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TIME-DELAYED ACTIVE CONTROL OF STRUCTURES

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ABSTRACT

The effects of time delays on collocated as well as noncollocated point control of classically damped discrete dynamic systems have been examined. Controllers of PID type have been considered. Analytical estimates of time delays to maintain/obtain stability for small gains have been obtained. It is shown that undamped structural systems cannot be stabilized with pure velocity (or integral) feedback without time delays while using a controller which is not collocated with the sensor, when the mass matrix is diagonal. However, with the appropriate choice of time delays, for certain classes of commonly occurring structural systems, stable noncollocated control can be achieved. Analytical results providing the upper bound on the controller's gain necessary for stability have been presented. The theoretical results obtained are illustrated and verified with numerical examples.

KEYWORDS

time delayed active control, noncolocation of sensors and actuators, PID controllers, finite gains.

1. INTRODUCTION

The methodologies for active control of structural and mechanical systems, which are modeled by linear matrix differential equations, lend themselves to a wide range of applications in civil, mechanical and aerospace engineering. Examples such as the control of tall building structures, bridges and nuclear power plants, to strong earthquake ground motion, the control of robot manipulators and the vibration control of Large Space Structures, are some examples where the proper control of structural systems to disturbances becomes essential to the continued usefulness and functionality of the systems concerned.

Many such systems are spatially distributed and due to their flexibility, low and closely spaced natural frequencies, and low damping coefficients, advanced structural control techniques are required to suppress the level of vibration of these structural systems to within acceptable limits. The interrelationship among the various disciplines affecting the analysis and design of structural systems shows that the aspect of modeling of the structures, control system design, and the decision regarding the number and location of sensors and actuators are closely related and play an important role in the problem of structural control. In this paper, we will focus on the problem of control system design which can be mainly divided into two major parts; (a) modeling, and (b) controller design. Though, in the past, a great deal of research effort has been devoted to the control of distributed parameter structural systems,

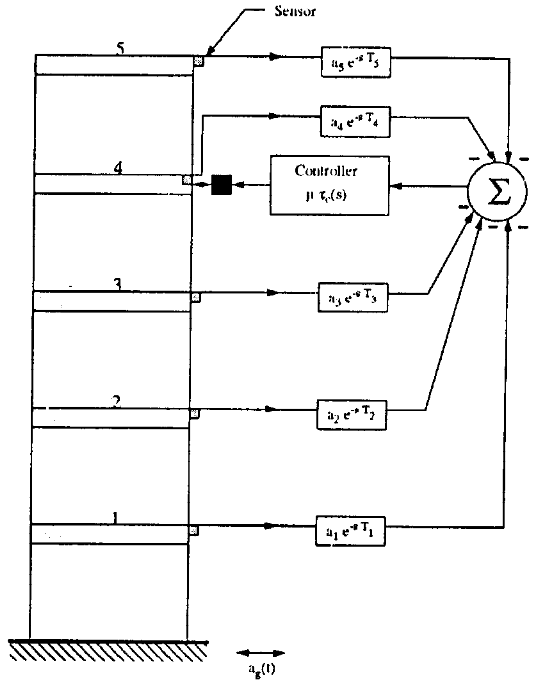

Figure 1: Shear frame building structure and control methodology.

most of these evolved control methodologies are applicable to only very simple systems. On the other hand, the control techniques, which deal with the finite element approximations of the real physical structural systems can be applied to a wide variety of real life structural problems.

In the field of active control of structural systems, it is a general notion that direct velocity feedback control of discrete dynamic systems, when the actuators and sensors are collocated, is stable for all values of controller gain (Balas (1979), Aubrun (1980)). This stable behavior for an undamped shear frame building (five degree-of-freedom system) structure (see Fig. 1), is shown in Fig. 2(a). The mass and the stiffness of each story are 1 and 1600 (taken in SI units). The system may also be thought of as a finite dimensional representation of a bar undergoing axial vibrations. The mass matrix is the identity matrix. With these system parameters, the undamped natural frequencies are calculated as 11.39, 33.23, 52.39, 67.30 and 76.76 rad/sec. In Fig. 2(a), the root loci of the closed loop poles correspond to when the actuator and the sensor are collocated at mass 4. The manner in which each pole of the system moves, as the controller gain is increased from zero are shown. We see that the system, as expected, is stable, since all the poles are in the left half s−plane.

However, in practical feedback control systems, small time delays in the control action are inevitable because of the involved dynamics of the actuators and sensors; such time delays become especially large when the control time histories call for relatively large force levels and/or high frequencies. Let us introduce a small time delay ($\Delta/T_1 \approx 0.045$, where T_1 is the fundamental period) of $\Delta = 0.025$ seconds in the collocated velocity feedback control action. Fig. 2(b) shows the root loci of the closed loop poles for the same configuration as used for Fig. 2(a) except that now we have introduced a small time delay in the control action. We see that the fourth and fifth closed loop poles start moving in the right half s−plane as the gain starts increasing from zero to even some small (infinitesimal) positive value.

We thus find that collocated velocity feedback control—a control methodology well established in the literature for its stability—turns out to become unstable, even at infinitesimal levels of gain, in the presence of small time delays!

Often, collocation of the actuator with the sensor is not possible. This is more true especially for large structural systems. It has been proved in the past that when actuator(s) and sensors are dislocated, feedback control of structural systems may become unstable (Cannon and Rosenthal (1984)). Fig. 3(a) shows the root loci of the closed loop poles for velocity feedback noncollocated control of the undamped

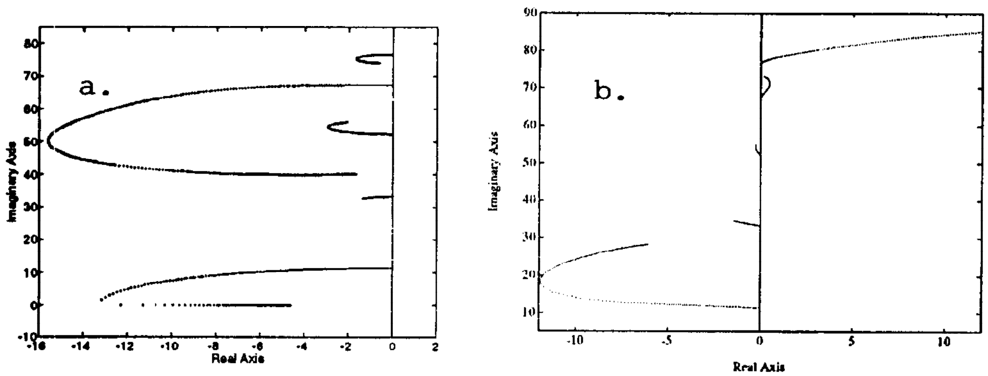

Figure 2: Root loci of closed loop poles of the velocity feedback collocated control system for time delay (a) $T_4 = 0.0$ sec (b) $T_4 = 0.025$ sec.

system of Fig. 1. Here, the controller is placed at mass 4 and fed with a direct velocity signal from location 5. We note that even for vanishingly small gains, as expected, the third, fourth and fifth closed loop poles start moving in the right half $s-$plane, and hence cause instability. However, with the appropriate choice of time delay, i.e., $T_5 = 0.04$ seconds, all the closed loop poles remain in the left half $s-$plane up to a certain value of the controller's gain μ. This is illustrated in Fig. 3(b). Thus, these time delays which had adverse effects for the collocated control and which are inherent in the system can be turned to our advantage in making the noncollocated control system stable!

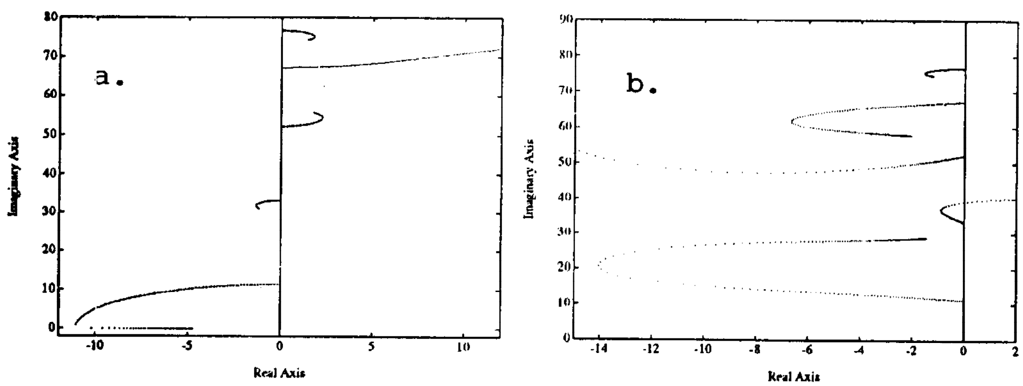

Figure 3: Root loci of closed loop poles of the velocity feedback noncollocated control system for time delay (a) $T_5 = 0.0$ sec (b) $T_5 = 0.04$ sec.

Therefore, the effects of time delays on the stability of closed loop control systems designed for large structures should be examined in depth. That is, how time delays affect the closed loop poles of the collocated control system and how these delays can be utilized to our advantage to handle noncollocated control configurations. These time delays are inevitable because of bandwidth limitations of any controller. Thus, a general theory, specifically answering questions such as: (1) what are the values of time delays which can be accommodated without losing the stability of the collocated feedback control systems and (2) what should be the values of intentional time delays (partly made of inherent time delays) which can help stabilize noncollocated feedback control, has to be evolved. The aim of this paper is to provide a general formulation for noncollocated feedback control of discrete systems and investigate some of the aforementioned issues. For a detailed literature review, the reader may refer to Kumar (1993).

Specifically, we model a structural or mechanical system as a classically damped multi-degree-of-freedom system and direct our attention to the general noncollocated feedback control design using several sensors and one controller. The multiple sensors collect response signals at various locations in the structural system. The control is taken to be of the PID type. Both collocated and noncollocated sensor-actuator positions are considered.

2. SYSTEM MODEL

Consider a linear classically damped structural system whose response $x(t)$, is described by the matrix differential equation

$$M\ddot{x}(t) + \tilde{C}\dot{x}(t) + \tilde{K}x(t) = g(t); \; x(0) = \dot{x}(0) = 0, \tag{1}$$

where, M is a positive definite, symmetric, $n \times n$ mass matrix, $\tilde{C}$ is the symmetric damping matrix and $\tilde{K}$ is the positive definite, symmetric, stiffness matrix. The n-vector, $g(t)$, is considered to be the forcing function. Substituting $y(t) = M^{1/2}x(t)$ yields

$$\ddot{y}(t) + C\dot{y}(t) + Ky(t) = f(t); \; y(0) = \dot{y}(0) = 0, \tag{2}$$

where $C = M^{-1/2}\tilde{C}M^{-1/2}$, $K = M^{-1/2}\tilde{K}M^{-1/2}$ and $f(t) = M^{-1/2}g(t)$.

Using another transformation $y(t) = Tz(t)$, we get

$$\ddot{z}(t) + \Xi\dot{z}(t) + \Lambda z(t) = T^T f(t); \; z(0) = \dot{z}(0) = 0, \tag{3}$$

where $\Xi = \text{Diag}\{2\xi_1, 2\xi_2, 2\xi_3, \ldots, 2\xi_n\}$, $\Lambda = \text{Diag}\{\lambda_1^2, \lambda_2^2, \lambda_3^2, \ldots, \lambda_n^2\}$ and the matrix $T = [t_{ij}]$ is the orthogonal matrix of real eigenvectors of K. We note that the simultaneous diagonalization of C and K is implied by the fact that the system is classically damped. Taking Laplace Transform we obtain

$$\hat{x}(s) = M^{-1/2}\hat{y}(s) = M^{-1/2}T\hat{z}(s) = M^{-1/2}T\Theta T^T M^{-1/2}\hat{g}(s), \tag{4}$$

where the hats indicate transformed quantities and the matrix

$$\Theta = \text{Diag}\{(s^2 + 2s\xi_1 + \lambda_1^2)^{-1}, (s^2 + 2s\xi_2 + \lambda_2^2)^{-1}, \ldots, (s^2 + 2s\xi_n + \lambda_n^2)^{-1}\}. \tag{5}$$

The open loop poles of the system are, therefore, given by the roots of the equations

$$s^2 + 2s\xi_q + \lambda_q^2 = (s - \gamma_{+q})(s - \gamma_{-q}) = 0, \; q = 1, 2, \ldots, n. \tag{6}$$

We have denoted the poles by $\gamma_{\pm q}$, $q = 1, 2, \ldots, n$, where the plus (minus) indicates the positive (negative) sign taken in front of the radical in solving the quadratic equations given in equation set (6). In this paper we shall always assume that the open loop poles are <u>all</u> distinct.

3. GENERAL FORMULATION FOR FEEDBACK CONTROL

We utilize p responses $x_{s_k}(t)$, $k = 1, 2, \ldots, p$, in our feedback control design. Each response $x_{s_k}(t)$ could, in general, be time-delayed by T_{s_k} and then linearly combined with other such time delayed responses before being then fed to a controller which generates the desired feedback control force. The control methodology, applied to a building structure, is as shown in Fig. 1. The actuator causes a force to be applied to the system thereby affecting the jth equation in the equation set (1).

When $j \notin \{s_k : k = 1, 2, \ldots, p\}$ we obtain a situation where the sensors and the actuator are noncollocated. If $j \in \{s_k : k = 1, 2, \ldots, p\}$ the sensor and the actuator are collocated.

Denoting the transfer function of the controller by $\mu\tau_c(s)$, where μ is the non-negative control gain, the closed loop system (in Laplace domain) is defined by

$$\hat{A}(s)\hat{x}(s) = [Ms^2 + \tilde{C}s + \tilde{K}]\hat{x}(s) = \hat{g}(s) - \mu\tau_c(s)\sum_{k=1}^{p} a_{s_k} x_{s_k}(s)\exp[-sT_{s_k}]e_j, \tag{7}$$

where e_j is the unit vector with unity in its jth element and zeros elsewhere. The real numbers a_{s_k} provide a linear combination of the responses which are fed to the controller. Moving the second term on the right hand side of (7) to the left, we obtain

$$\hat{A}_1(s)\hat{x}(s) = \hat{g}(s). \tag{8}$$

Therefore, the closed loop poles are the zeros of $\det[\hat{A}_1(s)]$.

We now assume that the following set of conditions is satisfied:

$$\left.\begin{array}{lll} (1) & \tau_c(\gamma_{\pm m}) \neq 0 & \text{for } m = 1,2,\ldots,n, \\ (2) & \sum_{k=1}^{p} a_{s_k} \exp[-\gamma_{\pm m} T_{s_k}] t_{s_k,m}^{(M)} \neq 0 & \text{for } m = 1,2,\ldots,n \text{ and} \\ (3) & t_{j,m}^{(M)} \neq 0 & \text{for } m = 1,2,\ldots,n, \end{array}\right\} \qquad (C1)$$

where $\gamma_{\pm i}$ are the open loop poles of the system and $t_{s_k,r}^{(M)} = \sum_{u=1}^{n} m_{s_k,u}^{-1/2} t_{u,r}$ and we have denoted the (i,j)th element of the symmetric matrix $M^{-1/2}$ by $m_{i,j}^{-1/2}$. The first condition in $C1$ requires that the zeros of the controller transfer function do not coincide with the open loop poles of the system; the second condition is a generalized observability condition, which requires that all mode shapes be observable from the summed, time-delayed, sensor measurements; and, the third condition is a controllability condition, and requires that the controller cannot be located at any node of any mode of the system.

When the open loop system has distinct poles and condition set $C1$ is satisfied, the open loop and closed loop systems do not have any pole in common, and the closed loop poles for $\mu > 0$ are given by those values of 's' which satisfy the relation

$$1 + \mu \tau_c(s) \sum_{k=1}^{p} \sum_{i=1}^{n} a_{s_k} \exp[-sT_{s_k}] \left[\frac{t_{s_k,i}^{(M)} t_{j,i}^{(M)}}{s^2 + 2s\xi_i + \lambda_i^2} \right] = 0. \qquad (9)$$

For details, the reader may refer to Kumar (1993).

4. SMALL GAIN STABILITY OF THE FEEDBACK CONTROL

A sufficient condition for the closed loop system to remain stable for infinitesimal gains is that:

$$\operatorname{Re} \left\{ \left. \frac{ds}{d\mu} \right|_{\substack{\mu \to 0 \\ s \to \gamma_{\pm r}}} \right\} < 0, \quad r = 1,2,\ldots,n. \qquad (10)$$

This condition requires the root loci of the closed loop poles to move towards the left half s-plane and hence stability is ensured. For undamped systems, relation (10) is a necessary and sufficient condition since the open loop poles now lie on the imaginary axis of the s-plane. In what follows, we derive general expression for the term inside the curly brackets in inequality (10). These results will be particularized in the next section.

When the controller gain, μ, equals zero, the system becomes open loop and the poles of the system are the open loop poles. Multiplying (9) by $(s^2 + 2s\xi_r + \lambda_r^2)$ and then differentiating with respect to μ and letting $s \to \gamma_{\pm r} = -\xi_r \pm i\sqrt{\lambda_r^2 - \xi_r^2}$ and $\mu \to 0$, we obtain

$$\left. \frac{ds}{d\mu} \right|_{\substack{\mu \to 0 \\ s \to \gamma_{\pm r}}} = -\frac{\tau_c(\gamma_{\pm r})}{\pm 2i\sqrt{\lambda_r^2 - \xi_r^2}} \left[\sum_{k=1}^{p} a_{s_k} \exp[-\gamma_{\pm r} T_{s_k}] t_{s_k,r}^{(M)} \right] t_{j,r}^{(M)}. \qquad (11)$$

We note that if the actuator is located at a node of the rth mode then the position of the rth open loop pole will not be affected by the feedback control because $t_{j,r}^{(M)} = 0$ and the right hand side of equation (11) vanishes. Therefore, system is not controllable. Furthermore, if the sensors are located such that

$$\sum_{k=1}^{p} a_{s_k} \exp[-\gamma_{\pm r} T_{s_k}] t_{s_k,r}^{(M)} = 0 \qquad (12)$$

for any particular r, then the rth open loop pole is not affected by the control because the system is not observable. We also note that when using multiple sensors (i.e., $p > 1$) even when the sensors are not located at any of the nodes of the rth mode, the sensor outputs could be so combined that (12) is satisfied. This will leave the rth mode unobservable and thus the rth open loop pole unaffected by the feedback control.

5. PID FEEDBACK CONTROL

We now particularize the controller's transfer function to be

$$\tau_c(s) = K_0 + K_1 s + \frac{K_2}{s}; \quad K_0, K_1, K_2 \geq 0. \tag{13}$$

The first term on the right refers to proportional control, the second to velocity control and the third to integral control. PID controllers are easily realizable and are commonly used in control system designs.

5.1 Results for Undamped System. For an undamped system, $C = 0$ and $\gamma_{\pm r} = \pm i\lambda_r$. For these system parameters, a necessary and sufficient condition for vanishingly small gains stability for general configuration can be deduced after substituting relation (11) into (10). Particularly, when using one sensor, collocation of the sensor with an actuator will cause PID feedback control to be stable (for small gains) for an undamped system if and only if

$$a_j \left\{ -\frac{K_0}{\lambda_r} \sin(\lambda_r T_j) + (K_1 - \frac{K_2}{\lambda_r^2}) \cos(\lambda_r T_j) \right\} > 0, \quad \text{for } r = 1, 2, \ldots, n. \tag{14}$$

For velocity feedback control the above condition will be satisfied for any $a_j > 0$ as long as the time delay is such that $T_j < \frac{\pi}{2\lambda_{max}}$, where λ_{max} is the highest undamped natural frequency of the system, i.e., $T_j < \frac{T_{min}}{4}$, where T_{min} is the smallest period of vibration of the system.

It should be noted that small gains stability is <u>not</u> ensured when using a number of sensors, *one* among which is collocated with the actuator, even when using no time delays. Once the stability is ensured for vanishingly small gains, we can show next that the velocity feedback control will be stable as long as

$$\mu < \frac{1}{a_j K_1 \eta_0 \sum_{i=1}^{n} \frac{[t_{j,i}^{(M)}]^2}{\eta_0^2 - \lambda_i^2}}, \tag{15}$$

where $\eta_0 = \frac{\pi}{2T_j}$. It can also be proved that if any of the closed loop poles cross the imaginary axis at $s = \pm i\eta$, then $\eta > \lambda_{max}$, where λ_{max} is the highest natural frequency of vibration. For a proof and similar stability bounds for integral and proportional feedback, the reader may refer to Kumar (1993).

Now considering the control of noncollocated systems, we have shown earlier with the help of an example problem that in the *absence* of time delays velocity feedback control is unstable. In general, for a PID controller it can be proved mathematically that if matrix M is diagonal and $K_1 > \frac{K_2}{\lambda_{min}^2}$ or $K_1 < \frac{K_2}{\lambda_{max}^2}$, it is impossible to stabilize such systems for small gains. Such feedback control is guaranteed to destabilize atleast one mode of the system. But for simple chain-like undamped structural systems when using an ID controller, where

(1) conditions $C1$ are satisfied,
(2) one sensor is used and it is not collocated with the actuator,
(3) the sign change in sequence $\{t_{s_1,r}^{(M)} t_{j,r}^{(M)}\}_{r=1}^{n}$ occurs when $r = m$,
(4) time delay, $T_{s_1} = \left(\frac{\pi}{2\lambda_{m-1}} - \epsilon\right)$, where ϵ is a small positive quantity and such that $(T_{s_1}\lambda_m) > \frac{\pi}{2}$,
(5) $\frac{\lambda_{max}}{\lambda_{m-1}} \leq 3$ and
(6) $K_1 > \frac{K_2}{\lambda_{min}^2}$, or $K_1 < \frac{K_2}{\lambda_{max}^2}$,

it is possible to stabilize noncollocated feedback control systems for small gains. Specifically, the velocity feedback control of such systems will be stable as long as $\mu < G$, where G is the minimum of all positive B_l, for $l = 0, 1, 2, \ldots$, where

$$B_l = \frac{-1}{a_{s_1} K_1 \eta_l \sin(\eta_l T_{s_1}) \sum_{i=1}^{n} \frac{t_{s_1,i}^{(M)} t_{j,i}^{(M)}}{\lambda_i^2 - \eta_l^2}}, \tag{16}$$

and $\eta_l = \frac{(2l+1)\pi}{2T_{s_1}}$. Details pertinent to these results are provided in Kumar (1993).

5.2 Results for Underdamped System. When using PID control for underdamped systems, $\xi_i < \lambda_i$, $i = 1, 2, \ldots, n$, a sufficient condition for the closed loop system to be stable for small gains can be derived directly from relations (10) and (11). For one sensor-actuator collocated pair, if $\left(K_1 - \frac{K_2}{\lambda_r^2}\right) \neq 0$, for all r, this sufficient condition takes the form

$$a_j \left(K_1 - \frac{K_2}{\lambda_r^2}\right) \cos(\sqrt{\lambda_r^2 - \xi_r^2}\, T_{s_k} + \phi) > 0, \quad \text{for } r = 1, 2, \ldots, n, \tag{17}$$

where

$$\phi = \tan^{-1}\left[\frac{K_0 - \left(K_1 + \frac{K_2}{\lambda_r^2}\right)\xi_r}{\left(K_1 - \frac{K_2}{\lambda_r^2}\right)\sqrt{\lambda_r^2 - \xi_r^2}}\right]. \tag{18}$$

And if we use pure velocity feedback, the closed loop poles (for small gains) will move to the left in the s-plane as long as the time delay

$$T_j < \min_{\forall\, r} \left[\frac{\frac{\pi}{2} + \phi}{\sqrt{\lambda_r^2 - \xi_r^2}}\right], \tag{19}$$

where

$$\phi = \tan^{-1}\left[\frac{\xi_r}{\sqrt{\lambda_r^2 - \xi_r^2}}\right]. \tag{20}$$

Similar bounds on time delays can be found for P, I and PID controllers.

For lightly damped systems ($\xi_r << 1$) whose mass matrix is diagonal, noncollocated PID control with no time delays will most likely destabilize the system when either $K_1 > \frac{K_2}{\lambda_{min}^2}$, or $K_1 < \frac{K_2}{\lambda_{max}^2}$. Since under these provisions, the condition necessary for the closed loop poles to move towards left (for small gains) is violated. Thus, as we increase the value of μ from 0 to 0+, some of the poles will most likely start moving towards right in the left half $s-$plane and quickly make the system unstable because the open loop system has very small amount of damping. An approximate bound on the gain to ensure stability can be found.

6. NUMERICAL RESULTS AND DISCUSSION

Two examples of structural response are provided in this section, serving as verification of our theoretical results. For integration, we use the fourth order Runge-Kutta scheme. The time step for integration, Δt, has been so taken that Δt (= 0.004 sec) $< T_{min}/20$, where T_{min} is the minimum period of vibration of the structure. For response results, a very small amount of damping has been introduced in each mode of vibration of the structure so that smooth integration can be carried out. For all the root loci plots in this paper, the controller's gain μ has been varied from 0 to 100 units. Response and force time history plots are shown only for first 10 seconds.

Example. In Section 1, we have shown that velocity feedback collocated control, when the time delay $T_4 = 0.025$ seconds (greater than $\frac{\pi}{2\lambda_{max}}$), is unstable for all $\mu > 0$. Appropriately taking the value of this time delay, $T_4 = 0.018$ seconds, we show in Fig. 4 that root loci of the closed loop poles of the collocated control system remain in the left half $s-$plane as long as the gain μ is less than 37.9 units. We observe that the 5th pole crosses the imaginary axis of the $s-$plane at $\eta = 87.27$ rad/sec. These numerically obtained values of the gain μ and the cross over frequency η are exactly those obtained from theoretical expressions given in Section 5.1.

Fig. 5(a) shows the displacement time history of mass 5 relative to the base, when this structure is subjected to the S00E component of the Imperial Valley Earthquake 1940 ground motion. The response has been shown for $\mu = 0$ and $\mu = 20.0$ units. Fig. 5(b) contains the time histories of the incoming force per story (i.e., negative of story mass times ground acceleration) and the control force required when the controller's gain $\mu = 20.0$ units.

7. CONCLUSIONS

Several results related to both collocated and noncollocated time delayed control of undamped as well as underdamped multi-degree-of-freedom systems have been presented. While the results are

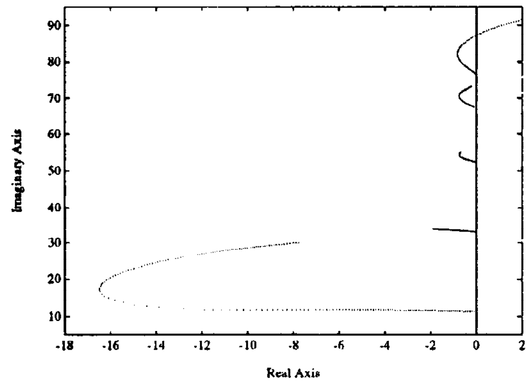

Figure 4: Root loci of closed loop poles of the velocity feedback collocated control system ($j = 4,\ s_1 = 4,\ a_4 = 1,\ T_4 = 0.018$ sec).

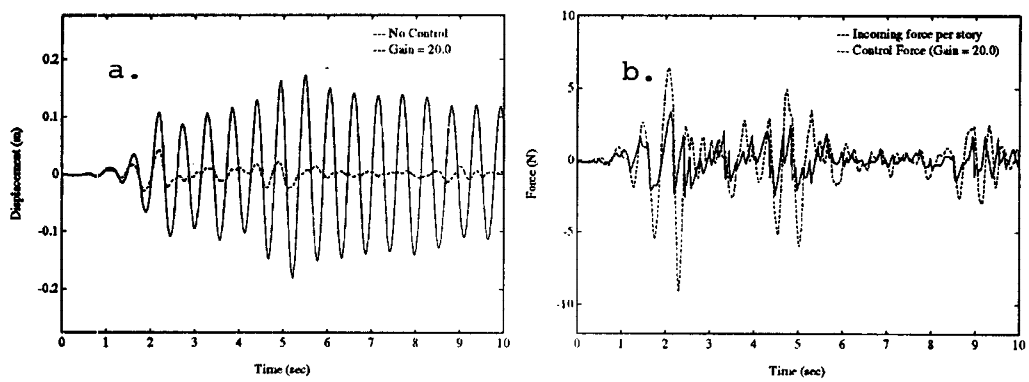

Figure 5: For collocated stable control, time variation of (a) relative displacement of mass 5 (b) incoming and control forces.

specifically related to PID controllers, the general approach provided in this paper can be used for all finite dimensional controllers. It is shown that time delays, which make collocated control systems unstable, can help stabilize noncollocated control systems. Easy-to-compute analytical expressions for time delays to maintain/obtain small gains stability and for upper bound on the controller's gain are provided for both collocated and noncollocated control configurations.

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