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TIME INTEGRATION ALGORITHMS FOR SEISMIC RESPONSE OF SOFTENING STRUCTURES

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ABSTRACT

The seismic response of concrete dams under severe ground motion is presented using Damage Mechanics Theory to simulate cracking of the concrete. Two time-marching algorithms are compared, the HHT-α and the modified Rosenbrock. Both algorithms are stable and control the undesirable high frequency response. However, the second algorithm produces less error when computing the global energy balance.

KEYWORDS

Concrete dams, Damage mechanics, Time integration, Seismic response.

INTRODUCTION

Concrete dams are designed to resist two levels of earthquakes: A design base earthquake (DBE), where the dam is expected to remain in the elastic range, and a maximum credible earthquake (MCE) where cracking and structural damage is expected as long as the reservoir is contained. Considerable work has been performed in the linear range on concrete dams. However, for the last twenty years significant effort has been invested in the numerical modelling of cracks for unreinforced concrete with emphasis on dam applications. In this paper the seismic response including cracking is presented using Damage Mechanics Theory (Ghrib and Tinawi, 1995), coupled with nonlinear fracture mechanics concepts. This is based on changes in the constitutive laws governing the concrete rather than creating a discontinuity in the continuum and resorting to remeshing techniques for the simulation of the crack propagation.

When using the local approach in fracture, the problem of mesh objectivity arises. The mesh-dependant hardening modulus results in the introduction of a length scale in the constitutive law. For large structures, this technique offers acceptable results regarding crack orientation and fracture energy dissipation under static loads. The proposed model appears to be particularly suited to include some other material formulations and presents reasonable expectations for the nonlinear seismic analysis of dams.

Unreinforced concrete dams exhibit softening and a decrease in stiffness when subjected to strong earthquakes. In particular, the strain softening is directly related to the tensile fracture of the concrete. Therefore, implicit time integration algorithms are often preferred to explicit methods because they exhibit favourable stability conditions in a nonlinear response. While for linear systems, stability conditions can be established analytically, the optimum time-marching algorithm is not always obvious for nonlinear systems. Often extensive numerical comparisons are performed to appreciate the characteristics of the algorithm to be used.

When evaluating the nonlinear dynamic response with an implicit time-marching algorithm, an iterative process is required to achieve convergence. In the presence of severe strain softening, Newton-type algorithms exhibit difficulties in converging. In addition, the closing-opening of the cracks induce high frequency shock waves which, if they are not annihilated rapidly, affect the stress history and the solution tends to diverge.

In this paper, two numerical algorithms are compared: the Hilber-Hughes-Taylor (1977) method (HHT-α) and the modified Rosenbrock (Piché, 1995). Koyna dam is used for calibration purposes in this study. Focus is placed on the convergence criteria of the two schemes and the stability of the results is controlled by the energy balance in the system.

DAMAGE MODEL FOR CONCRETE

The concept of damage assumes that the material degradation is induced by microcracks which leads to a reduction of the net area capable of supporting stresses. The loss of rigidity follows, as a consequence of microcracks, in defining a fictitious undamaged material in relation to the damaged one through the concept of energy equivalence. The strain energy in the damaged material W_d and the elastic strain energy in the equivalent undamaged material W_0 provide a possible definition of the damage variable d such that:

$$d = 1 - \sqrt{\frac{W_d}{W_0}} \tag{1}$$

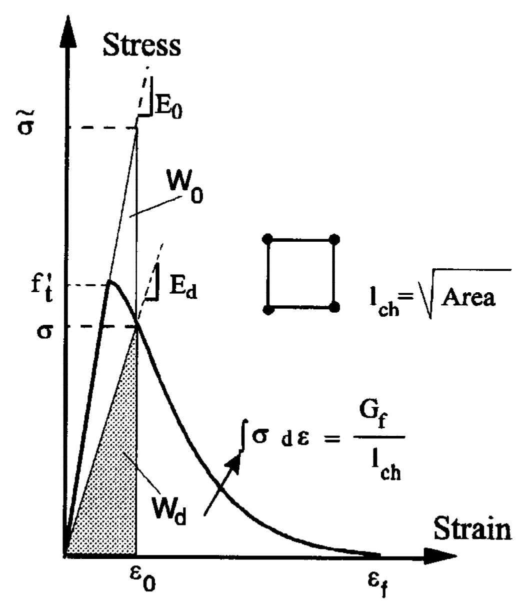

Fig. 1. Damage mechanics model.

If the material is in the elastic range, $W_d = W_0$ and therefore d=0. After complete cracking, no energy can be stored in the material, hence $W_d = 0$ and consequently d = 1 which implies total loss of rigidity (see Fig. 1).

The fracture energy of the concrete is defined as:

$$G_f = l_{ch} \int \sigma d\epsilon \tag{2}$$

where l_{ch} is the characteristic length related to the finite element size. If an exponential-like strain softening curve is adopted, the damage parameter can be expressed as

$$d = 1 - \left\{ \left(\frac{\epsilon_0}{\epsilon} \right) [2\exp(-b(\epsilon - \epsilon_0) - \exp(-2b(\epsilon - \epsilon_0))] \right\}^{\frac{1}{2}} \quad (3)$$

where

$$b = \frac{3}{\epsilon_0 \left(2E_0 G_f / l_{ch} f_t^2 - 1 \right)} \geq 0 \quad (4)$$

When the strain is increasing the damage will also increase. During cycling loading, the unloading path is a function of d and under compression the material recovers its initial stiffness. This model presented by Ghrib and Tinawi (1995) has the advantage of requiring only three parameters: the elastic modulus E_0, the tensile strength of concrete f_t and its fracture energy G_f.

IMPLICIT TIME INTEGRATION ALGORITHMS

In selecting a time marching algorithm, Hilber and Hughes (1978) provide the following attributes that a method should posses: (i) at least second-order accuracy; (ii) unconditionally stable for linear systems; (iii) contollable algorithmic damping in the higher frequency modes; (iv) self starting; and (v) no more than one set of an implicit system of equations should have to be solved at each step.

Numerous algorithms have been developed to satisfy these criteria. The main difference between these algorithms is the technique to control algorithmic damping. Numerical damping, for high-frequency control, is desired since a time-marching algorithm is used within a Finite Element context. This high frequency response is an artifact of the spacial discretization process and not representative of the behaviour of the governing equations. It is therefore desirable to remove the participation of the high-frequency mode components.

For nonlinear analysis and especially for problems dominated by strain-softening, removing or at least controlling the high frequency modes is necessary to achieve convergence. In these cases when fracture is simulated by continuum models, "weak" zones will develop, which are characterized by zero stiffness, whereas the undamaged material retains its initial stiffness. As a consequence, the natural periods of the model may differ significantly. For such situations, spurious oscillations would enter into the constitutive model and affect the computed response.

As pointed out by Hulbert (1991), ideally any high-frequency mode should be eliminated after one step, which is referred to as asymptotic annihilation of these components. The methods which possess this property are called L-stable.

Most of the popular time-marching scheme used for dynamic analysis of structures introduce a parameter to incorporate some amount of "algorithmic damping". In this paper two algorithms are compared based on the analysis of a full scale gravity dam subjected to an earthquake: (i) the HHT-α of Hilber Hughes and Taylor (1977); and (ii) modified Rosenbrock algorithm (Piché 1994). They are briefly summarised below.

HHT-α method

The equations of motion of a discretized structure is written in the following form :

$$[M]\{a_{i+1}\} + (1+\alpha)[C]\{v_{i+1}\} - \alpha[C]\{v_i\} + (1+\alpha)\{r_{i+1}\} - \alpha\{r_i\} = (1+\alpha)\{f_{i+1}\} - \alpha\{f_i\} \quad (5)$$

where [M] and [C] are the mass and damping matrices, {r} is the nodal restoring force and {f} is the external force excitation. To solve equation (5), the Newmark scheme is used. The displacement $\{u_{i+1}\}$ and the velocity $\{v_{i+1}\}$ are given in terms of the acceleration $\{a_{i+1}\}$ such that:

$$
\begin{aligned}
u_{i+1} &= u_i + dt\,v_i + dt^2\left[(\frac{1}{2}-\beta)a_i + \beta\, a_{i+1}\right] \\
v_{i+1} &= v_i + dt\left[(1-\gamma)a_i + \gamma\, a_{i+1}\right]
\end{aligned}
\qquad (6)
$$

α, β and γ are parameters that govern the numerical properties of the algorithm ($\alpha \in [-1/3, 0]$, $\beta = (1-\alpha)^2/4$ and $\gamma = (1-2\alpha)/2$). The solution process for nonlinear problems using HHT-α scheme is based on Newton type algorithms. The implementation in conjunction with a damage model is described elsewhere (Ghrib and Tinawi, 1995).

Modified Rosenbrock algorithm

A two-stage time step marching algorithm adapted from a modified Rosenbrock technique to solve dynamic structural problems was presented recently (Piché 1995). This method is L-stable and has a second-order accuracy with a small relative phase error compared to the average acceleration scheme. Its time marching scheme is presented in an algorithmic format (Piché 1995).

The dynamic equation is written in the following form :

$$[M]\{a(t)\} + \{R^{int}(v,u)\} = \{f(t)\} \qquad (7)$$

where t is time, [M] is the mass matrix and {f(t)} is the exciting force and $\{R^{int}\}$ is the internal forces in the structures which for linear problems is equal to [C]{v} + [K]{u}. For nonlinear behaviour, including damage, the internal forces will depend on the damage variable d. Let γ be a constant equal to $(1 - 1/\sqrt{2})$, to move from a time step i to i+1, the algorithm has five steps :

1. Evaluate the vectors $\{\nabla f\} = \{f_{i+1}\} - \{f_i\}$ and $\{r_0\} = \{R^{int}(v_i, u_i)\}$
2. If necessary, form the stiffness and damping matrices [K] and [C]
   Form the "equivalent mass matrix" $[M^*] = [M] + dt\gamma[C] + dt^2\gamma^2[K]$
3. **First stage**
   Solve the linear system
   $$[M^*]\{\tilde{e}\} = dt\,[\{f_i\} - \{r_0\} + \gamma\{\nabla f\} - \gamma dt[K]\{v_i\}\,]$$
   Compute the vector $\{\tilde{u}\} = dt\,(\{v_i\} + \gamma\{\tilde{e}\})$
4. **Second stage**
   Form the vectors
   $$\{f_{1/2}\} = \{f(t_i + 0.5dt)\},$$
   $$\{r_{1/2}\} = \{R^{int}(v_i + 0.5\,\tilde{e}, u_i + 0.5\tilde{u})\}$$
   Solve the linear system
   $$[M^*]\{\delta v\} = dt[\{f_{1/2}\} - \{r_{1/2}\} + dt\gamma(2\gamma - 1/2)[K]\{\tilde{e}\} + \gamma\,[C]\{\tilde{e}\}\,]$$
   Compute the vector $\{\delta u\} = dt\,(\{v_i\} + (0.5 - \gamma)\{\tilde{e}\} + \gamma\{\delta v\})$
5. Move the solution a step forward
   $$\{u_{i+1}\} = \{u_i\} + \{\delta u\}$$
   $$\{v_{i+1}\} = \{v_i\} + \{\delta v\}$$

The main difference between this algorithm and the classical methods used lies in the evaluation of δu and δv which are based on the Jacobian of the internal forces in the structure and not a linear multi-step formula. In the case of HHT-α, where Newmark formula is used with $\beta = 1/4$ and $\gamma = ½$, it is implicitly assumed that the acceleration is constant from time step i to time step i+1. Presented in this form, the implementation of this algorithm is relatively easy. The extension to nonlinear problem is straight forward since it is based on the evaluation of the internal forces in the structure.

In terms of computational effort this method is competitive with other one-step algorithms. For linear problems with a fixed time step, dt, the algorithm requires a single assembly of the tangential stiffness and damping matrices which remain constant during the analysis. For each time step, the forward solution uses two sets of linear systems instead of one for the HHT-α algorithm. For nonlinear problems the computational cost is governed by the number of formation of stiffness and damping matrices as well as the decomposition of the M* matrix, which is similar to other implicit methods.

Control of the stability of the numerical scheme

To control the stability of the numerical solution using the HHT-α and modified Rosenbrock algorithms, the seismic energy balance criterion is used together with equilibrium to ensure convergence. The energy balance of the system is computed as the absolute error between the input energy and the energy dissipated in the structure.

$$Err = \left| \frac{(E_q + E_p) - (E_k + E_d + E_r)}{(E_q + E_p)} \right| 100\% \quad \textbf{(8)}$$

Where E_q is the energy due to earthquake input, E_p is the deformation energy due to pre-seismic loads, E_k is the kinetic energy, E_d is the viscous damping energy and E_r is the elastic deformation energy in the structure.

NUMERICAL EXAMPLE

The damage model presented in this paper is implemented within a finite element code (Ghrib, 1994). The two time-marching algorithms discussed above can be used. The seismic analysis of Koyna dam is discussed as a numerical example as well as a benchmark for validation of numerical models. The numerical experiments have indicated that the cracking response is strongly depending on the material constitutive model (isotropic or orthotropic) as well as the viscous damping representation. In this paper, only orthotropic damage model and damage proportional damping is considered. In this case, the damping matrix is defined as follows:

$$[C_d] = \zeta [K_d] \quad \textbf{(9)}$$

The parameter ζ is calibrated to provide 5% of damping on the initial fundamental mode. The model and the finite element mesh are presented in Fig. 2. The foundation is considered rigid and the hydrodynamic effect of the reservoir is simulated using Westergaard added masses. Fig. 2 shows the horizontal and vertical components of the accelerograms.

The mass density of the concrete is $\rho = 2600$ kg/m^3, E= 30000 MPa, $\nu = 0.2$. The fracture resistance is defined mainly by the tensile strength $f_t = 1.8$ MPa and the fracture energy $G_f = 180$ N/m.

When using Newmark algorithm (HHT-α with α=0) the analysis failed and the energy error increased rapidly at about 4.5 sec. Newmark algorithm is free of any numerical damping and therefore there is no control on the high-frequency components. The spurious oscillations alter dramatically the solution.

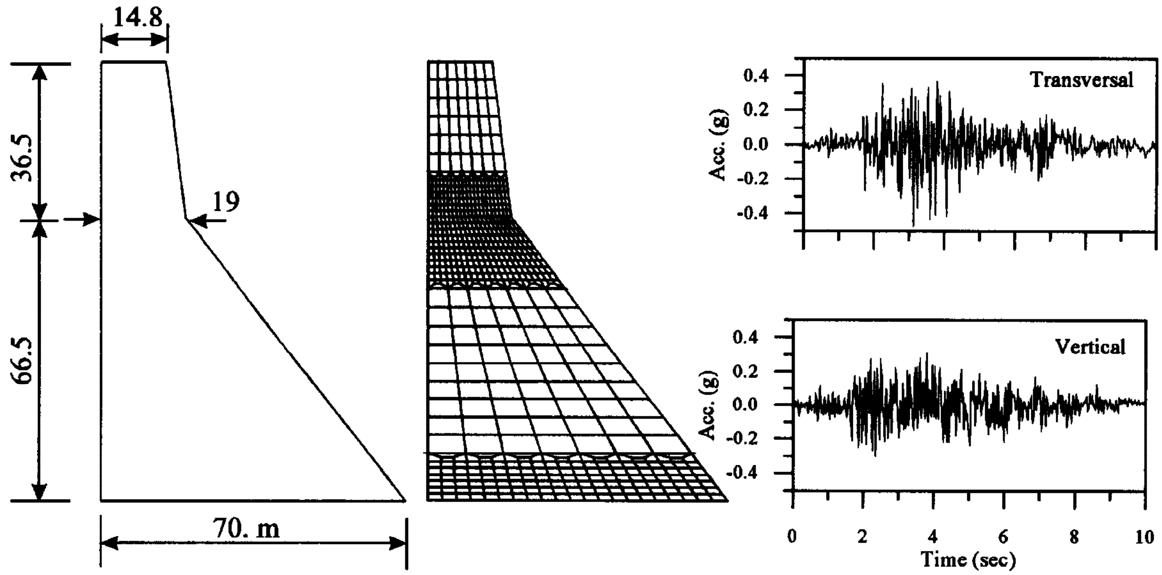

Fig. 2 Koyna dam model and seismic input.

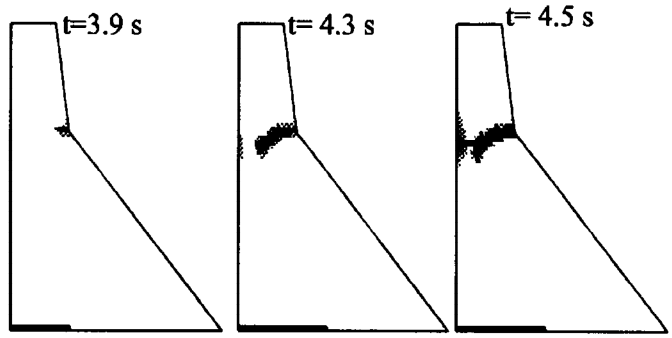

Fig. 3 Damage evolution in Koyna dam.

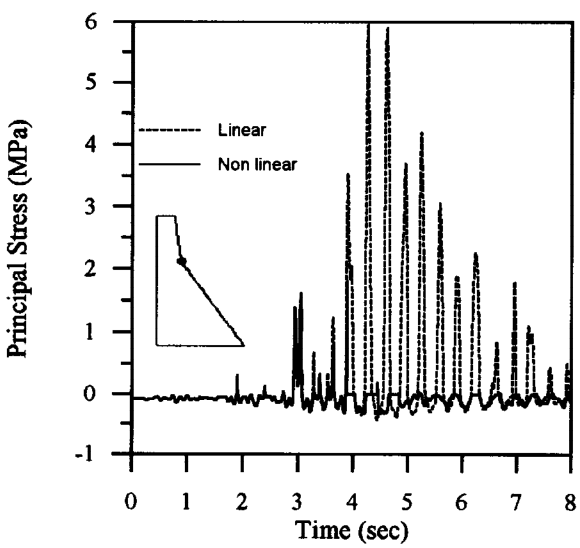

Fig. 4 Principal stresses at change of slope.

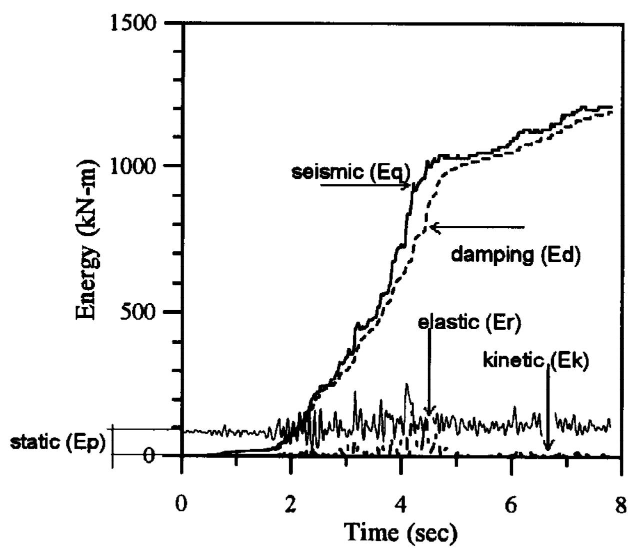

Fig. 5 Energy compoments.

Controlling the high-frequency modes by using non-zero α parameter (α = -0.1) eliminated the problem and convergence is achieved with an energy error less than 1%. The damage map showing the evolution of cracking in the structure is shown in Fig. 3. The global crack profile is comparable to what was observed in the field and simulated by experimentation. To show the efficiency of the constitutive model in simulating fracture, the principal stress of a selected element is presented in Fig. 4. Linear analysis shows an excessive stress and the damage mechanics model removed completely the tensile stress transmitting capability of the element after cracking. The different components of the energy balance is shown in Fig. 5. The viscous damping mechanism dissipates most part of the energy transmitted to the dam.

The dam was analysed using the modified Rosenbrock algorithm. In this case, no adjustment parameter is needed. Because of its "asymptotic annihilating" properties, this scheme eliminates the high-frequency components from the response. Fig. 6 shows a comparison of the crest displacement obtained by HHT-α (α=-0.1) and modified Rosenbrock methods. The response is almost the same. The stress history of a selected element at the downstream face of the dam, presented in Fig. 7, confirms the similarity of the results. By examining the time history of the energy error of the two algorithms, Fig. 8 shows that the modified Rosenbrock algorithm is smoother than HHT-α and produces a smaller global error.

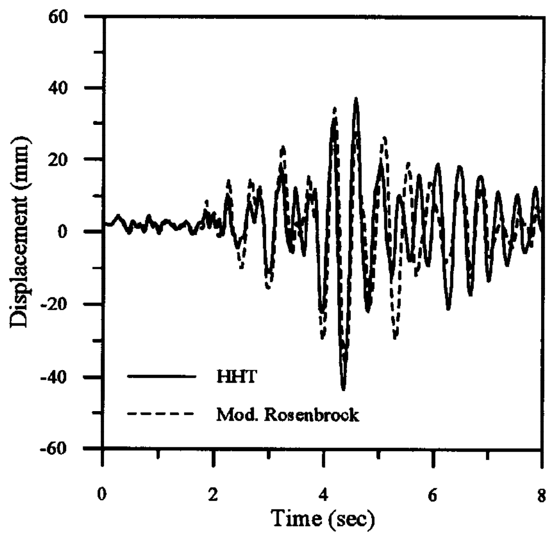

Fig. 6 Comparaison of the crest displacement.

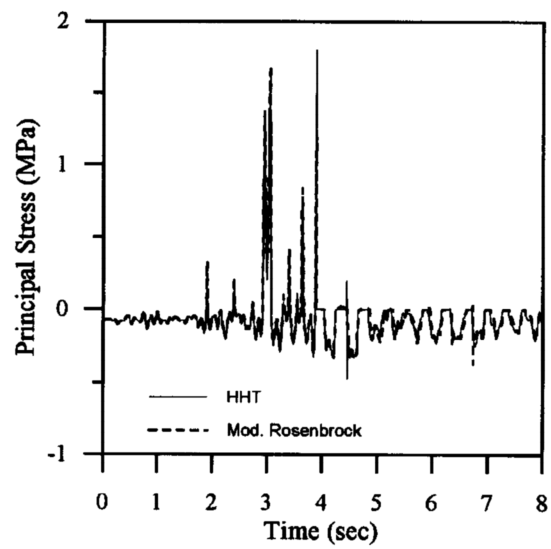

Fig. 7 Comapaison of principal stresses.

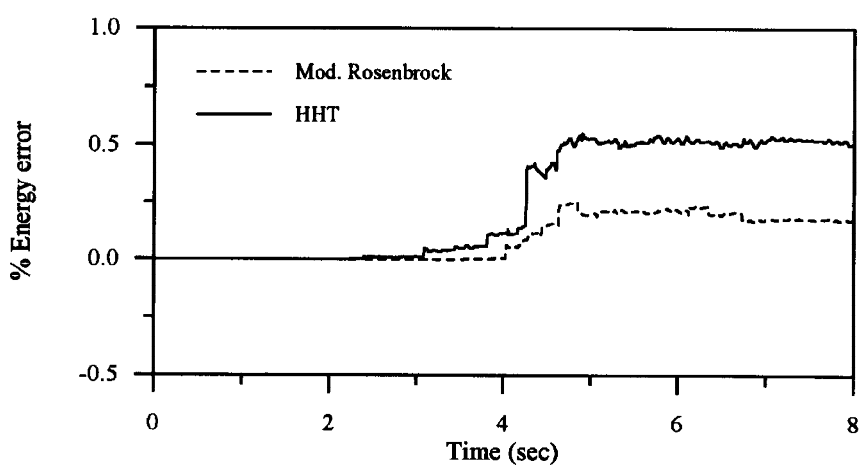

Fig. 8 Energy error of Koyna dam analysis using the two methods.

CONCLUSIONS

A Damage Mechanics model for the seismic response of concrete dams has been presented taking into consideration cracking of the material. This model offers distinct advantages over discrete crack models. A comparison between the HHT-α and the modified Rosenbrock algorithm shows that the second one appears to produce less errors in the global energy balance check. In addition, no parametric investigation for α is required to ensure numerical stability.

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