SIMULATION OF THREE DIMENSIONAL SEISMIC MOTION

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ABSTRACT

A direct method is presented for the conditional simulation of Gaussian spatially incoherent three-dimensional time histories of earthquake ground motion. The method is shown to be applicable when no restrictions are placed on the prescribed cross-correlation or cross-spectral density functions, except that they are positive definite, as well as in simpler situations resulting from restrictions sometimes used in earthquake engineering. The major computational effort in the simulation procedure is in the simultaneous solution of linear algebraic equations for required filtering functions. Particular attention is also given to the substantial reductions in computational effort which result from certain simplifying assumptions about the cross-spectral density terms. The method is shown to be almost four times as efficient as a method in the literature which involves inverting a matrix of cross-spectral densities. It is shown, though, that by taking advantage of a simplifying assumption which is sometimes adopted in the earthquake engineering, one can simulate three components of seismic motion with essentially the same effort as is required for only one component. Numerical examples demonstrate that the highly efficient direct method is effective even in situations with very high correlation between the time histories of motion at different points.

KEY WORDS

Conditional simulation; earthquake simulation; spatially incoherent seismic motion; three-dimensional random fields; scalar cross-spectral correlation; real incoherence

INTRODUCTION

Seismic ground motion involved in earthquake engineering problems can be reasonably modeled as a three-dimensional stochastic random field. Simulation of random variates and single random processes has been common for many years, but simulation of spatially incoherent random fields has received relatively less attention until quite recently (Hoshiya, 1995; Kameda and Morikawa, 1993,1994; Vanmarke *et al.*, 1991; Shinozuka and

Deodatis, 1991). The term conditional simulation is used for any technique in which one simulates new time histories which have prescribed correlation with a set of known time histories, as well as other prescribed properties. It has been noted, however, that most conditional simulation techniques (Kameda, 1993, 1994; Hoshiya, 1995) involve huge computations, even for simulation of a one dimensional random field. The emphasis here is to develop an efficient, simplified simulation algorithm. The major computation involved in the conditional simulation of a Gaussian, spatially incoherent random field using the formulation presented here is in solving a set of linear algebraic equations. The method extensively reduces the computational effort and can match any specified correlation of the random field. A numerical example is presented to illustrate the advantages of the proposed simulation technique. It is based on typical earthquake recordings and spatial incoherence information from recent analysis of array data.

BASIC CONCEPT OF CONDITIONAL SIMULATION

The problem is to simulate a three-dimensional time history $\xi^n(t) = \{\xi_x^n(t), \xi_y^n(t), \xi_z^n(t)\}^T$ for a new location n which has prescribed spectral properties with given time histories $\xi^1(t), \xi^2(t),..., \xi^{n-1}(t)$ which are also three-dimensional at the previous n-1 locations. The desired properties of the time histories can be given by stationary correlation matrices defined by

$$\mathbf{R}^{lm}(\tau) = E[\xi^l(t)\,\xi^{m\,T}(t-\tau)] \tag{1}$$

in which $\xi^l(t)$ is the stationary vector random process truncated to be non-zero only on a time interval of length T_l and the superscript T denotes a transpose. Note that the l and m superscripts denote locations, while subscripts will be used to designate the components of the random field at a particular location. In the conditional simulation of three-dimensional random fields, we will choose to simulate the components at location n in the order $\xi_x^n(t)$, then $\xi_y^n(t)$, then $\xi_z^n(t)$. Each component will be simulated by adding its conditional mean to a random deviate which is independent of all prior components. The situation is simplified by the fact that the conditional mean of a Gaussian random process is always a linear function of the given information. Let $g_r^n(t)$ and $\psi_r^n(t)$ represent the conditional mean and the random deviate corresponding to the component $\xi_r^n(t)$ (r= x, y, z) of the seismic random motion such that

$$\xi_r^n(t) = g_r^n(t) + \psi_r^n(t) \qquad \text{for } r = x, y, z \tag{2}$$

This relation can be converted into the frequency domain by applying the Fourier transform of eq. (2) as

$$\Xi_r^n(\omega) = G_r^n(\omega) + \Psi_r^n(\omega) \qquad \text{for } r = x, y, z \tag{3}$$

Although the mean zero, Gaussian random deviates $\psi_r^n(t)$ or $\Psi_r^n(\omega)$ are independent of the prior time histories they must be described by the required conditional spectral density functions given in the following sections. Thus, to simulate a three-dimensional random field, one must determine conditional mean values and spectral density functions.

DIRECT EVALUATION OF CONDITIONAL STATISTICS

Conditional Mean The conditional mean of the $\xi^n(t)$ vector will be written as $\mathbf{g}^n(t)$, defined as

$$\mathbf{g}^n(t) = \begin{pmatrix} g_x^n(t) \\ g_y^n(t) \\ g_z^n(t) \end{pmatrix} = \begin{pmatrix} E[\xi_x^n(t)\,|\,\{\xi^1(t)\},\{\xi^2(t)\},...,\{\xi^{n-1}(t)\}] \\ E[\xi_y^n(t)\,|\,\{\xi^1(t)\},\{\xi^2(t)\},...,\{\xi^{n-1}(t)\},\{\xi_x^n(t)\}] \\ E[\xi_z^n(t)\,|\,\{\xi^1(t)\},\{\xi^2(t)\},...,\{\xi^{n-1}(t)\},\{\xi_x^n(t)\},\{\xi_y^n(t)\}] \end{pmatrix} \tag{4}$$

The most general form for the linear dependence of $\mathbf{g}^n(t)$ on the given information can be written as a sum of convolution integrals:

$$\mathbf{g}^n(t) = \frac{1}{2\pi}\sum_{l=1}^{n}\int_{-\infty}^{\infty}\mathbf{w}^{nl}(\alpha)\,\xi^l(t-\alpha)\,d\alpha \tag{5}$$

with the weighting matrices $\mathbf{w}^{nl}(\alpha)$ being full 3×3 matrices for $l<n$, and $\mathbf{w}^{nn}(\alpha)$ being zero expect for all terms below the major diagonal. The problem of finding the conditional mean of $\xi^n(t)$ is now to find all these unknown $w_{jk}^{nl}(\alpha)$ $(j,k=x,y,z)$ weighting functions. A general property of conditional expectation is used to derive equations describing these weighting functions,. This can be stated as

$$E[Yh(\mathbf{X})] = E[E[Yh(\mathbf{X})|\mathbf{X}]] = E[E[Y|\mathbf{X}]h(\mathbf{X})] \tag{6}$$

for any random variate Y and any function h of the random vector $\mathbf{X}$. This relation can be utilized in evaluating terms of the form $R_{jk}^{nm}(\tau)=E[\xi_j^n(t)\xi_k^m(t-\tau)]$. This involves replacing Y by $\xi_j^n(t)$, $h(\mathbf{X})$ by $\xi_k^m(t-\tau)$, and $\mathbf{X}$ by the appropriate given information for $\xi_j^n(t)$, as given in eq.(6). The given information for the $\xi_r^n(t)$ component always includes $\xi_k^m(t)$ for $k=x,y,z$ and m=1 to n-1. For $r=y$ it also includes $\xi_x^n(t)$, and for $r=z$ it includes both $\xi_x^n(t)$ and $\xi_y^n(t)$. We can now use eq.(6) to simplify the necessary expressions for correlation by writing

$$\begin{aligned} R_{rk}^{nm}(\tau) &= E[\xi_r^n(t)\,\xi_k^m(t-\tau)] = E[g_r^n(t)\,\xi_k^m(t-\tau)] \\ &= \frac{1}{2\pi}\sum_{l=1}^{n}\sum_{j=x}^{z}\int_{-\infty}^{\infty}w_{rj}^{nl}(\alpha)\,R_{jk}^{lm}(\tau-\alpha)\,d\alpha \end{aligned} \tag{7}$$

In principle, these simultaneous integral equations can be solved to determine the $w_{rj}^{nl}(\alpha)$ weighting functions, but in practice it is more convenient to use the Fourier transform to obtain algebraic equations:

$$S_{rk}^{nm}(\omega) = \sum_{l=1}^{n}\sum_{j=x}^{z}W_{rj}^{nl}(\omega)\,S_{jk}^{lm}(\omega) \tag{8}$$

The filtering functions $W_{rk}^{nl}(\omega)$ (r , k = x, y, z ; l =1, 2, ..., n) are simply the Fourier transforms of the corresponding weighting functions $w_{rk}^{nl}(t)$. The $S_{rk}^{nm}(\omega)$ and $S_{rk}^{nl}(\omega)$ terms are the desired spectral density functions of the motion. The fact that the $\mathbf{w}^{nl}(t)$ and $\mathbf{W}^{nl}(\omega)$ matrices have non-zero terms only below the diagonal assures that only the prior components are included as conditioning information in eqs.(7) and (8). Note also that eq.(8) shows that the $W_{rk}^{nl}(\omega)$ functions are uncoupled from the $W_{jk}^{nl}(\omega)$ $(j\neq r)$ functions. In principle, one can find the conditional mean $\mathbf{g}^n(t)$ by solving eq.(8) for the required filtering functions. After these required filtering functions have been computed, it is convenient to find the conditional mean $\mathbf{g}^n(t)$ by using the Fourier transform of eq.(5), giving

$$\mathbf{g}^n(t) = \sum_{l=1}^{n}\int_{-\infty}^{\infty}\mathbf{W}^{nl}(\omega)\,\Xi^l(\omega)\,e^{i\omega t}d\omega \tag{9}$$

in which $\mathbf{W}^{nl}(\omega)$ is the 3×3 filtering function matrix with complex elements $W_{rk}^{nl}(\omega)$.

Conditional Spectral Density Eq. (3) indicates that, the spectral density corresponding to the random deviate $\psi_r^n(t) = \xi_r^n(t) - g_r^n(t)$ $(r=x, y, z)$ [or equivalently, in the frequency domain as $\Psi_r^n(\omega) = \Xi_r^n(\omega) - G_r^n(\omega)$] can be directly obtained by

$$\begin{aligned} S_r^n(\omega) &= \frac{2\pi}{T}E[\Psi_r^n(\omega)\Psi_r^{n*}(\omega)] = \frac{2\pi}{T}E[\{\Xi_r^n(\omega)-G_r^n(\omega)\}\Psi_r^{n*}(\omega)] \\ &= \frac{2\pi}{T}[S_{rr}^{nn}(\omega)-\sum_{l=1}^{n}\sum_{k=1}^{s}W_{rk}^{nl}(\omega)\,S_{kr}^{ln}(\omega)] \end{aligned} \tag{10}$$

This spectral density, along with the conditional mean $G_r^n(\omega)$, gives sufficient information to allow one to

simulate the time history $\xi_r^n(t)$ for $r = x, y, z$ according to eqs.(2) or (3). In particular, to simulate the random deviates the complex random process $\Psi_r^n(\omega)$ can be chosen as

$$\Psi_r^n(\omega) = B_r e^{i\theta_r} \qquad \text{for } r = x, y, z \tag{11}$$

with B_r and θ_r being independent real random variates, with B_r having $E[B_r^2]=E[\Psi_r^n\Psi_r^{n*}]$ and being Rayleigh distributed and θ_r being uniformly distributed on $[0, 2\pi]$. This results in the real and imaginary components of $\Psi_r^n(\omega)$ having independent Gaussian distributions with mean zero and the appropriate value for the spectral density. Based on the definition given by eq.(3), the relationship between B_r and $\Psi_r^n(\omega)$ is

$$E[B_r^2] = \frac{T}{2\pi} S_r^n(\omega) \qquad \text{for } r = x, y, z \tag{12}$$

It is obvious that the term $E[B_r^2]$ can also be considered to be the conditional variance of the frequency domain random process $\Xi_r^n(\omega)$. After the random process $\Psi_r^n(\omega)$ has been obtained one can determine the Fourier transform of the motion according to eq.(3) and the time histories of the random field can be evaluated by performing the inverse Fourier transformation.

FINDING FILTERING FUNCTIONS USING UPDATING ALGORITHM

Prior to simulating any particular component of motion it is necessary to evaluate the pertinent filtering functions by solving a set of simultaneous algebraic equations with a Hermitian coefficient matrix. The dimension of the matrix is equal to the number of prior components. For the simulation of a three-dimensional random field at a total of N locations this involves the sequential solution of matrix equations of dimension j for j=1 to $3N$-1. The computational effort can be quantified by counting the number of floating point operations (flops) involved (Golub and Van Loan, 1989). The number of flops to solve a matrix equation of dimension j is proportional to j^3, so that if no advanced algorithm is used the sequential solution of the matrix equations associated with all the $3N$ components is proportional to $(3N)^4$. This would require a huge amount of computational effort when N is large. Fortunately, the form of the matrix and vectors involved is such that it is possible to use more efficient procedures in which the number of flops is only proportional to the cubic power of the number of components, as has previously been done for one-dimensional random fields (Kameda and Morikawa, 1993,1994; Lutes *et al.*, 1996). The method used here is an efficient algorithm in which the Cholesky decomposition for any particular j values is found by updating the decomposition used at the previous stage.

To simplify the presentation, let eq.(8) be rewritten as

$$\mathbf{A}_{[j]}(\omega)\,\mathbf{X}_{[j]}(\omega) = \mathbf{a}_{[j]}(\omega) \tag{13}$$

in which all the elements are complex, the vector $\mathbf{X}_{[j]}(\omega)$ consists of the unknown filtering functions, and the square matrix $\mathbf{A}_{[j]}(\omega)$ and the vector $\mathbf{a}_{[j]}(\omega)$ consist of specified spectral density components. The subscripts give the dimension of each array. Efficient solution of the sequential equations is possible because of the following two facts: the coefficient matrix $\mathbf{A}_{[j]}(\omega)$ is exactly the matrix $\mathbf{A}_{[j-1]}(\omega)$ from the previous stage with one column and one row added, and the first $(j-1)$ elements of the added column are exactly the vector $\mathbf{a}_{[j-1]}(\omega)$ from the previous stage. Thus, the new elements introduced at stage j are the vector $\mathbf{a}_{[j]}(\omega)$ and the (j,j) element of $\mathbf{A}_{[j]}(\omega)$. Since the matrix $\mathbf{A}_{[j]}(\omega)$ is Hermitian in nature it can be decomposed by using the modified Cholesky decomposition procedure as $\mathbf{A}_{[j]}(\omega) = \mathbf{L}_{[j]}(\omega)\,\mathbf{L}_{[j]}^{T}(\omega)$ in which $\mathbf{L}_{[j]}(\omega)$ is a lower triangular matrix. Solution of eq.(13) then involves solving two triangular equations: $\mathbf{L}_{[j]}(\omega)\,\mathbf{Y}_{[j]}(\omega) = \mathbf{a}_{[j]}(\omega)$

and $\mathbf{L}_{[j]}^{*T}(\omega)\mathbf{X}_{[j]}(\omega)=\mathbf{Y}_{[j]}(\omega)$. Furthermore, the special nature of the problem allows one to write

$$\mathbf{L}_{[j]}(\omega) = \begin{pmatrix} \mathbf{L}_{[j-1]}(\omega) & \mathbf{0} \\ \mathbf{Y}_{[j-1]}^{*T}(\omega) & l_{[j]}(\omega) \end{pmatrix} \tag{14}$$

in which the two left hand submatrices come directly from the Cholesky solution from the previous stage. The sclalar element in the lower right corner is found by noting that the (j,j) element of $\mathbf{A}_{[j]}(\omega)$ must be given by $\mathbf{Y}_{[j-1]}^{*T}(\omega)\mathbf{Y}_{[j-1]}(\omega)+l_j(\omega)l_j^*(\omega)$. Thus, one can write

$$l_j(\omega) = \sqrt{A_{j,j}(\omega) - \mathbf{Y}_{[j-1]}^{*T}(\omega)\mathbf{Y}_{[j-1]}(\omega)}\, e^{i\theta} \tag{15}$$

in which θ is an arbitrary phase angle.

The major computation involved for solving a real linear equation with a $j\times j$ coefficient matrix using Cholesky decomposition is about $2(j-1)^2$ flops. This gives a total about $18N^3$ flops to perform the required computations for finding $\mathbf{X}_j(\omega)$ for $j=2,...,3N-1$. Since the effort to solve complex simultaneous equations is about 4 times that required for solving a similar array of real equations (Lutes *et al.*, 1996), the major computation involved in evaluating all required filtering functions for simulation of a general three-dimensional random field at a total of N locations is approximately $72N^3$ flops. A summary of this efficient updating algorithm is given in Table1.

Table 1 Updating algorithm for solving filtering functions

Given $\mathbf{A}_{[2]}(\omega)$, perform the Cholesky decomposition: $\mathbf{A}_{[2]}(\omega)=\mathbf{L}_{[2]}(\omega)\mathbf{L}_{[2]}(\omega)^{*T}$

Find $\mathbf{Y}_{[2]}(\omega)$ and $\mathbf{X}_{[2]}(\omega)$ by solving $\mathbf{L}_{[2]}(\omega)\mathbf{Y}_{[2]}(\omega)=\mathbf{a}_{[2]}(\omega)$, $\mathbf{L}_{[2]}^{*T}(\omega)\mathbf{X}_{[2]}(\omega)=\mathbf{Y}_{[2]}(\omega)$

For $j=3$ to $3N$-1

Form $\mathbf{L}_{[j]}(\omega)$ [eq.(14)] by using $\mathbf{L}_{[j-1]}(\omega)$, $\mathbf{Y}_{[j-1]}(\omega)$ and $l_{[j]}(\omega)$

with $l_j(\omega) = \sqrt{A_{j,j}(\omega) - \mathbf{Y}_{[j-1]}^{*T}(\omega)\mathbf{Y}_{[j-1]}(\omega)}\, e^{i\theta}$

Find $\mathbf{Y}_{[j]}(\omega)$ and $\mathbf{X}_{[j]}(\omega)$ by solving $\mathbf{L}_{[j]}(\omega)\mathbf{Y}_{[j]}(\omega)=\mathbf{a}_{[j]}(\omega)$, $\mathbf{L}_{[j]}(\omega)^{*T}\mathbf{X}_{[j]}(\omega)=\mathbf{Y}_{[j]}(\omega)$

end of j loop

Output $W_{rk}^{nl}(\omega)$ $(r,k=x,y,z;\ l=1,2,...,N)$ according to $\mathbf{X}_{[j]}(\omega)$ $(j=2,...,3N-1)$

SIMPLIFICATIONS FOR EARTHQUAKE MODEL

Simplication resulting from a scalar cross-spectral relationship The results derived so far have no restrictions placed on the form of the $\mathbf{R}^{lm}(\tau)$ correlation matrices or, equivalently, the $\mathbf{S}^{lm}(\omega)$ spectral density matrices to be matched in the simulation. However, significant simplification of the computation can be achieved by considering some special features of models used to approximate seismic motions in earthquake engineering research. First, let us consider a commonly used seismic model (Haricahandran and Vanmarcke, 1984) in which the cross-spectral density relationship between the seismic motions is essentially scalar in nature:

$$\mathbf{S}^{lm}(\omega) = q^{lm}(\omega)\mathbf{S}^0(\omega) \tag{16}$$

in which $q^{lm}(\omega)$ for locations l and m is a scalar function (real or complex) of frequency ω which satisfies $q^{lm}(\omega)=1$ for $l=m$, and $\mathbf{S}^0(\omega)$ is a given target spectral density matrix with elements $S_{jk}^0(\omega)$ $(j,k=x,y,z)$. It can be shown (Jin *et al.*, 1995) that substituting eq.(16) into eq.(8) gives $W_{rr}^{nl}(\omega)$ as being the same for $r=x$,

y, z. This is true for l=1,...n-1, and these filtering functions are the solutions of

$$q^{nm}(\omega) - \sum_{l=1}^{n-1} q^{lm}(\omega) W_{rr}^{nl}(\omega) \qquad \text{for } m=1,\ldots,n-1; \tag{17}$$

The other filtering functions for $l<n$ are given by

$$W_{xy}^{nl}(\omega)=W_{xx}^{nl}(\omega)=W_{yx}^{nl}(\omega)=0$$

$$W_{xx}^{nl}(\omega) = - W_{xx}^{nm}(\omega) W_{rr}^{nl}(\omega), \quad W_{xy}^{nl}(\omega) = - W_{xy}^{nm}(\omega) W_{rr}^{nl}(\omega), \quad W_{yx}^{nl}(\omega) = - W_{yx}^{nm}(\omega) W_{rr}^{nl}(\omega) \tag{18}$$

and for $l=n$ the results are

$$W_{yx}^{nn}(\omega)=S_{yx}^{0}(\omega)/S_{xx}^{0}(\omega)$$

$$W_{xx}^{nn}(\omega)=[S_{xx}^{0}(\omega)S_{yy}^{0}(\omega)-S_{xy}^{0}(\omega)S_{yx}^{0}(\omega)]/[S_{xx}^{0}(\omega)S_{yy}^{0}(\omega)-S_{xy}^{0}(\omega)S_{yx}^{0}(\omega)]$$

$$W_{xy}^{nn}(\omega)=[S_{xx}^{0}(\omega)S_{xy}^{0}(\omega)-S_{xx}^{0}(\omega)S_{yx}^{0}(\omega)]/[S_{xx}^{0}(\omega)S_{yy}^{0}(\omega)-S_{xy}^{0}(\omega)S_{yx}^{0}(\omega)] \tag{19}$$

In this formulation the only simultaneous equations to be solved are the set of dimension n-1 in eq.(17).This reduces the computational effort by approximately a factor of 27 as compared to solving eq.(8), since it is of dimension 3(n-1).

Simplication resulting from a real incoherence structure The computational task can be further reduced when one adopts another simplification commonly used in earthquake engineering (Haricahandran and Vanmarcke, 1984; Luco and Wong 1986; Vanmarke *et al.*, 1993) in which the incoherence of the seismic random components can be described by a real function. In particular, let the scalar cross-spectral density term of eq.(16) be a product of two terms, one of which is real and represents the amplitude of $q^{lm}(\omega)$ and, the other of which is complex with unit absolute value, representing only a phase difference between the frequency ω components of seismic motion at l and m:

$$q^{lm}(\omega) - b^{lm}(\omega)\phi^{lm}(\omega) \tag{20}$$

with $b^{lm}(\omega)$ being real and $|\phi^{lm}(\omega)|=1$. The term $b^{lm}(\omega)=|q^{lm}(\omega)|$ gives the random incoherence between the motion at points l and m, while the term $\phi^{lm}(\omega)$ gives the phase change from one location to the other. If one now imposes the condition that $\phi^{nl}(\omega)\phi^{lr}(\omega)=\phi^{nr}(\omega)$, as when $\phi^{nl}(\omega)$ represents coherent wave passage across the site, the result can be written as

$$\sum_{l=1}^{n-1} b^{lm}(\omega) Z^{nl}(\omega) = b^{nm}(\omega) \qquad \text{for } m = 1, \ldots, n-1 \tag{21}$$

in which the unknowns are the real quantities $Z^{nl}(\omega)=\phi^{ln}(\omega)W_{rr}^{nl}(\omega)$ for l=1,...,n-1. Thus, the problem of finding all the required filtering functions involved in the simulation has now been reduced to that of solving n-1 real scalar simultaneous linear equations, plus simple multiplication operations involving complex numbers. The resulting reduction in computation due to the use of the real incoherence structure is approximately a factor of 4. This is in addition to the factor of 27 resulting from the use of eq.(17).

As an alternative to the direct method presented here, it should be noted that one can perform the conditional simulation by using Gaussian conditional probability density functions. This method was implemented for one-dimensional processes by Kameda and Morikawa (1994), with the major computation being the inversion of a covariance matrix. It has been shown (Lutes *et al.* 1996) that their algorithm requires almost four times as much computation as in the direct method presented here, but that the efficiency of the probability density method can be improved by using a matrix inversion approach suggested by Hoshiya (1995) or by an approach in which only the needed elements of the matrix inverse are computed. In this latter form, the probability density method has been shown to be as efficient as the direct method for the one-dimensional problem, and it also could be efficiently adapted to three-dimensional problems.

NUMERICAL EXAMPLE

As a numerical example we simulate a three-dimensional seismic random field based on the recordings of a typical earthquake (El Centro 1940) and the available information about the spectral incoherence relationship between seismic motions at different surface locations (Haricahandran and Vanmarcke, 1984). The seismic spectral incoherence structure of the earthquake ground motion has been defined by the most popularly used scalar form with real incoherence as suggested by Haricahandran and Vanmarcke (1984):

$$\phi^{lm}(\omega) = \exp\left(i\omega (\mathbf{r}_l - \mathbf{r}_m)\cdot \mathbf{L}/c_s \right) \tag{22}$$

$$b^{lm}(\omega) = \exp\left(-[\gamma\omega |\mathbf{r}_l - \mathbf{r}_m|/v_s]^{\mu}\right) \tag{23}$$

where $\mathbf{L}$ is a horizontal unit vector oriented in the direction of apparent wave propagation, $\mathbf{r}_m$ or $\mathbf{r}_l$ is the position vector at the surface of the soil medium and $c_s = v_s/\sin\alpha$ is the apparent velocity of horizontal seismic wave propagation due to seismic wave propagation, with v_s being the wave propagation speed of the soil medium and α the angle of the wave propagation direction relative to the vertical axis as shown in the Fig.1, The incoherence parameter γ has been taken to range within 0.0~0.5 and μ has been given a value of 1.9 (Lutes *et al.*, 1996). The time histories of the El Centro earthquake recording include three components of accelerations have been utilized to evaluate the spectral density functions and to provide the target, local stochastic information for the random field to be simulated. Using the present simulation method, the three components of the displacement time histories of the seismic random field have been generated using v_s=100m/s, α=45°,γ=0.3 and β=0° at a total N=100 surface locations. Fig.2 shows a portion of the plots of the components of the displacement time histories at 10 equally spaced surface locations along the x axis.

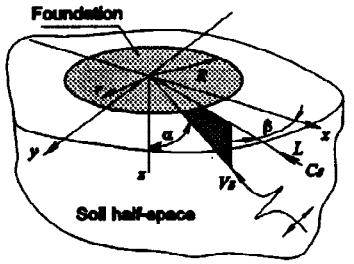

Fig. 1. Definition of the parameters involved in description of seismic ground motion

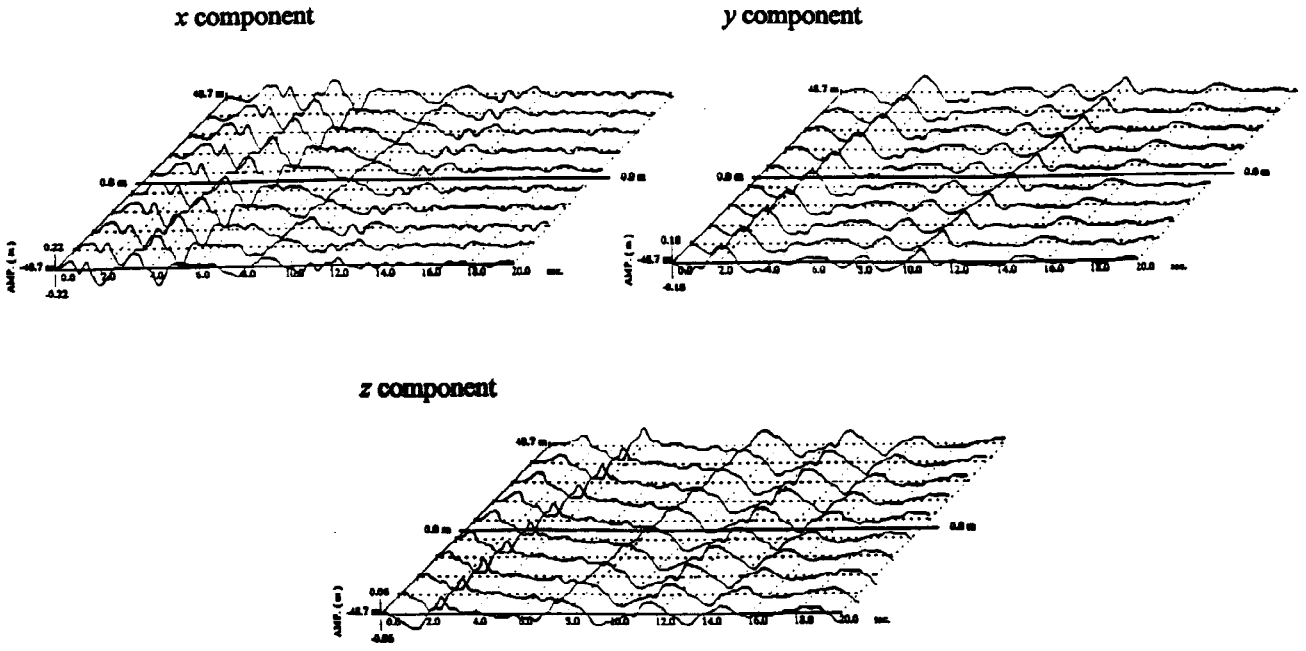

Fig. 2. Simulated displacement time histories of the seismic ground motion at 10 linearly arranged, equally spaced surface locations (v_s=100m/s, $\alpha=\pi/4$,β=0, γ=0.3)

SUMMARY AND CONCLUSIONS

An efficient method for simulation of multi-dimensional, spatially incoherent random fields has been presented using filtering functions derived from the conditional statistics of the seismic random field. The simulation of a three-dimensional random field can be performed as a simple procedure of evaluating the required filtering functions and using these functions to obtain the desired time histories. It has been found that for a popularly used seismic excitation model with a scalar cross-spectral correlation and a real spectral incoherence function, the solution of the required filtering functions using the presented method is particularly simple and computationally efficient.

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