

# STRESS-SLIP MODEL OF THE LONGITUDINAL REINFORCING BARS IN A REINFORCED CONCRETE COLUMN JOINT

YASUO MURAYAMA, KUMIKO SUDA, TOSHIMICHI ICHINOMIYA, and HIROSHI SHIMBO

Civil Engineering Department, Kajima Technical Research Institute Tobitakyu 2-19-1, Chofu-shi, Tokyo 182, JAPAN

#### ABSTRACT

A series of experiments were performed to investigate the slippage of longitudinal reinforcement at column base for the different loading histories including reversed loading. The relationships of stress - strain of the reinforcement, the rebar/concrete bond stress - slip, and stress - slip at loading end were investigated. As a result, it was found that differences in loading history have a considerable effect on the stress - strain and bond stress - slip relationships. However, due to the tendency of the influence of load history on each relationship cancelling each other, differences in loading history have almost no effect on the reinforcement stress - slip relationship. Based on these results, a reinforcement stress - slip model was established that can be easily incorporated into an RC column load - deformation analysis program able to take into consideration the effects of end rotation.

### **KEYWORDS**

Constitutive Law; Cyclic Load; Fiber Model; Reinforcing Bar; Slip.

#### INTRODUCTION

For the accurate seismic response analysis of RC structures, the precise expression of the hysteretic load - displacement characteristics is required. It is generally known that the load displacement relationship of RC structures is significantly influenced by joint rotation caused by the slippage of longitudinal reinforcement. It is therefore necessary to accurately take into account this phenomenon under the action of load reversals in the analysis. Until now two methods have been used to simulate analytically the effect of reinforcement slippage. One of them is based on the reinforcement stress - strain relationship and the bond stress - slip relationship, and the other makes direct use of a model of the reinforcement stress - slip relationship. However, the former method involves expensive and time consuming calculations, and the latter does not take adequate account of the effect of yielding of the reinforcement.

Experiments were conducted using scaled models of PC cable bridge piers and towers, to investigate the mechanism of reinforcement slippage under the action of load reversals. Based on the results of this investigation, a reinforcement stress - slip model is proposed that considers the effect of yielding of the reinforcement.

## Outline of the Experiments

The dimensions of the specimen and the loading set-up are depicted in Fig. 1. The set of five D10 (#3) rebars embedded in the concrete were welded to the steel block at the top of the concrete block. The mechanical properties of the concrete and the rebars are summarized in Table 1. The stress - strain curve for the reinforcement steel is shown in Fig. 2. Three types of specimens were prepared: the first type for the measurement of strain distribution of the rebars (numbered 1), the second type for measurement of the stress distribution (numbered 2), and the third type for the measurement of both stress and strain (numbered 3).

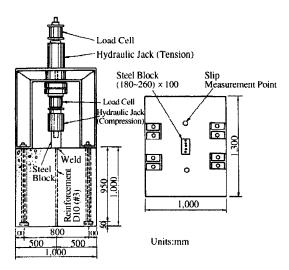


Fig. 1. Specimen Dimensions and Loading Set-up

Table 1. Material Properties

	Concrete					
	Aggregate Max Dia (mm)	Test Piece (cm)	28 day Strength (kgf/cm <sup>2</sup> )  Standard Curing		Strength at Test (kgf/cm <sup>2</sup> )	
Season						Water Cured On Site
			Compression	Tension	Compression	
Winter	20	φ 10×20	289	29	375	351
			Reinfo	rcement		
	Bearing Area				Strain Gauge Reading	
Туре	Nominal Dia	Coefficient BA	Yield Point	Tensile Strength	Yield Strain	Strain Hardening Point Strain
	(mm)	(%)	(kgf/mm <sup>2</sup> )	(kgf/mm <sup>2</sup> )	$(\times 10^{-6})$	(%)
D10	9.53	8.1	36.9	52.4	2,031	3.0

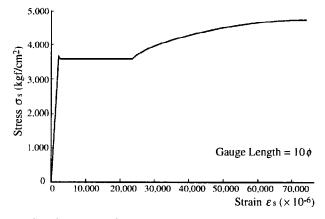


Fig. 2. Stress Strain Curve for Rebar (by Displacement Meter)

The loads were applied by two centerhole jacks, one for tension and one for compression, with a reaction frame. The three different loading patterns (M), (R) and (R'), as shown in Fig. 3, were employed in applying the loads. Monotonic cyclic tensile loading was represented by the loading pattern (M), the reversed loading was denoted by loading pattern (R) with 0kgf/cm² corresponding axial force of the RC column. A modified reversed loading pattern (R') assumes about 10kgf/cm² corresponding axial force. In the loading process, load was controlled prior to yield of the rebars and the displacement was controlled subsequent to yield. After the slippage reached 1mm (0.1 × rebar dia), in the case of load pattern (R) the slippage was increased by 1mm increments and the each loading repeated 10 times. On the other hand, in the case of load pattern (R') the slippage was increased by 1/3mm increments and the load applied 3~4 times. All test cases are summarized in Table 2.

## Method of Measurement

Strain Distribution. Strain gauges were attached to the middle 3 rebars at various depths for measuring the plastic strain to determine the depthwise strain distribution of the reinforcement by superposition of the various results.

Stress Distribution. In order to determine the stress after the initiation of plastic deformation, the yield strength of certain portions of the reinforcing bars were increased by 1.3 times by localized heat treatment, and strain gauges attached in the strengthened region. The stress was then calculated from the measured values of strain and the stress distribution was obtained in the same way as the strain distribution described above.

Slip Distribution. Slip measuring wires were attached at the points where the stress and strain measurements were made. Displacement meters were attached between the ends of wire and the lower surface of the concrete block to measure the relative displacement.

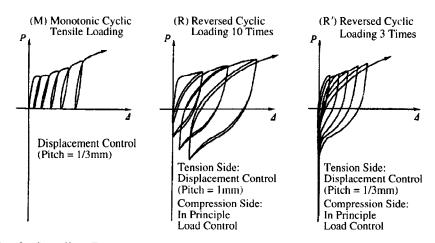


Fig. 3. Loading Patterns

Table 2. Test Arrangement

Designation	Load Pattern Symbol	Measurement Category	
GM1	(M)	Strain Specimen	
GM2	(141)	Stress Specimen	
GR1	(R)	Strain Specimen	
GR2	(K)	Stress Specimen	
GR'I		Strain Specimen	
GR'2	(R')	Stress Specimen	
GR'3	(8)	Combined Stress/ Strain Specimen	

Slip at Loading End. The slip of the reinforcement at the loading end was determined by measuring the relative displacement between the steel block and the top surface of the concrete block using a displacement meters.

## Data Processing

Local  $\tau$  - s Relationship. The bond stress was determined from the difference in stress values at adjacent points and the distance between the measuring points. The value of slip used was the average of the measured values of slip at the two adjacent measuring points.

 $\sigma_s - \varepsilon_s$  Relationship. The bond stress distribution was determined by applying the  $\tau$ -s relationship defined above to the slip distribution obtained by integration of the strain distribution in the depth-wise direction. In addition, by the integration of this bond stress distribution in the depthwise direction, the stress was calculated. Measured values of strain were used directly.

#### Results and Discussion

Load - Slip Relationship. The test results of the relationship between load and slip for the case of GR1 are shown as an example in Fig.4. From this figure it can be seen that the influence of the cyclic loadings at each step have little effect on the skeleton curve and hysteresis curve. In Fig. 5 a comparison of the initial

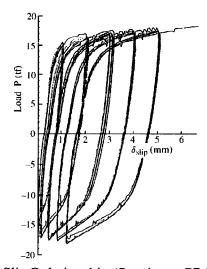


Fig. 4. Load - Slip Relationship (Specimen GR1)

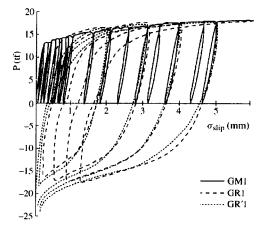


Fig. 5. Comparison of First Hysteresis Curves

hysteresis curve for the various specimens is made. From this figure it can be seen that for the load - slip relationship, regardless of the differences in loading history, almost the same skeleton curve and hysteresis curve were obtained.

Local  $\tau$ -s Relationship. The obtained  $\tau$ -s relationship is shown in Fig. 6. It can be clearly seen from Fig. 6 that the effect of the loading history on the  $\tau$ -s relationship is considerable after yielding of the reinforcement. Due to the action of the load reversals and axial force, it can be seen that the bond stress has become smaller for the same value of slip. The fact that the bond stress rapidly drops during the range of slippage between  $0.02 \sim 0.05$   $\phi$  ( $\phi$  = bar diameter), is due to the fact that the reinforcement has yielded.

 $\sigma_s - \varepsilon_s$  Relationship. The envelope of the curves for  $\sigma_s - \varepsilon_s$  for each of the measuring locations was investigated. The reinforcement stress was obtained using the  $\tau$ -s relationship shown as a solid curve in Fig. 6. The result of this is shown in Fig. 7. From the figure it can be clearly seen that the loading history has a major influence on the  $\sigma_s$ - $\varepsilon_s$  relationship after yielding of the reinforcement. It can be seen that the reinforcement stress tends to become larger for the same value of strain due to the action of the load reversals and compressive axial force.

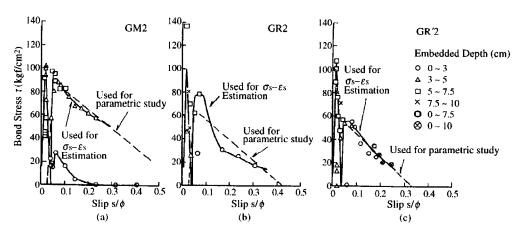


Fig. 6. Bond Stress - Slip Relationship

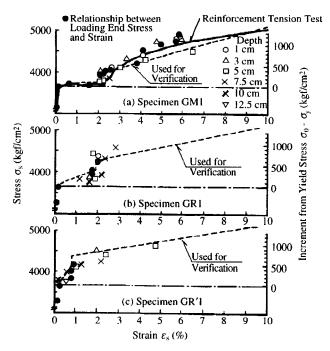


Fig. 7. Stress - Strain Relationship of Reinforcement

## ANALYTICAL STUDY OF THE $\sigma_{_{\!s0}}$ - $\delta_{_{\!s1}}$ SKELETON

In many cases the  $\tau$ -s model and the  $\sigma_s$ - $\varepsilon_s$  model are used for the analytical method to obtain the relationship between the loading end reinforcement stress and the slip  $(\sigma_{s0} - \delta_{s1})$ . The effect of the difference in the analytical model on the  $\sigma_{s0}$ - $\delta_{s1}$  relationship was investigated. The  $\tau$ -s model and the  $\sigma_s$ - $\varepsilon_s$  model used for the analysis were represented by straight lines as shown in Fig. 8, and a parametric study focussed on  $\tau_s$  and  $E_{s2}$  was carried out. The results are shown in Fig. 9.

From Fig. 9, it can be seen that even the slightest change in  $\tau_2$  or  $E_{s2}$  has a major influence on the analytical result. This indicates that a small difference in the  $\sigma_s$  -  $\varepsilon_s$  relationship or in the  $\tau$  - s relationship used for the input model will result in a large error being introduced in the analytical result for  $\sigma_{s0}$  -  $\delta_{s1}$ . However, under the action of reversed loading or axial force on the RC column, although  $\tau_2$  becomes small and  $E_{s2}$  becomes large, it can be seen with respect to slip that these two tend to cancel each other out. From this, as can be confirmed by experiment, the effect of reversed loading or axial force on the  $\sigma_{s0}$  -  $\delta_{s1}$  relationship is small.

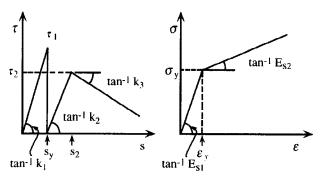


Fig. 8. Modeling for Calculation

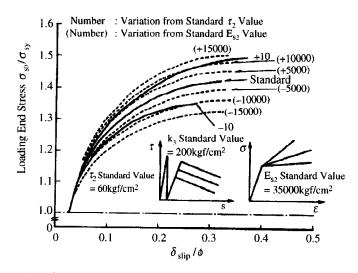


Fig. 9. Results of Parametric Study for Slippage

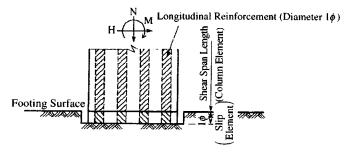


Fig. 10. Slip Element for Fiber Model Based Analysis

As the  $\sigma_{s0}$  -  $\delta_{s1}$  relationship is virtually unaffected by the loading history, it is effective to use the  $\sigma_{s0}$  -  $\delta_{s1}$  relationship model directly for the deformation analysis of an RC column. Therefore, a  $\sigma_{s0}$  -  $\delta_{s1}$  model was established that can easily be incorporated in the same way as the  $\sigma_{s}$  -  $\varepsilon_{s}$  model into a fiber model based analysis program.

Since the slip for a given reinforcement stress is proportional to the diameter of the rebars, the slip  $\delta_{s1}$  divided by the rebar diameter  $\phi$ , which represents the equivalent strain  $\varepsilon_{s1}$  is used for model description. For this reason, as shown in Fig. 10, reinforcing bar elements of length of  $1\phi$  are specially provided as an analytical model.

Details of the Model. The model is represented as a combination of the  $\sigma_s$  -  $\varepsilon_{s1}$  skeleton and internal hysteretic curves as shown in Fig. 11. The skeleton is as follows:

 $0 \le \varepsilon_{s1} \le \varepsilon_{s1,y}$ : Curve passing through the yielding point  $(\varepsilon_{s1,y}, \sigma_{s,y})$  and proportional to  $\varepsilon_{s1}^{2/3}$ .  $\varepsilon_{s1,y} \le \varepsilon_{s1} \le \varepsilon_{s1,h}$ : Straight line passing through the point  $(\varepsilon_{s1,y}, \sigma_{s,y})$  and the hardening point  $(\varepsilon_{s1,h}, \sigma_{s,h})$ .  $\varepsilon_{s1,h} \le \varepsilon_{s1} \le \varepsilon_{s1,0.5}$ : Hyperbolic curve passing through  $(\varepsilon_{s1,h}, \sigma_{s,h})$  and the 0.5 strain point  $(\varepsilon_{s1,0.5}, \sigma_{s,0.5})$ .

Unloading - Tension Side.

 $0 \le \varepsilon_{sl} \le \varepsilon_{sl,x}$ : Straight line in the direction of the origin.

 $\varepsilon_{\text{sly}} \le \varepsilon_{\text{sl}} \le \varepsilon_{\text{sl},0.5}$ : Straight line. For the slope, the secant modulus slope at yield point or reduced value.

Loading - Compression Side. A hyperbolic curve joining the starting point of the compressive side loading on the  $\varepsilon_{st}$  axis ( $\varepsilon_{st,0}$ ,0) to the point (0,  $\sigma_{s,0}$ ) on the  $\sigma_{s}$  axis is used. For the initial slope, the tension side unloading gradient or reduced value is used. Here,  $\sigma_{s0}$  is proportional to the 3rd root of  $\varepsilon_{st,0}$  when  $\varepsilon_{st}$  is below 0.1, and linearly proportional to  $\varepsilon_{st,0}$  when  $\varepsilon_{st}$  is greater than 0.1.

Unloading - Compression Side. A straight line parallel to the tension unloading curve is adopted.

Reloading - Tension Side. A hyperbolic curve toward the maximum previous slip point on the  $\sigma_{s0}$  -  $\varepsilon_{s1}$  curve is taken. For the initial slope, the tension side unloading gradient or reduced value is used. Also, for the unloading from the tension side reloading curve, a straight line parallel to the straight unloading curve from the skeleton is used, and a hyperbolic curve is taken for the compression side reloading curve.

Curve for Reinforcement Penetration. For columns in which the axial force is small, the reinforcement hardly penetrates into the concrete, and even in cases where the axial force is large, the column concrete

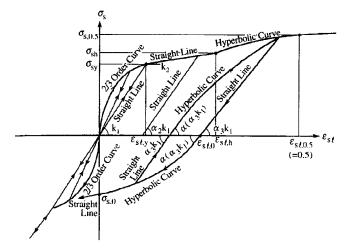


Fig. 11. Proposed Model for Reinforcement Stress - Slip Relationship

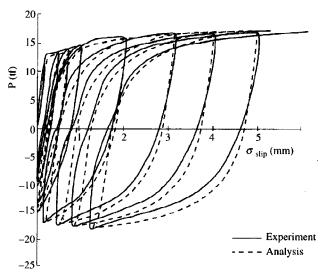


Fig. 12. Comparison of Load - Slip Experiment and Analysis (GR1)

tends to deteriorate before the reinforcement penetrates into the concrete, so the penetration is small. In addition, the rotation at the base of the column is influenced predominantly by the slip out rather than the penetration. Therefore, for the sake of convenience, a simplified penetration model is adopted.

### **EXAMPLE OF ANALYSIS**

A comparison was made between the analytical results of force - slip relationship based on the proposed model and experiment for the verification. The solid line in Fig. 12 represents the analytical results of the GR1 specimen under the following conditions:

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\begin{split} \sigma_{y} &= 3690 kgf/cm^{2} & \sigma_{s,h} = 4100 kgf/cm^{2} & \sigma_{s,0.5} = 4900 kgf/cm^{2} \\ \varepsilon_{si,y} &= 0.021 & \varepsilon_{si,h} = 0.1 & \varepsilon_{si,0.5} = 0.5 \\ k_{1} &= 25,000 kgf/cm^{2} & k_{2} = 5200 kgf/cm^{2} \end{split}
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The ratio of the slope for the compression side reloading and the tension side unloading was determined constant at 0.55.

The results of the analysis were found to provide a good approximation to the experimental results.

#### **CONCLUSIONS**

From the experimental and analytical study of the slip of longitudinal reinforcement in the RC column joint, the following conclusions can be drawn:

- 1) The bond stress slip relationship and the reinforcement stress strain relationship of the longitudinal reinforcement in the RC column joint are considerably influenced by the loading history.
- 2) However, the influence of loading history on the reinforcement stress slip relationship is small because the effects of the change in bond stress slip and stress strain relationship due to change in loading tend to cancel each other out.
- 3) Based on these observations a reinforcement stress slip model was proposed.