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OPTIMISATION OF SHAKING TABLE TESTING OF BRIDGE MODELS

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ABSTRACT

This paper describes the analytical studies that were performed to identify the best shaking table testing strategies to be used in the experimental study of bridge models. Bridge structures are idealised by a spatial model with 6 d.o.f. per node, assuming that non-linear behaviour will occur only in the piers due to bending, and so a fibre type model is used. The results of the application of the developed testing strategy to a model of a bridge structure with piers of unequal height are presented and discussed.

KEYWORDS

Shaking table testing; bridge structures; seismic behaviour.

INTRODUCTION

The principal characteristic of a shaking table test in the nonlinear range is that each model can be tested only one time, in principle. Under the assumption that tests are carried out to obtain "information" about the probability of failure, one may compare:
i) The "information" given by a fictitious set of models that are tested only one time and then discarded; different models are tested for different intensities;
ii) The "information" given by a single model that is tested for increasing intensities of shaking; those intensities are defined by a geometric series with a common ratio α of the values of the peak acceleration.

The "information" given by the tests is evaluated in a Bayesian framework in which "information" is associated to a probability distribution defined in the space of the vulnerability functions of the bridge under consideration. Results of the shaking table tests allow a sharpening of this *a priori* distribution through the Bayes theorem and the basic fact that the average value of the response, computed from several realizations of the stochastic process that idealizes earthquake vibration, has a distribution that is very nearly a gaussian distribution.

Sequences of case ii) tests with different values of α were simulated, and their "information" was compared with the information from a case i) test, in order to identify the best value for α. After that, the sequences were analyzed again considering that the model at the beginning of each test is already damaged.

BAYESIAN METHODOLOGY FOR THE QUANTIFICATION OF THE PROBABILITY OF FAILURE

According to the methodology developed at LNEC the computation of the probability of failure is based on the knowledge of the vulnerability function. This function is a non-linear function which relates the parameters describing the severity of earthquake actions (h) with the variables that describe their effects in the structures (c). The direct estimation of the vulnerability function requires heavy computational effort (Vaz et al, 1996) and so an alternative procedure, based on the application of the theorem of Bayes has been developed.

The theorem of Bayes relates *a priori* probabilities with *a posteriori* probabilities, taking into account the occurrence of some event. Let **S** = $\{s_1, s_2, s_3, \dots\}$ be a probability space in which states s_1, s_2, s_3, ... are mutually exclusive, $P(s_i)$ the *a priori* probability of state s_i and $P(r \mid s_i)$ the conditional probability of an event r, if state s_i stands. Then the *a posteriori* probability $P(s_i \mid r)$ of state s_i, if event r happens, is given by

$$P(s_i|r) = \frac{P(s_i)\,P(r|s_i)}{\sum_i P(s_i)\,P(r|s_i)} \qquad (1)$$

Therefore changing from *a priori* to *a posteriori* probabilities corresponds to the changing of the knowledge due to the occurrence of event r.

Representation of knowledge

Assuming total absence of information, the vulnerability function may be any function subjected to the restriction of being a non-decreasing function, since it seems reasonable that an increasing in the loads will correspond to an increasing in the load effects. Since, from a practical viewpoint, it doesn't make sense to find the "true" vulnerability function, the first step consists in generating a number of functions which, according to Duarte, (1991), is finite. One of the functions in this set will match the "true" vulnerability function under an appropriate norm.

The robustness of the final estimates will depend on the qualities of the functions contained in this set. To control the robustness two partial sets are selected (Duarte, 1991). The first one is constituted by analytical functions expressed as

$$c = \alpha h + \beta h^2 + \gamma h^3 \qquad (2)$$

with appropriate values of α, β and γ being chosen to generate the selected number of functions. The second partial set is composed by realisations of discrete Markov processes where the value c_i of the control variable, corresponding to a value h_i of the load intensity, is given by

$$c_i = (1 + \delta x)\, c_{i-1} \qquad (3)$$

with δ constant and x a random variable uniformly distributed in the interval [0, 1]. This discrete functions are transformed into continuous functions by linear interpolation on a bi-logarithmic plot. The values of δ are selected to generate a partial set with the target characteristics. Results obtained by considering separately the partial sets allow the evaluation of the robustness. The set of functions is probabilised by associating to each function V_i a probability value p_i. Those probabilities must obey to the condition $\Sigma\, p_i = 1$. Each set of values **p** = $\{p_1, p_2, p_3, \dots\}^T$ represents a state of knowledge. There is a good state of knowledge when all the functions in the set are associated to very small probabilities with exception of those ones which are close to the "true" vulnerability function.

As above referred, at the beginning the state of knowledge is non-informative, represented by a constant probability density in a logarithmic scale, i.e., the probability of the probability of failure lying in the interval (10^{-3}, 10^{-4}) is equal to the probability of it lying in the interval (10^{-4}, 10^{-5}), and similarly for the other similar intervals. Values of p_i may be easily computed to ensure approximately a non-informative distribution. The uncertainty associated to each state of knowledge may be quantified by the ratio between the 5% and the 95% fractiles of the probability distribution of the probability of failure.

Bayesian analysis

When a non-linear computation is performed, the value of the control variable is just an estimate of the "true" value of the vulnerability function, since earthquake actions are idealised by a stochastic model. However, several realisations (time histories of acceleration) of the stochastic process may be used and, consequently, a sample of values of the control variables is obtained and the sample mean value obviously is a better estimate of the "true" value of the vulnerability function.

The influence of the sample size constitutes an important issue but, according to Duarte, (1991) and Bairrão et al. (1995) the sample mean value approximately follows a gaussian distribution with a mean value equal to the mean value of the response to one realisation and a variance equal to the variance of the response to one realisation divided by the number of elements in the sample. Several past studies (Vaz, 1992, 1994 and 1995) have shown that variance may be assumed to correspond to a coefficient of variation c.o.v.= 0.3.

Therefore it is possible to compute the conditional probability $P(c\,|V_i(h))$ of obtaining a mean value c of the control variable, if function $V_i(h)$ is the "true" vulnerability function, by

$$P(c|V_i(h)) = G\left(V_i(h)\,,\frac{\text{c.o.v.}^2\;V_i^2(h)}{n}\right) \tag{4}$$

where $G(\mu\,,\sigma^2)$ represents a gaussian distribution with mean value μ and variance σ^2 and n is the number of realisations. This result allows the computation of the *a posteriori* probabilities $P(V_i(h)\mid c)$ such that

$$P(V_i(h)|c) = \frac{P(V_i(h))\,P(c|V_i(h))}{\sum\limits_i P(V_i(h))\,P(c|V_i(h))} \tag{5}$$

which represent a new state of knowledge with a new value of the probabilities associated to each function in the set $p_i = P(V_i(h)\mid c)$.

Preposteriori analysis

The value h of the intensity of earthquake vibration to be used in the non-linear computations may be selected to provide an optimal increase in knowledge through a *preposteriori* analysis. This basically consists in a probabilistic evaluation for a large number of h values whose change in the state of knowledge is expectable if the computations are performed for that value. This evaluation is carried out by computing for each value the 5% and 95% fractiles of the probability distribution of obtaining a value of the response, as may be computed from the probabilities p_i of the functions in the set, for each intensity. The optimal increase in the state of knowledge will correspond to the minimum value of the ratio between the 5% and 95% fractiles, the corresponding value of h being selected to perform the next series of non-linear analysis.

STRATEGY OF ANALYSIS

Bridge description

The bridge to be analysed is a regular bridge structure with 3 piers, with 50 m long spans making a total length of 200 m. The piers have hollow circular section with diameter 3.0 m and thickness 0.40 m. Lateral piers are 14 m high and the height of the central pier is 21 m. Piers have been assumed to be built in on the deck and on the foundations. Longitudinal displacements are free in all cases; however at the abutments it was assumed that transverse displacements to the bridge longitudinal axes are restrained. The general layout of the bridge and the typical deck section are schematically shown in Figure 1.

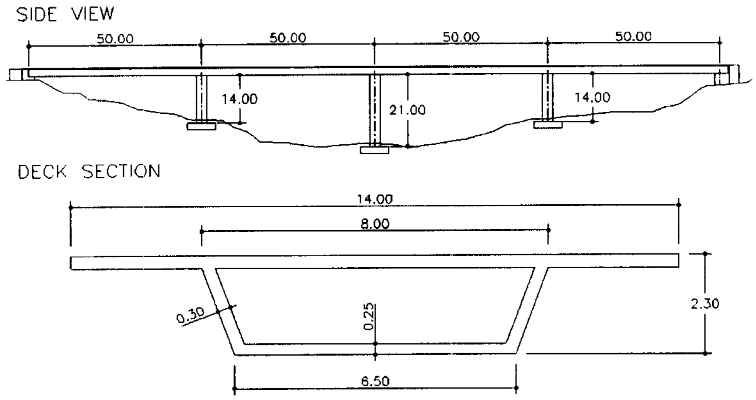

Figure 1 - General layout of the bridge.

The first step in the analysis consisted on the dynamic characterisation of the structure, i.e., on the computation of natural frequencies and corresponding mode shapes. Fundamental frequencies in the longitudinal and transverse directions are respectively f_L= 1.38 Hz and f_T= 0.89 Hz.

The seismic response was computed considering the response spectrum for soil condition B, as prescribed at the Eurocode 8 (EC8, 1994) assuming a peak ground acceleration of 150 cm/s^2. An uniform value for modal damping ζ=5% was assumed. The longitudinal and transverse amounts of reinforcement in each critical section were obtained by combining the internal forces due to earthquake action with those resulting from the other actions (namely, dead loads and temperature), considering a value of the behaviour coefficient q=1.5 and a safety coefficient γ_E=1.0. The prescriptions of the Portuguese Code for Reinforced and Prestressed Concrete Structures (REBAP, 1983),which is very similar to Eurocode 2 (EC2, 1991), have been considered.

Numerical models

The bridge structure was idealised by a spatial model with beam elements with 6 degrees of freedom per node. The weight of the deck is distributed along the span length, resulting an axial force of about 10000 kN at the top of each pier. It should be noted that the structural model used in the linear and non-linear analyses of the bridge is the same.

In what concerns the non-linear analyses, the main assumption is that the deck will behave as linear elastic and that the critical zones are located at the piers' extremities. In consequence, it was assumed that the energy dissipation mechanism is constituted by hysteretic hinges at the bottom of the piers. These hinges are represented by non-linear beam finite elements with a length equal to the equivalent plastic hinge length, which is estimated on the basis of the results presented by Priestley et al. (1984); in this particular study lengths of 1.5 m have been estimated for the plastic hinges at the bottom of the piers. The non-linear behaviour at the potential plastic hinges is quantified by moment versus curvature relationships determined by a fibre model. This model involves the discretization of the critical sections in a number of concrete "filaments" with uniaxial behaviour, the steel bars being considered one by one. The force versus deformation loops for steel are based on the model proposed by Giuffrè et al. (1970), whereas for the concrete the model proposed by Kent et al. (1971) has been adopted.

Earthquake input and development of the analysis

Earthquake input consists of 5 sets of artificial accelerograms, constructed to match the design spectrum. Those accelerograms have a duration of 10 seconds, one of the accelerograms considered being shown in

Figure 2. Each set of accelerograms is constituted by 3 different accelerograms, corresponding to global X, Y and Z directions. With those sets of accelerograms a series of nonlinear analyses was performed, with the intensities (represented by the peak ground acceleration) ranging from 0.5 m/s^2 to 10 m/s^2, by appropriately scaling the accelerograms. For each intensity a "new" (undamaged) model is considered and the results of those analyses allowed the direct computation of the "true" vulnerability function of the structure (Vaz, 1996) and hence its probability of failure. For further reference this will be called "reference case".

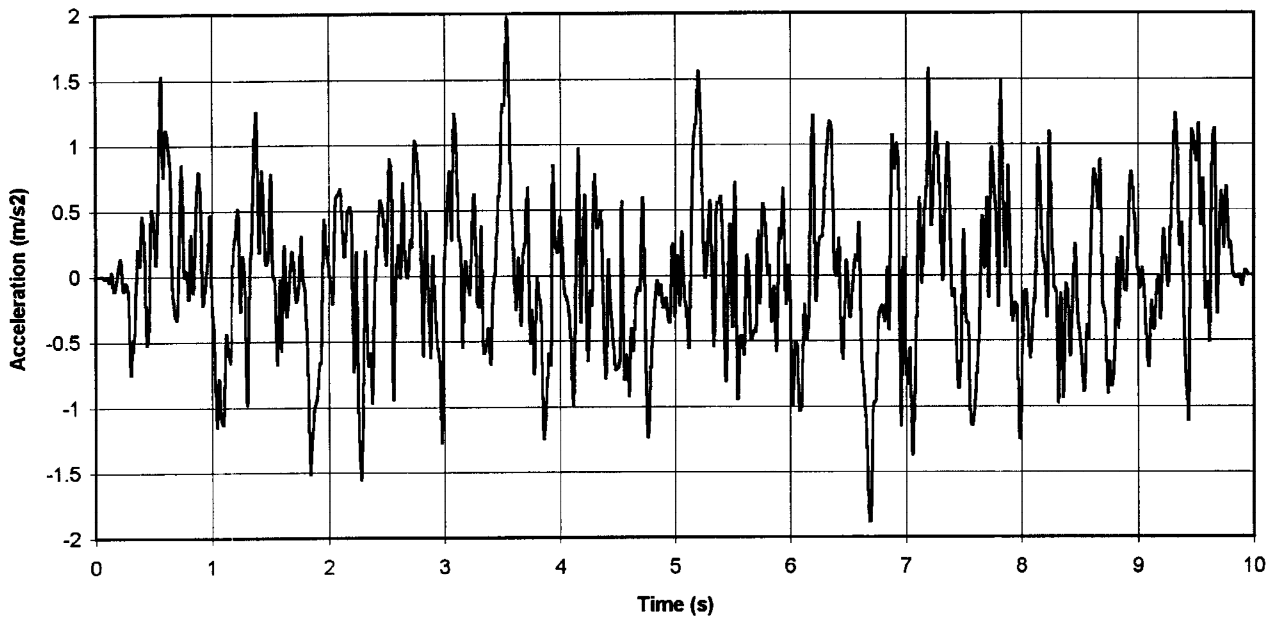

Figure 2 - Sample accelerogram matching EC8 response spectrum (Soil B condition).

The methodology adopted in the reference case is not useful when considering the experimental testing of models until collapse since it requires a considerable number of models and, in general, the number of models available is limited due to their high cost. In an extreme situation of having just one model available, the important issue is to define the testing strategy so that the maximum information can be retained, i.e., what should be the optimal increase of shaking intensity. In fact, the option for very small increments of the intensity probably will cause a premature damage of the model when the relevant intensities are reached; on the other hand, very large increments can lead to misrepresentative information, in the sense that the relevant intensities are overpassed.

Let α be the ratio between the intensities of 2 consecutive experiments on the same model. In this study values α=1.2, α=1.44 (1.2^2) and α=1.728 (1.2^3) have been considered, regarding 3 different models to be tested with different strategies. Assuming that for each model the starting intensity corresponds to a peak ground acceleration a_g=1 m/s^2, each one of these models will be tested considering the following intensities: 1.0, 1.2, 1.44, ... m/s^2 for the model with α=1.2, 1.0, 1.44, 1.44^2, ... m/s^2 for the model with α=1.44 and 1.0, 1.728, 1.728^2, ... m/s^2 for the model with α=1.728.

For each model, at the beginning of the test for a given intensity, the damage sustained by the model in the tests previously performed for lower intensities must be considered. This condition can be numerically fulfilled by remaking the analysis from the beginning and separating the scaled accelerograms corresponding to the different previous intensities with "blanks" of appropriate duration (in this study a time of 5 seconds was selected). Those gaps correspond to free vibrations of the structure and their duration is selected so that its amplitude at the beginning of the test for the current intensity is small, when compared with the maximum values of the response. This means that the duration of the analyses increases as far as higher intensities are considered. To illustrate this procedure the accelerogram corresponding to the fifth intensity analysis of the model tested with α=1.728 is shown in Figure 3.

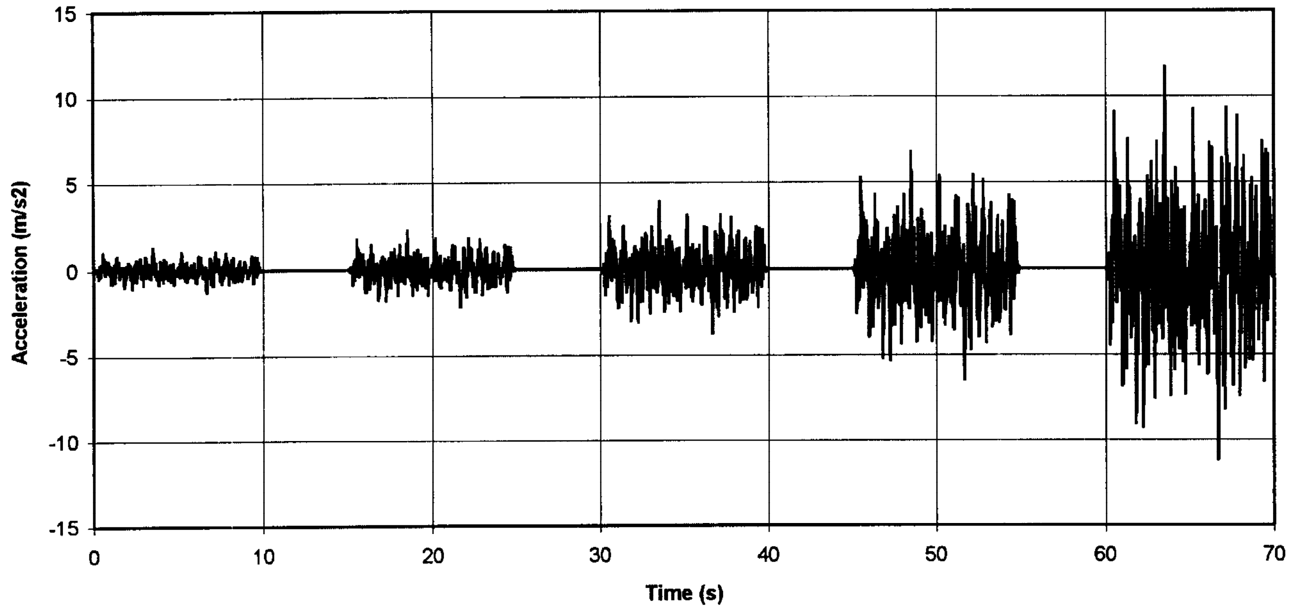

Figure 3 - Accelerogram corresponding to the 5th test (α=1.728).

RESULTS

Due to the high computational effort required by this type of procedure the *preposteriori* analysis above referred can play a fundamental role. In fact, particularly for low values of α, it allows a significant saving in the amount of computations because, taking into account the aims of the study, analyses need not to be performed for all the values of the intensity.

Assuming that the optimal value of α can be found considering both the estimate of the probability of failure (to be compared with that one corresponding to the reference case) and the 90% confidence interval, the relevant quantities to be controlled are the mean value of the probability distribution of the probability of failure and the corresponding values of the 5% and 95% fractiles.

The variation of the estimates of those quantities for the different values of α considered in this study are shown in Figures 4 to 6. From those figures fast convergence is evident. It should be noted that in those figures consecutive iterations do not correspond to consecutive (and increasing) values of the intensity but to the intensities "foreseen" as more informative according to the *preposteriori* analysis above referred.

The values of the mean value and for the 5% and 95% fractiles of the probability distribution of the probability of failure for the analyses performed are presented in Table 1 and shown in Figure 7.

Table 1 - Mean value and 90% confidence interval of the estimates of the probability distribution of the probability of failure for the different test strategies

Fractile	Reference case	α=1.2	α=1.44	α=1.728
95%	7.67×10^{-7}	4.70×10^{-6}	2.45×10^{-6}	1.92×10^{-6}
50%	1.04×10^{-5}	3.01×10^{-5}	2.30×10^{-5}	9.75×10^{-6}
5%	8.08×10^{-5}	2.98×10^{-4}	1.98×10^{-4}	3.31×10^{-6}

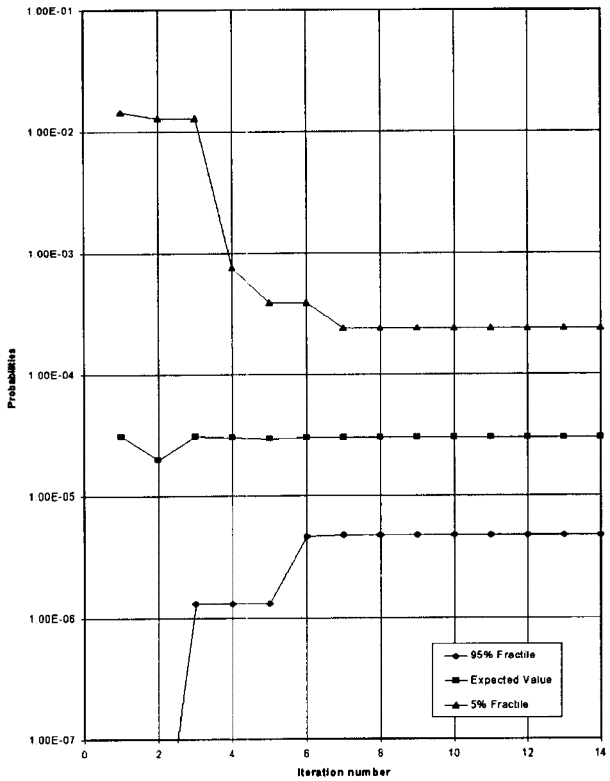

Figure 4 - Convergence of mean value and 5% and 95% fractiles of the probability distribution of the probability of failure for the model tested with α=1.2.

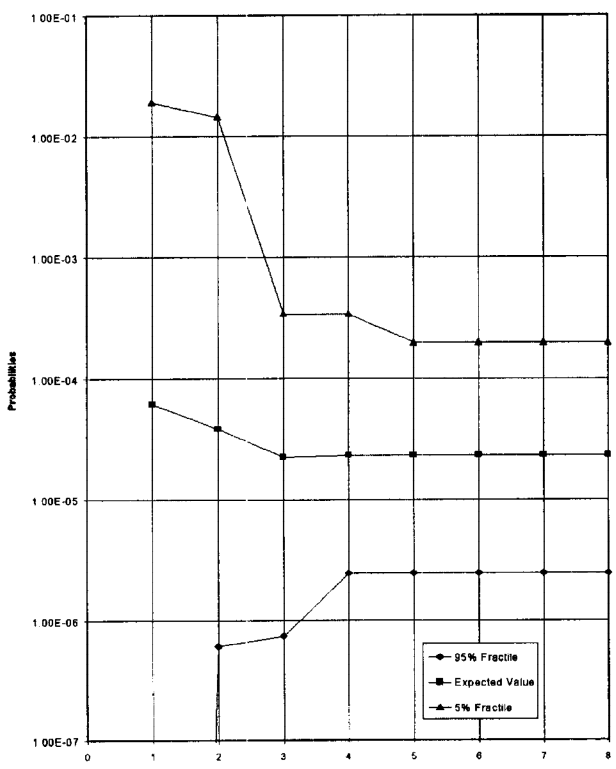

Figure 5 - Convergence of mean value and 5% and 95% fractiles of the probability distribution of the probability of failure for the model tested with α=1.44.

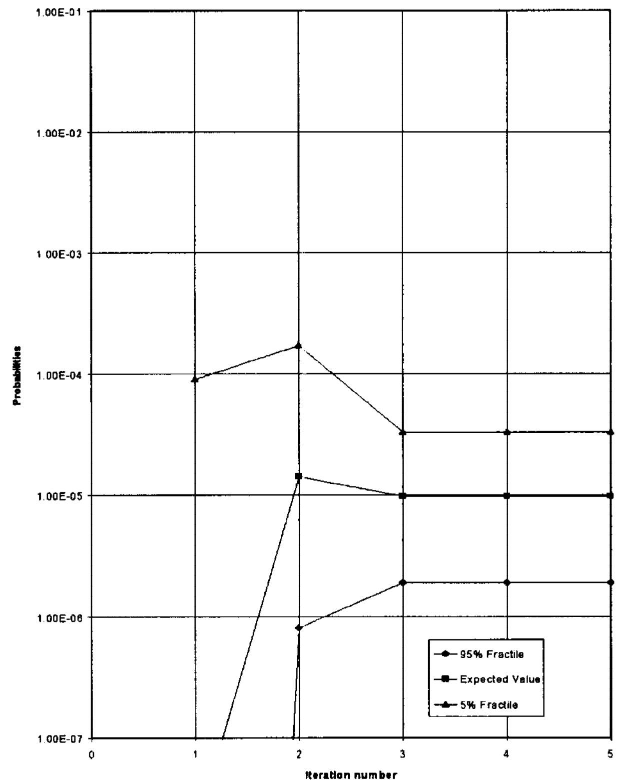

Figure 6 - Convergence of mean value and 5% and 95% fractiles of the probability distribution of the probability of failure for the model tested with α=1.728.

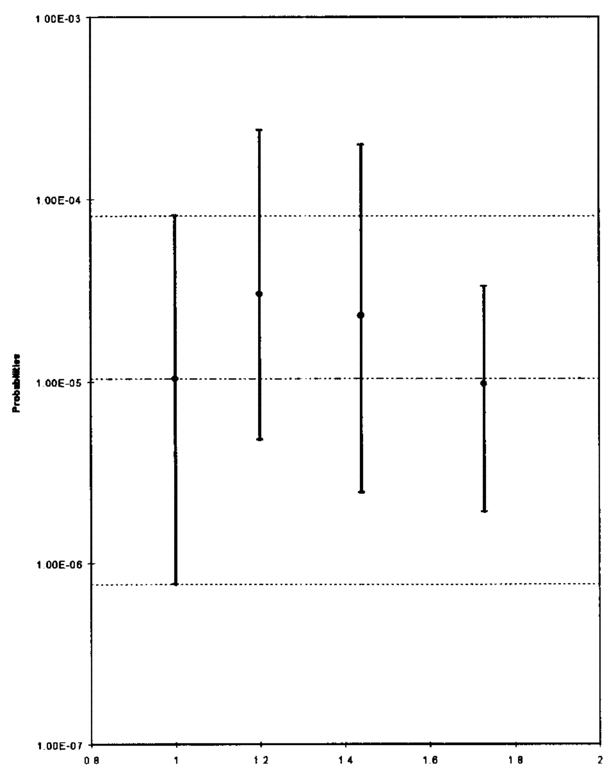

Figure 7 - Probabilities of failure obtained by numerical testing with a geometric progression of intensities with common ratio α.

CONCLUSIONS

In this paper a methodology aiming at the establishment of optimal shaking table testing strategies of bridge models was presented. This methodology is based on the Bayes theorem and was developed with the objective of optimising the information obtained from tests until the collapse.

For the bridge under study, the adoption of α=1.728 seems to be the optimal procedure since a good estimate of the probability of failure (regarding the reference case) is obtained with the lowest uncertainty. The extent of this kind of analysis is still very limited but the continuous increasing in the computational power makes possible to hope that this kind of procedure can be used case by case in a near future.

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