

## STUDY ON NONLINEAR SEISMIC RESPONSES OF PIPING SYSTEM WITH FRICTION

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### ABSTRACT

This report deals with the experimental and analytical study of seismic response behavior of piping systems in industrial facilities. Piping is generally set up on the supporting structures, therefore when seismic motion is given, frictional vibration should take place between the piping and supports. In this paper, the investigation is focused on the nonlinear dynamic responses of piping systems due to this frictional vibration. A three-dimensional mock-up piping and supporting structure model is excited by large scale shaking table. FEM model is made from the mock-up piping, and modal parameters of linearized model are calculated. The maximum responses and the quantity of dissipated energy due to friction are calculated for the modal model. And these values are compared with the results of vibration test.

### KEYWORDS

Piping system; Pipe support; Frictional response; Shaking test; Modal analysis; Seismic design.

### INTRODUCTION

The dynamic characteristics of various kinds of piping and supporting structures in industrial facilities have to be recognized as one of the significant issues from economic and safety design consideration. Piping systems are generally connected to supporting structures at several points. When attention is directed to the dynamic interaction between the piping and its supporting structure, they frequently behave nonlinear frictional vibration due to contact motion existing among piping and supporting structures. Such a kind of nonlinear behaviors could be expected to suppress the seismic responses under seismic condition. Therefore, several reports were published to observe these nonlinear dynamic effects caused by the frictional vibration (Kobayashi et al., 1987, Sone et al., 1990, Suzuki et al., 1992). This report addresses the response of piping and supporting system in which a single frictional support is installed to suppress both of first and second modal responses. Nonlinear response analysis is carried out by the modal model using modal

parameters such as natural frequencies and modal vectors calculated by eigenvalue analysis. The calculated maximum responses and dissipated energy caused by the frictional motion are compared with those obtained by the experiment.

## VIBRATION TEST

### Equipments for Test

An overview of the mock-up equipment for the test and the size of piping specimen is illustrated in Fig.1. The piping model made of carbon steel has a total length of 24.75 m, a diameter of 165.2 mm, thickness of 5 mm and total weight of 527 Kg. In order to simultaneously realize both of the first mode and second mode, axial direction of piping is located on the table at 45 degree with respect to excitation direction. A friction support is set up on the piping marking by F in (a) of Fig.1. Two dimensional frictional motion is expected to be realized on this support. One is X directional motion due to first mode and another is Z directional one due to second mode. Friction support is composed of a shoe and a plate, as shown in (b) of Fig.1. This shoe is made of carbon steel and has a diameter of 45 mm. The material of the plate is a fluorine rosin known as Teflon generally used for the actual support allowing the relief of piping thermal expansion. The vertical force acting on the friction device and friction forces for X and Z directions are measured by three direction load cell.

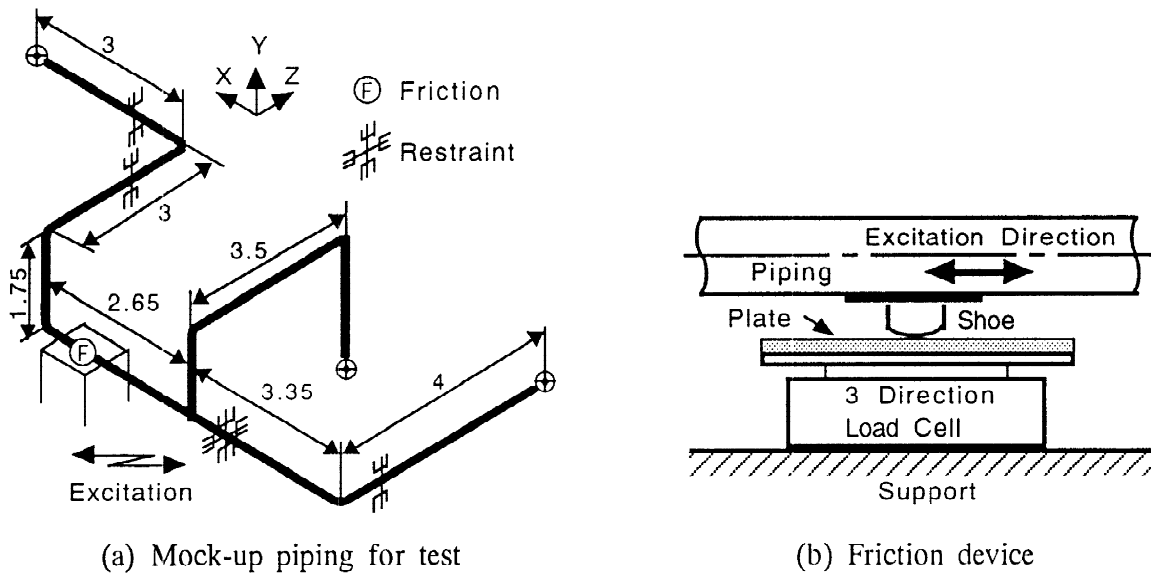


Fig. 1. Mock-up piping specimen and friction device

### Testing Procedure

The large-scale shaking table (15.0 m × 15.0 m) at the Japanese National Research Institute for Earth Science and Disaster Prevention is utilized as a testing facility. This table can produce a maximum displacement of 220mm. Two types of waves are taken as input motion for the test. One is sinusoidal wave

having the frequency just agreed with the natural frequency of the piping, and another is a so-called “combined” sinusoidal wave composed of two sinusoidal waves having the first modal and the second modal frequencies. This wave is superposed so that the first and the second modal responses at the friction part can be observed approximately equally. Particular attention must be paid to the identification of dynamic behavior between the piping and the supporting structure due to the combination between the first and the second modal vibration. Sensors are installed along with the axial direction of piping.

## TIME HISTORY RESPONSE ANALYSIS

### Analytical Approach

In the analytical study, the first and the second mode of motions are particularly taken into consideration mainly from the seismic design aspect. The displacement response of X and Z directions have to be especially noticed. Basic equation of motion of FEM model is shown as follows,

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{e\}\ddot{u}_g + \{f\}, \quad u = U - u_g \quad (1)$$

where,  $[M]$ ,  $[C]$ ,  $[K]$ ,  $\{u\}$ ,  $\{f\}$ ,  $\{q\}$  and  $\{e\}$  show mass matrix, damping matrix, stiffness matrix, actual displacement related to shaking table, friction force vector, modal displacement and excitation direction vector, respectively.

In order to simulate the same condition of the table motion,  $\{e\}$  has to be described as follows,

$$\{e\} = \left\{ -\cos(45^\circ), 0, \cos(45^\circ), 0, 0, 0 / \dots \right\}^T = \left\{ -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0, 0, 0 / \dots \right\}^T \quad (2)$$

As friction force vector  $\{f\}$  should be zero at the position where frictional force does not generate,

$$\{f\} = \left\{ 0, \dots, F_{fx}(t), 0, F_{fz}(t), 0, \dots \right\}^T \quad (3)$$

Sliding direction varies depending on time “t” and frictional force is directed as same as sliding velocity of X and Z. When the  $\theta$  is described as the angle of X direction with respect to sliding direction as in Fig. 2, the equation for the friction force can be written as follow.

$$F_{fx}(t) = -\mu N_f \frac{\dot{u}_{fx}(t)}{\sqrt{\dot{u}_{fx}(t)^2 + \dot{u}_{fz}(t)^2}} = -\mu N_f \text{sign}(\dot{u}_{fx}) \cos(\theta(t)) \quad (4)$$

$$F_{fz}(t) = -\mu N_f \frac{\dot{u}_{fz}(t)}{\sqrt{\dot{u}_{fx}(t)^2 + \dot{u}_{fz}(t)^2}} = -\mu N_f \text{sign}(\dot{u}_{fz}) \sin(\theta(t)) \quad (5)$$

$$\text{sign}(\dot{u}) = \begin{cases} 1 & : \dot{u} > 0 \\ 0 & : \dot{u} = 0 \\ -1 & : \dot{u} < 0 \end{cases} \quad (6)$$

Subscript "f" is friction point and subscripts "x" and "z" are x direction and z direction respectively.  $\mu$  and N indicate coefficient of friction and vertical force.

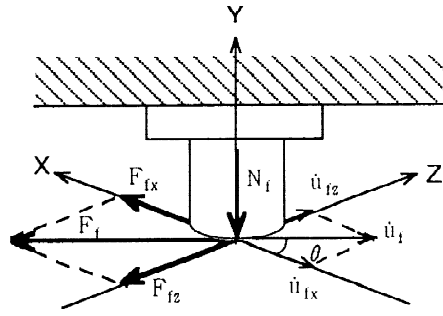


Fig.2. Sliding direction of velocity and friction force

By using following relation,

$$\{u\} = \sum_{i=1}^n \{\phi\}_i q_i = [\phi]\{q\} \quad (7)$$

where transpose of the modal matrix  $[\phi]^T$  is multiplied from left side of eq.(1), Equation (1) can be transformed into

$$[\phi]^T[M][\phi]\{\ddot{u}\} + [\phi]^T[C][\phi]\{\dot{q}\} + [\phi]^T[K][\phi]\{q\} = -[\phi]^T[M]\{e\}\ddot{u}_g + [\phi]^T\{f\} \quad (8)$$

where,

$$[M^*] = [\phi]^T[M][\phi], \quad [C^*] = [\phi]^T[C][\phi], \quad [K^*] = [\phi]^T[K][\phi] \quad \text{and} \quad [m^*] = [\phi]^T[M]\{e\} \quad (9)$$

Since friction force could excite both of the first and the second modal motions,

$$\{f^*\} = [\phi]^T\{f\} = -\mu N_f \begin{pmatrix} \phi_{fx1} \text{sign}(\dot{u}_{fx}) \cos(\theta(t)) + \phi_{fz1} \text{sign}(\dot{u}_{fz}) \sin(\theta(t)) \\ \phi_{fx2} \text{sign}(\dot{u}_{fx}) \cos(\theta(t)) + \phi_{fz2} \text{sign}(\dot{u}_{fz}) \sin(\theta(t)) \\ 0 \\ \vdots \end{pmatrix} \quad (10)$$

When  $\phi_{fx1} \gg \phi_{fz1}$  and  $\phi_{fx2} \gg \phi_{fz2}$  are assumed for the modal shape, friction vector  $\{f\}$  is modified as

$$\{f^*\} = -\mu N_f \begin{pmatrix} \phi_{fx1} \text{sign}(\dot{u}_{fx}) \cos(\theta(t)) \\ \phi_{fz2} \text{sign}(\dot{u}_{fz}) \sin(\theta(t)) \\ 0 \\ \vdots \end{pmatrix} \quad (11)$$

As the first and second modal responses have to be calculated, the velocity in the sign function is given as follows.

$$\dot{u}_{fx} = \phi_{fx1}\dot{q}_1 + \phi_{fx2}\dot{q}_2, \quad \dot{u}_{fz} = \phi_{fz1}\dot{q}_1 + \phi_{fz2}\dot{q}_2 \quad (12)$$

When  $\phi_{fx1} \gg \phi_{fx2}$  and  $\phi_{fz2} \gg \phi_{fz1}$  are assumed, eq. (8) can be transformed into

$$[M^*]\{\ddot{q}\} + [C^*]\{\dot{q}\} + [K^*]\{q\} = -\{m^*\}\ddot{u}_g - \mu N_f \begin{pmatrix} \phi_{fx1} \text{sign}(\phi_{fx1}\dot{q}_1) \cos(\theta(t)) \\ \phi_{fz2} \text{sign}(\phi_{fz2}\dot{q}_2) \sin(\theta(t)) \\ 0 \\ \vdots \end{pmatrix} \quad (13)$$

where,

$$\phi_{fx1} \text{sign}(\phi_{fx1}\dot{q}_1) = |\phi_{fx1}| \text{sign}(\dot{q}_1), \quad \phi_{fz2} \text{sign}(\phi_{fz2}\dot{q}_2) = |\phi_{fz2}| \text{sign}(\dot{q}_2) \quad (14)$$

$$\frac{K_i^*}{M_i^*} = \omega_i^2, \quad \frac{C_i^*}{M_i^*} = 2\zeta_i\omega_i, \quad \frac{m_i^*}{M_i^*} = \beta_i, \quad \frac{\mu N_f |\phi_{fx1}|}{M_1^*} = F_1^*, \quad \frac{\mu N_f |\phi_{fz2}|}{M_2^*} = F_2^* \quad (15)$$

Finally, eq. (13) can be described as follows,

$$\ddot{q}_1 + 2\zeta_1\omega_1\dot{q}_1 + \omega_1^2q_1 = -\beta_1\ddot{u}_g - F_1^* \text{sign}(\dot{q}_1) \cos(\theta(t)) \quad (16)$$

$$\ddot{q}_2 + 2\zeta_2\omega_2\dot{q}_2 + \omega_2^2q_2 = -\beta_2\ddot{u}_g - F_2^* \text{sign}(\dot{q}_2) \sin(\theta(t)) \quad (17)$$

### Sliding angle

Sliding angle  $\theta(t)$  in the equations(16) and (17) are the function of time. Either equation (16) or (17) can not be separately calculated because angle  $\theta(t)$  should be calculated by using  $\dot{q}_1$  and  $\dot{q}_2$ . Figure 3 shows the histograms of velocity angle and friction force angle under the combined sinusoidal excitation. These histograms are obtained by counting the number of angles on time history angle data calculated from X and Z directional responses. The dominant peak appears around  $60^\circ$  for both of velocity and friction force and similar characteristics are observed for both of linear (non frictional) and nonlinear frictional cases. Therefore, sliding angle  $\theta(t)$  can be determined as constant value of  $60^\circ$ .

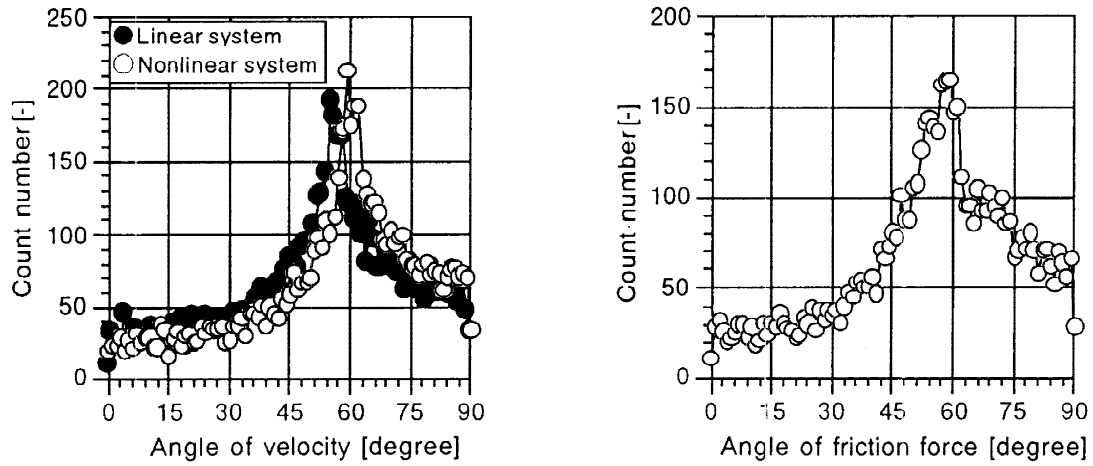


Fig.3. Histograms of velocity angle and friction force angle for combined sinusoidal input

Parameters for analysis

Time history response analysis is performed using parameters shown in Table 1. These values were obtained from the experiment and the eigenvalue analysis.

Table 1 Parameters for analysis

| Order                                 | First  | Second |
|---------------------------------------|--------|--------|
| Natural Frequency [Hz] FEM            | 4.66   | 7.84   |
| Natural Frequency [Hz] Experiment     | 4.65   | 7.05   |
| Damping Ratio                         | 0.015  | 0.013  |
| Modal Mass [Kg]                       | 291.84 | 149.81 |
| Stimulation Factor                    | -0.820 | 0.850  |
| Natural Mode (friction point)         |        |        |
| X direction $\phi_{fx}$               | 0.996  | 0.021  |
| Z direction $\phi_{fz}$               | 0.059  | 0.779  |
| Coefficient of Friction $\mu$         | 0.135  |        |
| Vertical Force at Friction Device [N] | 1470   |        |

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

Sliding motion trajectory

Figure 4 shows the sliding trajectories of the piping motion at the friction device. Figures (a) and (b) show experimental and analytical results respectively. Combined sinusoidal input was generated so that the displacement of the first (X direction) and the second mode (Z direction) of motion at friction part are approximately equal as shown in Fig.4. From this figure, almost similar sliding motion can be observed for both of analytical and experimental results.

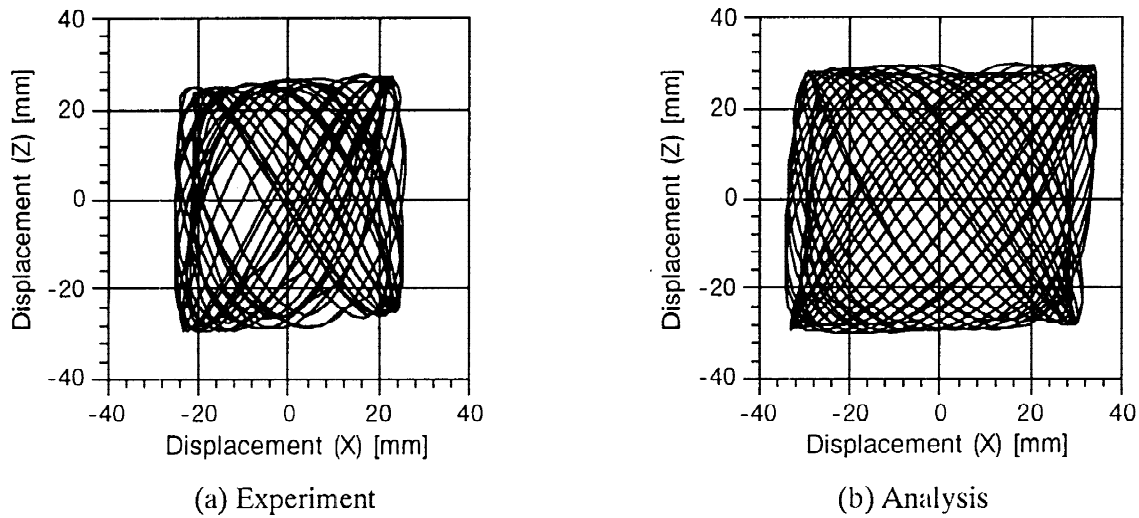


Fig.4 Sliding trajectories

### Maximum response

Evaluation of the maximum response of piping is important to be introduced to the seismic design, in particular, the maximum displacement response has to be significantly evaluated. Figure 5 shows the maximum displacement with respect to input level in the case of combined sinusoidal excitation, by demonstrating the comparison of analysis with experiment. Figures (a) and (b) depict X and Z directional responses, respectively. Up to 350 Gal of input level, results from analysis do not coincide with the responses from the experiment because resonant motion does not occur in the experiment due to frictional effect. However, results from the analysis approximately simulate the responses from experiment for higher input level, particularly for the Z directional motion.

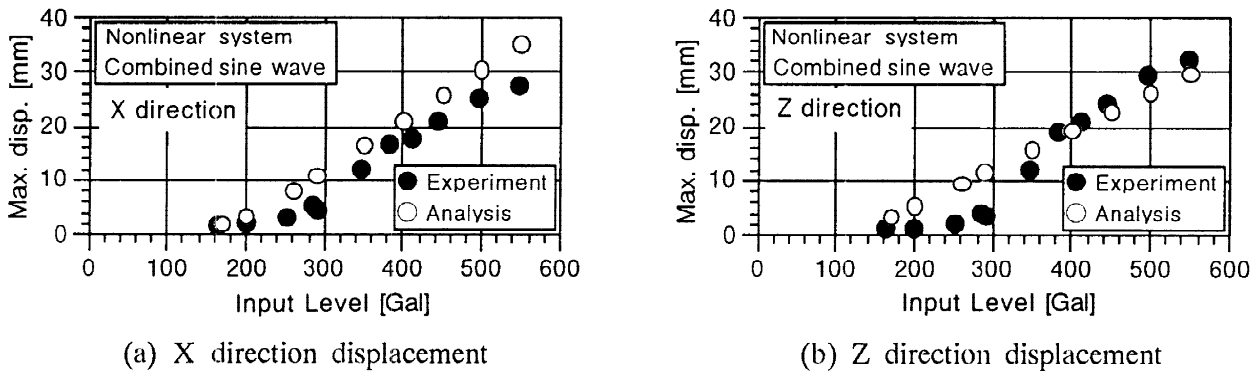


Fig.5. Maximum responses in the case of combined sine wave

### Dissipated energy

The energy dissipated caused by frictional motion is also important because this is closely related to the damping of the system. If the dissipated energy calculated by the analysis is different from the energy estimated from the experiment under the same maximum response condition, the analytical model used here

should be regenerated. Figure 6 shows the dissipated energy with respect to input level in the case of combined sinusoidal excitation. From this, it can be confirmed that the results from the analysis can quite fairly simulate the results from experiment.

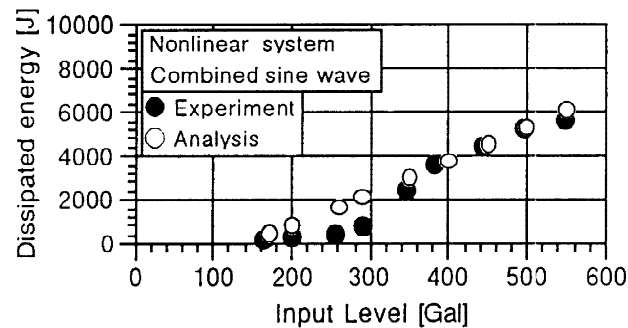


Fig.6 Dissipated energy at friction device

## CONCLUSIONS AND ACKNOWLEDGEMENTS

1. A large scale shaking test was performed for the mock up piping and supporting structural system having a frictional supporting device. Basic response properties such as time histories and frictional motion trajectories were obtained.

2. Time history analysis was carried out based on the modal model and the first and the second modal responses were calculated. Time history responses, maximum responses and dissipated energy were also calculated. By comparing results from analysis with those from the experiment, fairly good agreement was obtained. Therefore analytical procedure proposed in this study could be utilized in order to get the response properties for this kind of coupled piping and supporting systems.

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