral deflections for stiffened coupled shear walls subjected to a unit horizontal load at an arbitrary level.

LATERAL DEFLECTIONS

Consider a coupled shear wall system with a stiffening beam at level x_1 subjected to a unit lateral load at level H_p (Fig. 1c). By employing the continuum approach of analysis, the coupling beams above and below the stiffening beam may be replaced by a continuous distribution of laminae with equivalent stiffness. It is assumed that the points of contraflexure of the lintel beams coincide with the centerline of the laminae. A cut is made along the line of contraflexure as shown in Fig. 2, and a continuous distribution of shear force along the cut will be released. Let q denote the shear flow per unit height, the compatibility consideration of the vertical displacements above and below the stiffening beam requires that

$$l\frac{dy_a}{dx}-\frac{hb^3}{12EI_b}q_a-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\left[\int_{x_1}^{x}T_a dx+\int_0^{x_1}T_b dx\right]=0 \tag{5a}$$

$$l\frac{dy_b}{dx}-\frac{hb^3}{12EI_b}q_b-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^{x}T_b dx=0 \tag{5b}$$

where y_a, q_a, T_a and y_b, q_b, T_b are the lateral deflection, the laminar shear and the axial force in the walls above and below level x_1 respectively.

The moment-curvature relationships of the walls are

$$M_e=EI\frac{d^2y_a}{dx^2}+lT_a \quad \text{for } x>x_1 \quad \text{and} \qquad M_e=EI\frac{d^2y_b}{dx^2}+lT_b \quad \text{for } x\le x_1 \tag{6}$$

where $I=I_1+I_2$, and M_e is the external bending moment due to the unit lateral load, given by

$$M_e=0 \quad \text{for } x>H_p \qquad \text{and} \qquad M_e=H_p-x \quad \text{for } x\le H_p \tag{7}$$

The axial forces T_a and T_b in the walls are given respectively by

$$T_a=\int_x^H q_a dx \qquad \text{and} \qquad T_b=\int_{x_1}^{H}q_a dx+V_m+\int_x^{x_1}q_b dx \tag{8}$$

where V_m is the shear force in the stiffening beam.

Considering the compatibility at the point of contraflexure of the stiffening beam

$$l\frac{dy}{dx}-\frac{V_m b^3}{12E_m I_m}-\frac{1}{E}\left(\frac{1}{A_1}+\frac{1}{A_2}\right)\int_0^{x_1} T_b dx=0 \tag{9}$$

and by equating the corresponding terms of eqs. (5a) and (5b) at $x=x_1$, the shear force in the stiffening beam can be represented by

$$V_m=S_m Hq_a(x_1)=S_m Hq_b(x_1) \tag{10}$$

where $q_a(x_1)$ and $q_b(x_1)$ are the laminar shear flows at $x=x_1$, and S_m is the relative flexural rigidity of the stiffening beam given by

$$S_m=\frac{E_m I_m}{EI_b\, H/h} \tag{11}$$

in which H/h may be considered as the number of stories.

The laminar shear flow intensity q and the axial force T are related by

$$q=-\frac{dT}{dx} \tag{12}$$

Differentiating eq. (5) and combining eqs. (6) and (12) to eliminate the variables y and q yields the governing differential equations for the axial force T

$$\frac{d^2T_a}{dx^2}-\alpha^2T_a=-\gamma M_e \quad \text{and} \quad \frac{d^2T_b}{dx^2}-\alpha^2T_b=-\gamma M_e \tag{13}$$

where $\gamma=\dfrac{12I_b l}{hb^3 I}$ $\qquad \alpha^2=\gamma\left(l+\dfrac{AI}{A_1A_2l}\right)$ $\qquad A=A_1+A_2$

For $H_p \geq x_1$, the complete solutions to eq. (13) are as follows,

$$T_{a1}=B_1\cosh\alpha x+C_1\sinh\alpha x \qquad T_{a2}=B_2\cosh\alpha x+C_2\sinh\alpha x+\frac{\gamma}{\alpha^2}(H_p-x) \tag{14a}$$

$$T_b=B_3\cosh\alpha x+C_3\sinh\alpha x+\frac{\gamma}{\alpha^2}(H_p-x) \tag{14b}$$

where the subscripts 1 and 2 denote the positions above and below level H_p, respectively.

The corresponding expressions for the laminar shear above and below the position of the stiffening beam can then be derived by using eq. (12), and are given by

$$q_{a1}=-\alpha(B_1\sinh\alpha x+C_1\cosh\alpha x) \qquad q_{a2}=-\alpha\left(B_2\sinh\alpha x+C_2\cosh\alpha x-\frac{\gamma}{\alpha^3}\right) \tag{15a}$$

$$q_b=-\alpha\left(B_3\sinh\alpha x+C_3\cosh\alpha x-\frac{\gamma}{\alpha^3}\right) \tag{15b}$$

The values of the integration constants B_1, C_1, B_2, C_2, B_3 and C_3 can be determined by considering the following boundary conditions,

$$q_b(0)=0 \qquad q_{a1}(H_p)=q_{a2}(H_p) \qquad q_{a2}(x_1)=q_b(x_1) \tag{16a}$$

$$T_{a1}(H)=0 \qquad T_{a1}(H_p)=T_{a2}(H_p) \qquad T_{a2}(x_1)+V_m=T_b(x_1) \tag{16b}$$

Substituting eqs. (14) and (15) into eq. (16) yields

$$B_1=-\,C_1\tanh\alpha H \qquad B_2=B_1-C_3\sinh\alpha H_p \qquad B_3=\mu_1 B_2+\mu_2 C_3 \tag{17a}$$

$$C_1=C_2-C_3\cosh\alpha H_p \qquad C_2=\mu_4 C_3 \qquad C_3=\frac{\gamma}{\alpha^3} \tag{17b}$$

For $H_p \le x_1$, the complete solutions to eq. (13) are as follows,

$$T_a=B_4\cosh\alpha x+C_4\sinh\alpha x \tag{18a}$$

$$T_{b1}=B_5\cosh\alpha x+C_5\sinh\alpha x \qquad T_{b2}=B_6\cosh\alpha x+C_6\sinh\alpha x+\frac{\gamma}{\alpha^2}\left(H_p-x\right) \tag{18b}$$

The corresponding expressions for the laminar shear above and below the position of the stiffening beam can then be derived by using eq. (12), and are given by

$$q_a=-\,\alpha\left(B_4\sinh\alpha x+C_4\cosh\alpha x\right) \tag{19a}$$

$$q_{b1}=-\,\alpha\left(B_5\sinh\alpha x+C_5\cosh\alpha x\right) \qquad q_{b2}=-\,\alpha\left(B_6\sinh\alpha x+C_6\cosh\alpha x-\frac{\gamma}{\alpha^3}\right) \tag{19b}$$

The values of the integration constants B_4, C_4, B_5, C_5, B_6 and C_6 can be determined by considering the following boundary conditions,

$$q_{b2}(0)=0 \qquad q_{b2}(H_p)=q_{b1}(H_p) \qquad q_{b1}(x_1)=q_a(x_1) \tag{20a}$$

$$T_a(H)=0 \qquad T_{b2}(H_p)=T_{b1}(H_p) \qquad T_a(x_1)+V_m=T_{b1}(x_1) \tag{20b}$$

Substituting eqs. (28) and (29) into eq. (20) yields

$$B_4=-\,C_4\tanh\alpha H \qquad B_5=\varepsilon_1 B_4-\varepsilon_2 C_5 \qquad B_6=B_5-C_6\sinh\alpha H_p \tag{21}$$

$$C_4=\varepsilon_3 C_5 \qquad C_5=C_6\left(1-\cosh\alpha H_p\right) \quad C_6=\frac{\gamma}{\alpha^3} \tag{22}$$

The factors μ_1 to μ_4 and ε_1 to ε_3 are as follows:

$$\mu_1=\frac{\cosh\alpha x_1-\sinh\alpha x_1\tanh\alpha x_1}{S_m\alpha H\sinh\alpha x_1+\cosh\alpha x_1-\sinh\alpha x_1\tanh\alpha x_1} \qquad \varepsilon_1=\frac{\operatorname{sech}\alpha x_1}{S_m\alpha H\sinh\alpha x_1+\operatorname{sech}\alpha x_1}$$

$$\mu_2=\frac{S_m\alpha H\left(1-\cosh\alpha x_1\right)}{S_m\alpha H\sinh\alpha x_1+\cosh\alpha x_1-\sinh\alpha x_1\tanh\alpha x_1} \qquad \varepsilon_2=\frac{S_m\alpha H\cosh\alpha x_1}{S_m\alpha H\sinh\alpha x_1+\operatorname{sech}\alpha x_1}$$

$$\mu_3=1+\left[\mu_2-(\mu_1-1)\sinh\alpha H_p\right]\tanh\alpha x_1 \qquad \varepsilon_3=\frac{1-\varepsilon_2\tanh\alpha x_1}{1+\tanh\alpha H(\varepsilon_1-1)\tanh\alpha x_1}$$

$$\mu_4=\frac{\mu_3+(\mu_1-1)\cosh\alpha H_p\tanh\alpha H\tanh\alpha x_1}{1+\tanh\alpha H(\mu_1-1)\tanh\alpha x_1}$$

After determining the axial forces, the lateral deflections of the walls above and below the position of the stiffening beam can be derived by integrating eq. (6) twice.

For $H_p \geq x_1$, the expressions of the lateral deflections are

$$y_{a1} = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3x-H_p)\frac{H_p^{\,2}}{6} - \frac{l}{\alpha^2}(B_1 \cosh \alpha x + C_1 \sinh \alpha x) + D_1 x + F_1\right] \quad (H_p \leq x \leq H) \tag{23a}$$

$$y_{a2} = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3H_p - x)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_2 \cosh \alpha x + C_2 \sinh \alpha x) + D_2 x + F_2\right] \quad (x_1 \leq x \leq H_p) \tag{23b}$$

$$y_b = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3H_p - x)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_3 \cosh \alpha x + C_3 \sinh \alpha x) + D_3 x + F_3\right] \quad (0 \leq x \leq x_1) \tag{23c}$$

The integration constants D_1, F_1, D_2, F_2, D_3 and F_3 can be determined by satisfying the following compatibility requirements,

$$y_b(0) = 0 \qquad y_b(x_1) = y_{a2}(x_1) \qquad y_{a2}(H_p) = y_{a1}(H_p) \tag{24a}$$

$$y_b'(0) = 0 \qquad y_b'(x_1) = y_{a2}'(x_1) \qquad y_{a2}'(H_p) = y_{a1}'(H_p) \tag{24b}$$

This results in

$$D_1 = \frac{l}{\alpha}\left[(B_1 - B_2)\sinh \alpha H_p + (C_1 - C_2)\cosh \alpha H_p\right] + D_2 \tag{25}$$

$$D_2 = \frac{l}{\alpha}\left[(B_2 - B_3)\sinh \alpha x_1 + (C_2 - C_3)\cosh \alpha x_1\right] + D_3 \tag{26}$$

$$D_3 = \frac{l}{\alpha} C_3 \tag{27}$$

$$F_1 = \frac{l}{\alpha^2}\left[(B_1 - B_2)\cosh \alpha H_p + (C_1 - C_2)\sinh \alpha H_p\right] + (D_2 - D_1)H_p + F_2 \tag{28}$$

$$F_2 = \frac{l}{\alpha^2}\left[(B_2 - B_3)\cosh \alpha x_1 + (C_2 - C_3)\sinh \alpha x_1\right] + (D_3 - D_2)x_1 + F_3 \tag{29}$$

$$F_3 = \frac{l}{\alpha^2} B_3 \tag{30}$$

For $H_p \leq x_1$, the expressions of the lateral deflections are

$$y_a = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3x-H_p)\frac{H_p^{\,2}}{6} - \frac{l}{\alpha^2}(B_4 \cosh \alpha x + C_4 \sinh \alpha x) + D_4 x + F_4\right] \quad (x_1 \leq x \leq H) \tag{31a}$$

$$y_{b1} = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3x-H_p)\frac{H_p^{\,2}}{6} - \frac{l}{\alpha^2}(B_5 \cosh \alpha x + C_5 \sinh \alpha x) + D_5 x + F_5\right] \quad (H_p \leq x \leq x_1) \tag{31b}$$

$$y_{b2} = \frac{1}{EI}\left[\left(1-\frac{\gamma l}{\alpha^2}\right)(3H_p - x)\frac{x^2}{6} - \frac{l}{\alpha^2}(B_6 \cosh \alpha x + C_6 \sinh \alpha x) + D_6 x + F_6\right] \quad (0 \leq x \leq H_p) \tag{31c}$$

The integration constants D_4, F_4, D_5, F_5, D_6 and F_6 can be determined by satisfying the following compatibility requirements,

$$y_{b2}(0) = 0 \qquad y_{b2}(H_p) = y_{b1}(H_p) \qquad y_{b1}(x_1) = y_a(x_1) \tag{32a}$$

$$y'_{b2}(0)=0 \qquad y'_{b2}(H_p)=y'_{b1}(H_p) \qquad y'_{b1}(x_1)=y'_a(x_1) \tag{32b}$$

This results in

$$D_4=\frac{l}{\alpha}\left[(B_4-B_5)\sinh\alpha x_1+(C_4-C_5)\cosh\alpha x_1\right]+D_5 \tag{33}$$

$$D_5=\frac{l}{\alpha}\left[(B_5-B_6)\sinh\alpha H_p+(C_5-C_6)\cosh\alpha H_p\right]+D_6 \tag{34}$$

$$D_6=\frac{l}{\alpha}C_6 \tag{35}$$

$$F_4=\frac{l}{\alpha^2}\left[(B_4-B_5)\cosh\alpha x_1+(C_4-C_5)\sinh\alpha x_1\right]+(D_5-D_4)x_1+F_5 \tag{36}$$

$$F_5=\frac{l}{\alpha^2}\left[(B_5-B_6)\cosh\alpha H_p+(C_5-C_6)\sinh\alpha H_p\right]+(D_6-D_5)H_p+F_6 \tag{37}$$

$$F_6=\frac{l}{\alpha^2}B_6 \tag{38}$$

NUMERICAL INVESTIGATION AND CONCLUSIONS

To illustrate the effect of a stiffening beam on the dynamic characteristics of coupled shear walls, a typical structure is analyzed as an example. The dimensions and structural properties are given in Table 1, and the first ten natural frequencies of vibration are presented in Table 2.

Table 1. Dimensions and Properties of the example structure

wall section:	0.3×6m	$A_1=A_2=1.8\text{m}^2$	$I_1=I_2=5.4\text{m}^4$
coupling beam section:	0.3×0.3	$A_b=0.09\text{m}^2$	$I_b=0.000675\text{m}^4$
stiffening beam section:	0.3×1.5	$x_1/H=0.5$	$I_m=0.084375\text{m}^4$
$H=95$m	$h=3.8$m	$b=2$m	$l=8$m
$\rho=2400\,\text{kg/m}^3$	$E=E_m=2.76\times 10^7\ \text{kN/m}^2$		

Table 2. Natural frequencies of the example structure

structural	mode number									
form	1	2	3	4	5	6	7	8	9	10
N* (f_{1o})	0.6675	2.925	7.159	13.28	21.44	31.61	43.82	58.04	74.30	92.57
S* (f_1)	0.7632	2.926	8.120	13.29	22.46	31.61	45.14	58.04	75.54	92.57

* N and S denote normal coupled shear walls and stiffened coupled shear walls, respectively.

Figure 3 shows the effect of the stiffening beam on the free vibration characteristics of the structure. It can be seen that the addition of a stiffening beam can increase significantly the first natural frequency of vibration. The increase rate could reach 30% in some circumstances. Figure 4 presents the optimal positions for the stiffening beam to increase as far as possible the first natural frequency of vibration for different values of S_m. It is obvious that the optimal position is at level between 0.4 and 0.5 of the structural height, depending on the degree of coupling of the structure, αH, and the stiffness of the stiffening beam, S_m. This position is essentially the same as that required to minimize the lateral drift, since the first mode of vibration corresponds closely to the static deflected form of the structure.

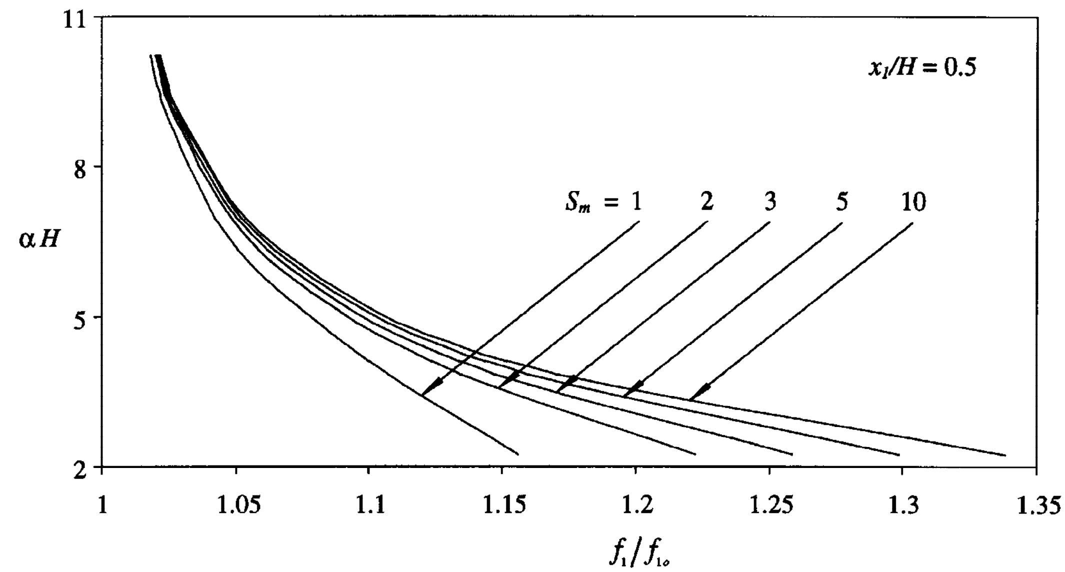


Fig. 3. Ratios of first natural frequency of stiffened coupled shear walls to that of normal coupled shear walls

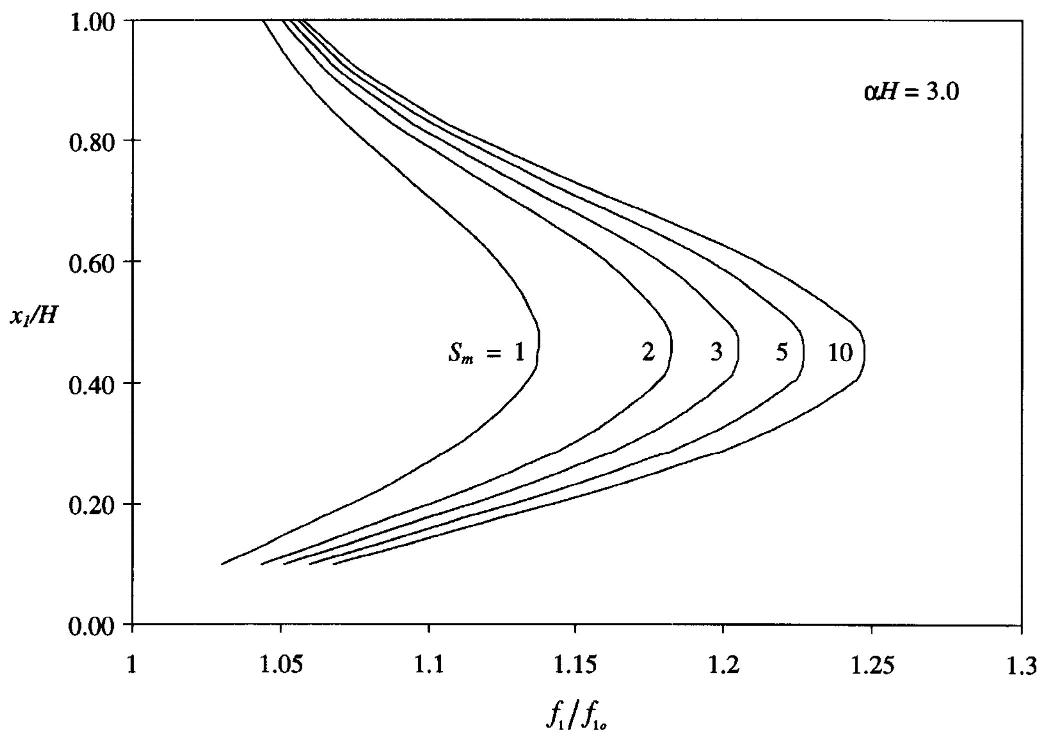


Fig. 4. Effect of stiffening beam location on first natural frequencies

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