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ANALYTICAL MODELING OF R/C BEAM COLUMN CONNECTIONS UNDER CYCLIC LOADS

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ABSTRACT

The present study deals with the behavior of connections under cyclic loads incorporating both equilibrium and compatibility conditions within the joint. It further considers stress softening of concrete and reinforcement details of the joint. The softened truss model theory has been adopted to establish the shear stress-shear strain relationship and to determine the shear capacity of the joint. The flexibility of joints is considered in the inelastic dynamic analysis of framed structure by developing a basic component model for flexural elements with joint elements at the ends. The proposed model is designed to identify joint shear failure and anchorage failure within the joint. The capabilities of the model are demonstrated through an illustrative study on an interior beam column joint.

KEYWORDS

Anchorage (structural); connections; cyclic loads; earthquake-resistant structures; joints; stirrups; slippage; shear strength; stress-strain relationships; structural analysis.

INTRODUCTION

Seismic hazard assessment of existing reinforced concrete frames depends on the inelastic behavior of its structural components- beam, column and the connecting element ***Joint***. Research studies through experiments have given better understanding on the inelastic behavior of flexure predominant elements rather than shear predominant elements. Experimental model study on behavior of joints with predominant effect of shear under cycling loading was difficult because of stringent scaling laws. Efficient mathematical models are available for flexural elements to represent the inelastic cyclic behavior. Uncertainty and differences of opinion still pervade almost every aspect of connection behavior which pose obstacles to evolve analytical model.

Framed structures not compliant with recent seismic codes, may have inadequate joint shear reinforcement and detailing. This may result in local shear failure at joints before other flexural elements reach their capacity. In seismic hazard assessment of such structures (where joint shear strength may govern the strength of the structure) detecting vulnerable joints and to include the effect on the total response of the

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structure are important. Most of the analytical tools, designed for evaluation of seismic damage of the structure, consider joints as rigid zones failing to include the effect of joint contribution.

This paper presents analytical procedures to include the effect joints along with flexural elements in analysis of reinforced concrete plane frames. This is effectively done after understanding the joint mechanics and implementing the model on IDARC(2.1) platform. The efficiency of the model is illustrated through a case study on interior beam column subassemblage.

JOINT BEHAVIOR

Joints in reinforced concrete frames are subjected to severe cyclic shear under seismic excitations. The joints, if not designed and detailed properly, will place the whole structure in jeopardy. They demand more attention and careful understanding. Experimental studies have been conducted extensively on statically determinate beam column joint subassemblages representing RC frame connections. They have attempted either to understand the joint shear behavior until its failure or to check the joint design with stated objective. The perceived responses of the specimens are affected by the vast range of influencing parameters adopted in the respective experiments. They include the nature of loading history that is applied to the specimens also, which is the additional cause for the diversity in the responses between various experiments.

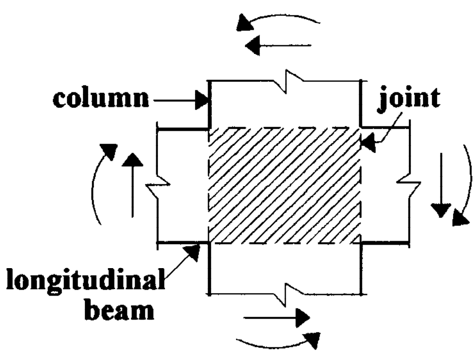

Fig. 1a. Actions at the boundaries of the joint

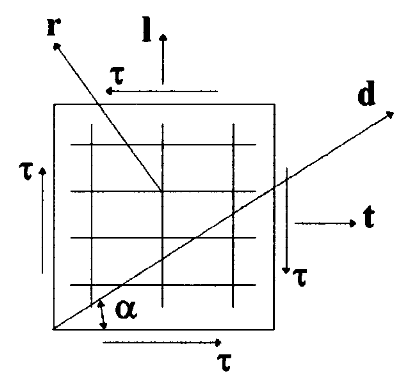

Fig. 1b. Joint idealisation with reinforcement

It has been recognized in experimental studies on exterior (Ehsani *et al.*, 1985) and interior beam column connections (Durrani *et al.*, 1985; Leon, 1990) that there is interaction between joint shear stress level and anchorage requirements during cycling in the inelastic range. The joints showed satisfactory performance with respect to anchorage failure when the joint shear stress is well below limiting shear stress. It is worth mentioning here that in many of the experiments, joint responses have been observed under coupled flexure-shear mode of failure. The behavior of shear predominant joints (panel zones) under cyclic loading could possibly be better understood only when the panel zone is subjected to pure shear failure. This is considered important if realistic force resisting mechanisms and capacity are to be understood. The effect of axial load variation on the panel zone shear capacity has been studied by Agbabian (1994) after ensuring pure shear failure mode within the joint. Increase of hoop reinforcement and axial load to an optimum level has contributed to the better performance of joints (Meinheit and Jirsa, 1981; Paulay *et al.*, 1978).

From the above discussion, it can be reckoned that various experimental results did not allow to draw universal conclusions. Code requirements for the design of joints are mainly in the form of empirical formula derived based either on the results from such experiments or on admissible equilibrium solution obtained from diagonal strut and truss mechanisms for the forces acting on joint boundaries as shown in Fig. 1a. This is why there is still no consensus on joint design recommendations among various countries. From the point of view of structural performance the joint deformation bears as much significance as the joint shear resistance. The approaches adopted in the codes do not consider the deformation of joints and it is very much necessary that both quantities be reflected in design recommendations.

It is realised that there are much more constraints and restrictions on the design of specimens to bring out a clear picture of joint mechanisms in experimental studies. However, analytical models can be developed comparatively with much less constraints that can help to interpret the responses fully. Several conceptual models have been developed based only on equilibrium requirements (Paulay, 1989). More recently, work has been done on the mathematical formulations for interior joints (Pantazopoulou and Bonacci, 1992) considering joint kinematics and material response.

JOINT MECHANICS

The observations from experimental studies support that joint can be modelled as a two dimensional panel in the direction of loading, reinforced in two orthogonal directions with column reinforcement in vertical direction and hoop reinforcement in transverse direction as shown in Fig. 1b. The element is acted upon by inplane shear stresses and normal stresses. To satisfy both equilibrium of stress resultants and compatibility of deformations within the joint the suitable softened truss model theory (Hsu, 1993) has been adopted. The algorithm establishes the shear stress-shear strain relationship of the joint after taking into account the constitutive law for softened concrete.

Joint Equilibrium

The stresses and strains are averaged over the dimensions of the entire joint. It is assumed that bond of beam and column reinforcements at the perimeter is good and the shear stress τ, is assumed to be uniformly distributed over the boundaries of the joint. The concrete stresses and steel stresses are added to get the stresses in reinforced concrete element after making an assumption that the steel reinforcement can take only axial stresses in the respective directions. The equilibrium condition of the truss model provides three basic equations. The normal stresses in l and t directions give two equations in terms of principal stresses in concrete(σ_d , σ_r) and smeared steel stresses. The third equation is given for shear stress which can be expressed as

$$\tau = (-\sigma_d + \sigma_r) \sin \alpha \cos \alpha \tag{1}$$

Joint kinematics

The compatibility condition of the truss model provides three basic equations. Since the stresses and strains within the joint are taken in an average sense, the overall geometry of the joint is described by average angle of shear distortion(γ). From the basic compatibility equations the strains in the column reinforcement and transverse hoop reinforcement ε_l, ε_t respectively.

$$\gamma = 2(-\varepsilon_d + \varepsilon_r) \sin \alpha \cos \alpha \tag{2}$$

$$\varepsilon_l = \varepsilon_r + \frac{\varepsilon_r - \varepsilon_d}{\sigma_r - \sigma_d}[\sigma_l - \sigma_r - \rho_l f_l] \tag{3}$$

$$\varepsilon_t = \varepsilon_r + \frac{\varepsilon_r - \varepsilon_d}{\sigma_r - \sigma_d}[\sigma_t - \sigma_r - \rho_t f_t] \tag{4}$$

where σ_l , σ_t are the normal stresses, ρ_l , ρ_t are the reinforcement ratios and f_l , f_t are the steel stresses in (l , t) co-ordinate system.

Constitutive Equations

The following constitutive equations give good estimate of stress-strain relationship for concrete and steel. The concrete stress in tension is negligible and is assumed to be zero.

The effective compressive stress-strain for softened diagonal concrete struts is given as follows. The ascending portion of the curve is expressed as

$$\sigma_d = \zeta f'_c \left[2\left(\frac{\varepsilon_d}{\zeta\varepsilon_o}\right) - \left(\frac{\varepsilon_d}{\zeta\varepsilon_o}\right)^2 \right] \qquad \varepsilon_d / \zeta\varepsilon_o \le 1 \qquad (5a)$$

The descending portion of the curve is expressed as

$$\sigma_d = \zeta f'_c \left[1 - \left(\frac{\varepsilon_d / \zeta\varepsilon_o - 1}{2/\zeta - 1}\right)^2 \right] \qquad \varepsilon_d / \zeta\varepsilon_o > 1 \qquad (5b)$$

where σ_d , ε_d are the stress and strain in the diagonal concrete struts, f_c is the maximum cylindrical compressive stress ε_o is strain at maximum compressive stress and ζ is the coefficient for softening effect. This coefficient is a function of tensile strain in the perpendicular direction to strut orientation which is expressed as

$$\zeta = \frac{0.9}{\sqrt{1 + 600\varepsilon_r}} \qquad (6)$$

The stress strain relationship of steel is assumed to be bilinear and is given by the following expressions

$$f_s = E_s \varepsilon_s \qquad \text{for} \qquad \varepsilon_s < \varepsilon_y \qquad (7a)$$
$$f_s = f_y \qquad \text{for} \qquad \varepsilon_s \ge \varepsilon_y \qquad (7b)$$

where f_s , ε_s are stress and strain in steel and f_y , ε_y are corresponding values at yield stress in steel. E_s is the Young's modulus of steel.

Role of Hoops before and after Yield

The closed form of hoops in combination with uniformly distributed longitudinal column reinforcements significantly enhances the shear resistance of the joints. This is true only till the yielding of hoops. Upon yielding of hoop steel, the effective confinement available for the joint concrete is considerably reduced. Equations (3) and (4) are used to monitor the strain in the steel.

After hoop yield the sensitivity of joint behavior to variations of stress states changes markedly. The shear stress at any level above that causes yield in hoop reinforcement applied in the joint will result in the failure of joint by concrete crushing. The failure can also occur due to yielding of steel in both directions before concrete crushes.

Evaluation of Shear Capacity of Joint

The shear capacity of the joint is limited by two possible modes of failure. Using this analytical procedures Agbabian (1994) test specimens are evaluated for shear capacity. The results are tabulated in Table 1 along with the reported experimental and analytical results and very good comparison is observed. The specimens designated are SA1, SA2, and SA3 which were subjected to axial column load of 5, 0, 10 per cent of the squash load respectively.

Table 1. Shear Capacity of Joint

Axial Compression	Agbabian Results		Present Study
	Experimental (kips)	Analytical (kips)	Analytical (kips)
(SA2) 0%	22.07	20.74	23.07
(SA1) 5%	24.16	21.53	24.61
(SA3) 10%	27.23	23.92	26.04

Shear Stress- Shear Strain History

An interior beam column specimen tested at the University of Tokyo designated as C1 (Otani *et al.*, 1985) has been taken for describing the shear stress-shear strain behavior up to the failure point. Fig. 2. shows the shear stress-shear strain history for the specimen. As mentioned earlier, better performance of the joint is ensured till the point of hoop yield. Hence at the threshold of yield point the design variables are evaluated using the present formulation and are compared with the reported analytical results (Bonacci and Pantazopoulou, 1993).

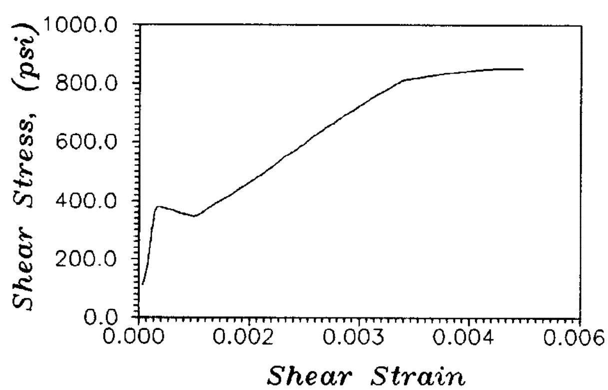

Fig. 2. Shear stress-shear strain history (specimen C1)

Table 2. Comparison of design variables at $\varepsilon_t = 0.00159$ ($0.99\ \varepsilon_y$)

Formulations	Design variables				
	ε_l	ε_r	ε_d	τ (psi)	γ
Bonacci *et al.* (1993)	0.008	0.0028	-0.00048	731.0	0.0032
Present	0.008	0.0029	-0.00058	793.8	0.0034

IMPLEMENTATION OF JOINT MODEL

The softened truss theory, as applied to the joints is found to be capable of assessing the capacity of the joint as well as shear stress-strain relationship with desired accuracy. Analytical procedures have been developed to include joint flexibility in evaluation of structural response. These procedures are implemented on IDARC (Park *et al.*, 1987) platform incorporating necessary modifications in the algorithm used.

Structural Model

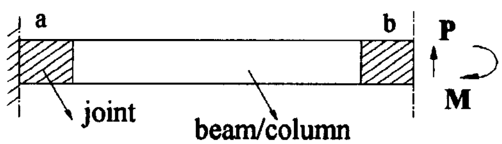

Fig. 3. Component model

An improved component model representing flexural elements with joint elements at the ends has been proposed and is shown in Fig. 3. The joint element is flexurally rigid and modelled as shear beam element acting in series with the adjoining flexural elements. The flexibility factor, 1/EI, is assumed to be linearly distributed between the two end faces of the flexural element and the point of contraflexure. The non-linear flexibility coefficients are derived from the integration of the M/EI diagram including the contribution from shear beam element. The flexural rigidity EI and shear rigidity G are monitored and updated throughout the analysis. The incremental force-deformation relationship obtained for the basic element can be expressed as follows.

$$\begin{pmatrix} \Delta\delta_b \\ \Delta\theta_b \end{pmatrix} = \begin{bmatrix} f_{11} f_{12} \\ f_{21} f_{22} \end{bmatrix} \begin{pmatrix} \Delta P_b \\ \Delta M_b \end{pmatrix} \tag{8}$$

where $\Delta\delta_b$, $\Delta\theta_b$ are the incremental deformations corresponding to the force increments ΔP_b, ΔM_b and f_{ij} are the flexibility coefficients.

Hysteresis Model

The analytical procedures based on softened truss model theory establishes the primary curve, for the joint element, defining shear stress-shear strain values at cracking, yielding and ultimate stages. This curve is used in the cyclic analysis. To account for the continually varying stiffness and energy absorbing characteristics the hysteresis rules described by origin oriented model are used which simulates heavy pinching due to crack closing and shear within the joint. The values for hysteretic parameters of flexural elements representing stiffness degradation, strength degradation and pinching effect are to be chosen as per the guidelines reported in the literature (Kunnath *et al.*, 1990).

Features of Joint Model

The joint model is designed to identify the joint shear failure under cyclic loads. In experimental studies, it is observed that the beam column joint test specimens have failed in any or combinations of the failure modes such as beam hinging, anchorage failure within the joint resulting in pull out of the beam reinforcement, and column hinging other than joint shear failure. In the present study, anchorage failure of the joint is checked and pull out of failure of beam reinforcement is identified. The bond index (BI) term (Otani, 1985), which is a nominally dimensionless measure of bond demand, is defined as the average bond stress that must develop over the column depth when beam bars yield in tension and compression at both column faces. This average stress is normalised by $\sqrt{f'_c}$ in appropriate units (the term BI actually carries units of $\sqrt{psi}$). When the bond demand exceeds the allowable value 20 (suggested form experimental observations when Grade 60 reinforcements are used) the joint has encountered with anchorage failure (Bonacci and Pantazopoulou, 1993). The effect produced on energy dissipating characteristics of the adjoining beam elements have been duly accounted for by incorporating heavy pinching moment curvature relationship.

Method of Analysis

The step by step non-linear inelastic dynamic/quasi-static analysis is used. The necessary flexibility coefficients are formulated on member by member basis. The structural stiffness matrix is assembled from the inverted flexibility matrices. The primary curves for all types of flexural elements and joints are established. Instantaneous stiffness for the elements are computed according to the hysteresis rules. The element stiffness matrix in each time/load step is updated only if there is a change of stiffness in the joint and the elements connected to that particular joint. Hence only a portion of the stiffness matrix is changed depending on the elements that change stiffness during a particular time/load step.For every time/load step the following steps are followed:

- The assembled stiffness matrix is solved for the deformations.
- Member moments/forces are calculated.
- Joint shear forces are arrived from the adjoining member forces and shear stresses are calculated.
- Instantaneous stiffness from hysteresis rules are calculated.
- The stiffness matrix is reassembled , if necessary.
- Next load step.

CASE STUDY

The capabilities of analytical model proposed are illustrated through an example of interior joint subassemblage. This particular example represents the test specimen designated as BCJ2 reported in the literature(Leon, 1990). The specimen for the given structural details has been analysed for its response applying cyclic displacements at the tip of the column up to 4 in. The joint region in the specimen has been reported to have failed in shear failure and anchorage failure within the joint. The joint model could identify these failure modes properly and reflect the loss of energy dissipating capacity, in the form of pinching, in load-deformation response (Ref. Fig. 4). The maximum load reached is 11 kips. which compares well with the reported value (10.95 kips). The corresponding joint shear strain is found to be 0.008 units. where the reported value is 0.0075 units. The cyclic response of the joint is shown in Fig. 5. in terms of shear stress and shear strain.

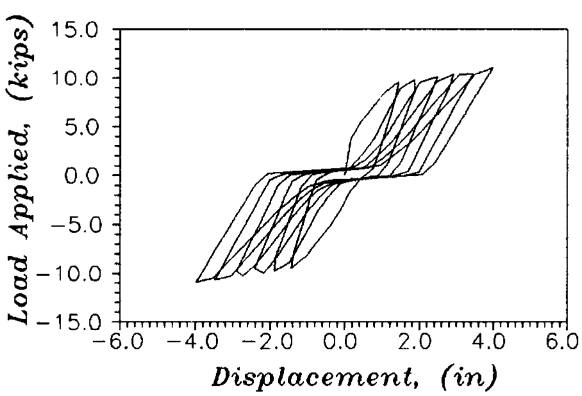

Fig. 4. Load-deflection response

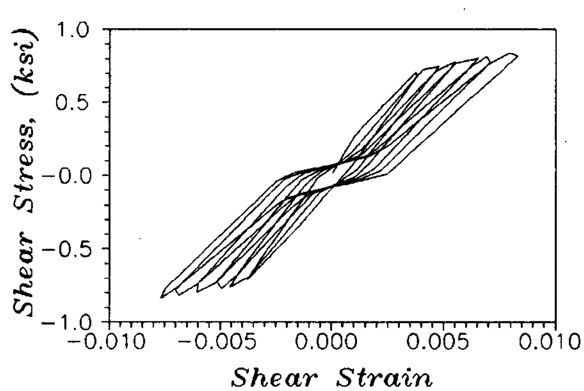

Fig. 5. Shear stress vs. shear strain of the joint

CONCLUSIONS

The present work describes the joint mechanics by considering the equilibrium and kinematics within the joint using softened truss model theory. The applicability of this theory is validated with respect to the estimation of shear capacity of joints and establishment of shear stress-strain relationship up to the failure of the joint. The analytical model , configuring the joint elements at the ends of flexural element, characterizes the effect of shear deformation and degradation of joints under cyclic loads. It is evident from the illustrative example that the analytical tool developed is capable of predicting shear failure and anchorage failure within the joint.

REFERENCES

Agbabian, M. S., E. M. Higazy and A. M. Abdel-Ghaffar (1994). Experimental observations on the seismic performance of r/c beam column connections subjected to varying axial column force. *Earthquake Engg. and Str. Dyn.*, 34, 859-876.

Bonacci, J. and S. Pantazopoulou (1993). Parametric investigation of joint mechanics. *ACI Str. J.*, 90, 61-71.

Durrani, A. J. and J. K. Wight (1985). Behavior of interior beam-to-column connections under earthquake-type loading. *ACI J.*, 82, 343-349.

Ehsani, M. R. and J. K. Wight (1985). Exterior reinforced concrete beam-to column connections subjected to earthquake-type loading. *ACI J.*, 82, 492-499.

Hsu, T. T. C. (1993). *Unified theory of reinforced concrete*. CRC Press, Ann Arbor.

Kunnath, S.K., A.M. Reinhorn and Y.J. Park (1990). Analytical modeling of inelastic seismic response of r/c structures. *J. Str. Eng. ASCE.*, 116, 996-1017.

Leon, R. T. (1990). Shear strength and hysteretic behavior of interior beam-column joints. *ACI Str. J.*, 87, 3-11.

Meinheit, D. F. and J. O. Jirsa (1981). Shear strength of r/c beam column connections. *J. Str. Eng. ASCE.*, 107, 2227-2244.

Otani, S., K. Kitayama and H. Aoyama (1985). Beam bar bond stress and behavior of reinforced concrete interior beam-column connections. *Proceedings*, U.S. -N.Z.-Japan Seminar on design of reinforced concrete beam column joints, Department of Architecture, University of Tokyo, 1-40.

Otani, S. (1991). Architectural Institute of Japan proposal of ultimate strength design requirements of r/c buildings with emphasis on beam-column joints. In: *Design of Beam Column Joints for Seismic Resistance, SP-123*, 125-144. American Concrete Institute, Detroit.

Pantazopoulou, S. and J. Bonacci (1992). Consideration of questions about beam-column joints. *ACI Str. J.*, 89, 27-36.

Park, Y. J., A.M. Reinhorn and S.K. Kunnath (1987). IDARC: Inelastic damage analysis of reinforced concrete frame-shear wall structures. Technical Report NCEER-87-0008, State University of Buffalo, New York.

Paulay, T., R. Park and M. J. N. Priestley (1978). Reinforced concrete beam column joints under seismic actions. *ACI J.*, 75, 585-593.

Paulay, T. (1989). Equilibrium criteria for reinforced concrete beam column joints. *ACI Str. J.*, 86, 635-643.