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Published by Elsevier Science Ltd 1996
Paper No. 1672. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

THEORETICAL EXPERIMENTAL STUDY OF CYLINDRICAL STEEL TANKS

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ABSTRACT

This paper presents a theoretical experimental study of cylindrical steel tanks to be used as wine storage vessels characterized by the $H/R \geq 2.0$ rate (height/radius).

The Caucete Earthquake (1977), in San Juan, Argentina, caused severe damages including the collapse of some tanks. The lack of an adequate earthquake resistant method on the tank dynamic characteristics and the real response, caused the tank failure. Besides, many of them presented failures produced by elastic instability (buckling).

In 1980, the first theoretical contribution which significantly improved the design of the tanks built before the 1977 earthquake was made (2).

The parametric parabola by N. Ungareano and A. L. Nogoita (10th WCEE) is herein introduced. Furthermore, in order to avoid buckling, the critic solicitations have been considered.

It has been proven that the most unfavourable situation to the seismic effect in these tanks occurs when they are completely full of liquid. In this case, the sloshing of the liquid have a null effect.

On the other hand, this study includes experimental determinations "in situ" of the natural vibration periods to be used in the tank seismic-resistant design.

KEYWORDS

Earthquake - Resistant; Seismic Coefficient; Seismic Stress; Buckling ; Natural Period.

INTRODUCTION

This paper deals with closed metallic cylindrical tanks used as wine vessels. They have a conical cover or a spherical cap with a transition to tangentially join their walls. The botton is flat, slightly tilted so as to facilitate its emptying and cleaning. They are anchored in their all perimeter to a reinforced concrete rigid and heavy foundation which is the real base. In these cases, the H/R radius is greater than 2.0, Fig. 1.

It has been proven that the most unfavourable situation to the seismic effect in these tanks occurs when they are completely full of liquid. In this case the sloshing of the water have a null effect.

SEISMIC COEFFICIENT

The parabolas proposed by N. Ungureano and A. L. Nogoita (4) have been adopted in the height distribution of the seismic coefficient, i.e:

$$C(x) = \frac{a(x)}{g} = \psi(x)\,\frac{a(H)}{g} = \psi(x)\,C(H)$$

where

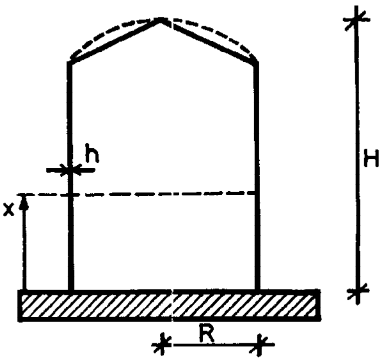

Fig. 1

- $C(x)$: variable seismic coefficient with x
- $a(x)$: variable seismic acceleration with x
- g : gravity acceleration
- $a(H)$: liquid level seismic acceleration
- $C(H)$: $\frac{a(H)}{g}$
- $\psi(x)$: $(\alpha - 1)\frac{x^2}{H^2} + (2 - \alpha)\frac{x}{H}$
- α : $- 0.266\ H/R + 1.133$ (base hinged wall)
- α : $0.286\ H/R + 0.857$ (base embedded wall)

On the other hand, being C_0 the seismic coefficient obtained from the codes and from which the total "seismic shear " at the base is obtained, we get the C (H) value :

$$H.\,Co = \int_0^H \psi(x)\,.\,C(H).\,dx = \int_0^H \left[(\alpha - 1)\frac{x^2}{H^2} + (2-\alpha)\frac{x}{H}\right] C(H)\,dx = C(H)\left[(\alpha - 1)\frac{H}{3} + (2-\alpha)\frac{H}{2}\right]$$

then:

$$C(H) = \frac{6\,.\,Co}{(4 - \alpha)}$$

and

$$C(x) = \Psi(x)\,\frac{6.\,Co}{4 - \alpha}$$

SEISMIC STRESS CALCULATION

The stress calculation is made according to the membrane theory of shell structures and adopting the following nomenclature. Fig. 2.

where

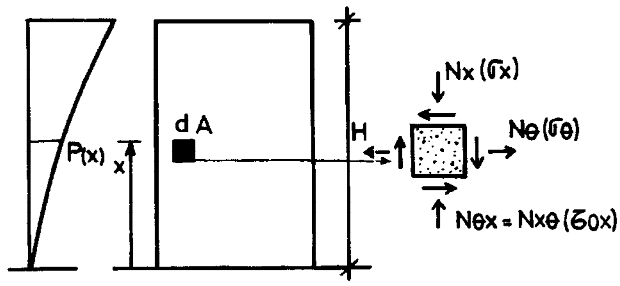

Fig. 2

- $N_x = \sigma_x\,.\,h$
- $N_\theta = \sigma_\theta\,.\,h$
- $N_{x\theta} = N_{\theta x} = \tau_{x\theta}.h$
- X, Y and Z = load components
- θ = circumferential coordinate
- x = vertical coordinate
- σ_x = axial stress

$\sigma\theta$ = hoop stress
$\tau\,\theta x = \tau x\theta$ = shear stress
h = wall thickness

SEISMIC STRESSES. EMPTY TANK CASE

In this case, the seismic force per wall unit of surface is:

$p(x) = \rho \,.\, h \,.\, 1 \,.\, C(x) = q \,.\, c(x)$

where

p (x) = mass density of the wall material
$q = \rho \,.\, h \,.\, 1$

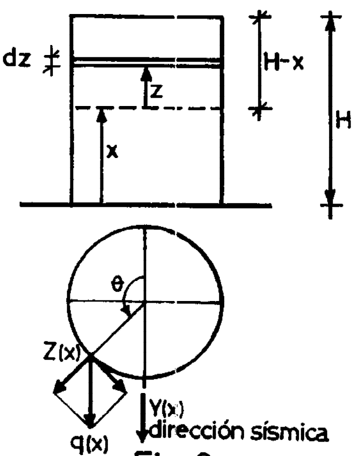

Fig. 3

From Fig. 3. The load components are deduced

$Y = q \,.\, C(x) \sin\theta$
$X = 0$
$Z = q \,.\, C(x) \cos\theta$

It is an asymmetrical load where

$$Nx = \frac{M(x) \,.\, y}{I} = \frac{M(x)\cos\theta}{\pi R^2} \qquad (1)$$

$$N\theta = ZR = -q \,.\, C(x) \,.\, R \,.\, \cos\theta \qquad (2)$$

$$N\theta x = Nx\theta = Nx\theta_1 \,.\, \sin\theta \qquad (3)$$

where $Nx\theta_1$ is only an x function.

On the other hand, it is (Fig.3.)

$$M(x) = A \int_0^{H-x} \psi(x+z) \,.\, z \,.\, dz$$

where

$A = 2 \,.\, \pi \,.\, R \,.\, q \,.\, C(H)$

$$M(x) = A\frac{(\alpha-1)}{H^2}(H-x)^2\left[\frac{x^2}{2} + \frac{(H-x)\,.\,2}{3} + \frac{(H-x)^2}{4}\right] + A\frac{(2-\alpha)}{H}(H-x)\left[\frac{x}{2} + \frac{(H-x)}{3}\right]$$

$$Mmax\,.\,(x=0) = A\left[(\alpha-1)\left(\frac{2H}{3} + \frac{H^2}{4}\right) + (2-\alpha)\frac{H}{3}\right]$$

This expression which has been replaced in (1) allows the determination of Nx . Now, the $Nx\theta = N\theta x$ value has to be determined.

Being

$$Q(x) = A\int_x^H \Psi(x)\,dx = A\left[(\alpha-1)\frac{H}{3} + (2-\alpha)\frac{H}{2} - (\alpha-1)\,.\,\frac{x^3}{3H^2} - (2-\alpha)\frac{x^2}{2H}\right]$$

On the other hand, it is (Fig.4.)

$$Q(x) = R\int_{0}^{2\pi} Nx\theta \sin\theta \, d\theta = R\, Nx\theta_1 \int_{0}^{2\pi} \sin^2\theta \, d\theta = R \,.\, Nx\theta_1 \,.\, \pi$$

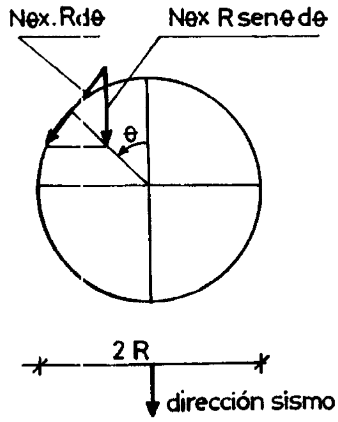

Fig. 4

where

$$Nx\theta_1 = \frac{Q(x)}{\pi R}$$

Then, replacing this value in (3), we get

$$Nx\theta = \frac{Qx}{\pi R}\sin\theta$$

SEISMIC STRESSES. FULL TANK CASE

The load components, X and Y, are null because the pressures are hydrostatic. The Z component is deduced from Fig. 5.

Where

$$Z(\theta,x) = \rho\omega \,.\, 1 \,.\, (R - R\cos\theta) \,.\, \Psi(x) \,.\, C(H) = \rho\omega \,.\, R \,.\, \Psi(x)\, C(H) - \rho\omega \,.\, R \,.\, \Psi(x) \,.\, c(H) \,.\cos\theta =$$

$$= ZA - ZB$$

where

Z A: symmetrical load
Z B : antisymmetrical load

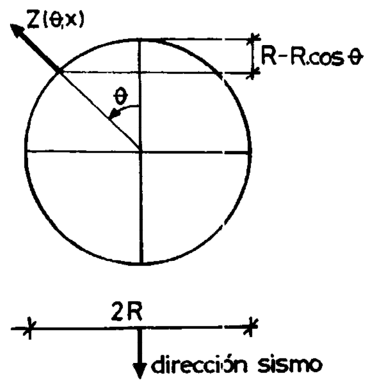

Fig. 5

then

$$P(x) = \rho\omega \,.\, R^2 \,.\, C(H) \,.\Psi(x) \int_{0}^{2\pi} \cos^2\theta \, d\theta =$$

$$\rho\omega \,.\, R^2 \,.\, C(H)\, \Psi(x) \,.\, \pi$$

and

$$M(x) = \int_{0}^{H-x} P(x+z) \,.\, z \,.\, dz = B\int_{0}^{H-x} \Psi(x+z) \,.\, z \,.\, dz$$

where

$$B = \rho\omega \,.\, R^2 \,.\, C(H) \,.\, \pi$$

The value of this integral has already been calculated when dealing with the dead load case.

$$M(x) = B\left(\frac{\alpha - 1}{H^2}\right)(H-x)^2\left[\frac{x^2}{2} + (H-x)\frac{2}{3} + (H-x)^2\frac{1}{4}\right] + B\left(\frac{2-\alpha}{H}\right)(H-x)\left[\frac{x}{2} + \frac{H-x}{3}\right]$$

$$M_{max}(x=0) = B\left[(\alpha - 1)\left(\frac{2H}{3} + \frac{H^2}{4}\right) + (2-\alpha)\frac{H}{3}\right]$$

and

$$Q(x) = \int_{o}^{H-x} P(x+z)\,.\,dz = B\int_{o}^{H-x} \Psi(x+z)\,.\,dz = B\ (\alpha-1)\,.\,\frac{1}{H^2}\left[\ x^2\ (H-x) + x\,(H-x)^2\right] + B\ (2-\alpha)\,\frac{1}{H}$$

$$\left[\frac{x\,.\,(H-x)}{H} + \frac{(H-x)^2}{2}\right]$$

$$Q_{max}\,(x = o) = B\ \left[(\alpha - 1)\,\frac{H}{3} + (2-\alpha)\,\frac{H}{2}\right]$$

Symmetrical load

$$Z_A = \rho\omega\,.\,R\,.\,C\,(H)\,.\,\Psi(x)$$

The stress state is:

$$Nx = 0$$
$$N\theta = -Z_A\,.\,R = -\rho\omega\,.\,R^2\,.\,C(H)\,.\,\Psi(x)$$
$$Nx\theta = 0$$

Antisymmetrical Load

$$Z_B = -\rho\omega\,.\,R\,.\,\Psi(x)\,.\,C\,(H)\,.\,\cos\theta$$

The stress state is

$$Nx = \frac{M\,(X)\,.\,y}{\pi R^3}$$

$$N\theta = -Z_B\,.\,R = \rho\omega\ R^2\,.\,C\,(H)\,.\,\cos\theta\,.\,\Psi(x)$$
$Nx\theta = N\theta x = Nx\,\theta_1\ \sin\theta$, where $Nx\theta_1$ is only an x function

And Q (x) should be:

$$Q(x) = R\int_{o}^{2\pi} Nx\theta\sin\theta\,d\theta = R\,Nx\theta_1\int_{o}^{2\pi}\sin^2\theta\,d\theta = R\,.\,Nx\theta_1\,.\,\pi$$

then

$$Nx\theta_1 = \frac{Q\,(x)}{\pi R}$$

and

$$Nx\theta = \frac{Q\,(x)}{\pi R}\,.\,\sin\theta$$

In sum:

$$Nx = \frac{M\,(x)\,.\,y}{\pi R^3}$$

$$N\theta = \rho\omega\,.\,R^2\,.\,C\,(H)\,.\Psi\,(x)\,(\cos\theta - 1)$$
$$Nx\theta = \frac{Q\,(x)}{\pi R}\,.\,\sin\theta$$

BUCKLING

As the thickness of the steel tanks is very small, it is necessary to take into account this phenomenon. The Caucete Earthquake (1977) in San Juan, Argentina, caused such phenomenon in many cases. It had not been considered when structurally analyzing these tanks.

In order to follow a practical procedure to determine the critical conditions, the formulas showed in "Ciencia de la Construcción" , Volume IV, Odone Belluzzi, were used.

The first one corresponds to the case of the buckling produced by a uniform axial compression. Though it is not exactly the case, it is applicable with enough approximation. The formula is as follows (Fig. 6.)

$$q_{crit} = \sigma x\ crit \,.\, h = 0.605\ \frac{E.h^2\ crit}{R} \quad \text{(metric units)}$$

$$\frac{Mcrit}{\pi \,.\, R^2} = 0.605\ \frac{E\,.\,h^2\ crit}{R}$$

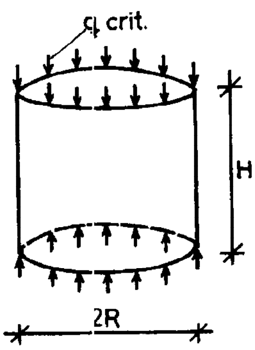

Fig. 6

and

$$h_{crit} = 0.0005\ \sqrt{\frac{Mcrit}{R}}$$

On the other hand, being $M = \frac{Mcrit}{3}$, then

$$h = 0.00087\ \sqrt{\frac{M}{R}}$$

The second formula corresponds to the buckling of the steel tanks which are submitted to flexion. It is as follows (Fig. 7.)

$$Mcrit = 1.219\ \frac{E : R\,.\,h^2}{\sqrt{1 - \nu^2}} = 1.34\ E.\,R\,.\,h^2\ (\nu = 0.3)$$

then

$$hcrit = 0.0006\ \sqrt{\frac{Mcrit}{R}}$$

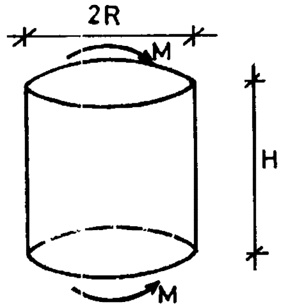

Fig. 7

being $M = \frac{Mcrit}{3}$

$$h = 0.00104\ \sqrt{\frac{M}{R}}$$

Basically, this formula is the one that best approximates to the real one than the previous formula.
From the H. Akiyama's formula (1) it is inferred that this phenomenon depends on the σθ / σp ratio, thus σθ is the hoop stress caused by the liquid - filled shell and the steel fluency stress σp , so

$$\sigma xcrit = \frac{0.8\,E}{\sqrt{3\,(1-\nu^2)}}\ \left(\frac{h}{R}\right)\left(i - \frac{\sigma\theta}{\sigma p}\right) = \frac{N\varphi}{h} = \frac{Mcrit}{h.\,\pi\,.\,R^2}$$

Taking $\nu = 0.3$ and cleaning h, it results in $h = 0.00097 \sqrt{\frac{M}{R(1 - \frac{\sigma x}{\sigma p})}}$

If $\frac{\sigma\theta}{\sigma p} = 0.57$ then the maximun value = $h = 0.00148 \sqrt{\frac{M}{R}}$

If $\frac{\sigma\theta}{\sigma p} = 0$ the minimun value = $h = 0.00097 \sqrt{\frac{M}{R}}$

If $\frac{\sigma\theta}{\sigma p} = 0.3$ the mean value more approximated to the real one is $h = 0.00116 \sqrt{\frac{M}{R}}$

This expression is practically the same as the previous one, and it is the formula that best approximates to the real one.

VIBRATION FUNDAMENTAL PERIOD.

The theoretical formula deduced for the filled and confined liquid foundation embedded tanks is

$$T_t = 0.5596\ H^2 \frac{\sqrt{q}}{\sqrt{E.\ I.g}}$$

and using

$$E = 2.1 \times 10^{10}\ kg/m^2$$
$$g = 9.81\ m/sec$$
$$T = 0.0000023\ (H/R)^2 \frac{\sqrt{q}}{\sqrt{h/R}}$$ (metric units: m , kg.)

where

$$q = \pi R^2 . \rho + 2 . \pi . R . h . \rho\omega$$
h = average thickness of the tank wall
$$I = h \pi R^3$$

On the other hand $Tr = K\ Tt$ where K is a factor which considers the foundation soil. Table I shows the values recommended in Reference (5).

Table I

Soil	Standard Penetration Number of Strikes	Shear Wave Velocity for Supporting Medium m/s	K
A	≥ 30	400 < Vc < 700	1.10
B	≤ 30	100 < Vc < 400	1.30
C	> 10	Vc < 100	1.50

EXPERIMENTAL VALUES OF THE FUNDAMENTAL PERIODS

The record of the forced microvibrations have allowed the determination of the experimental values of the existing full tanks . Table II shows the obtained results.

Table II

Capacity m^3	R. m	H . m	H/R	h Average mm	h/R	Soil	K	Tr. sec.	Tt. sec.	Wine Warehouse
178.	2.865	7.0	2.44	4.45	0.0016	B	1.3	0.090	0.100	Cepas Argentinas
60.	1.875	5.6	2.99	3.6	0.0019	B	1.3	0.061	0.067	Cepas Argentinas
200.	3.	7.m	2.33	5.6	0.0019	C	1.5	0.075	0.070	Melo
150.	2.0	6.7m	3.35	5.6	0.0028	C	1.5	0.096	0.083	Melo
300.	3.3	8.8	2.7	6.0	0.0018	C	1.5	0.130	0.111	Melo

As a conclusion it can be said that the formula to calculate the tank fundamental periods using these results is very reliable.

NOTATION

The following symbols are used in this paper:

- E : Young's Modulus of the tank material
- g : Gravitational acceleration
- H : Height of the tank
- h : Thickness of the tank wall
- I : Moment of inertia of the wall tank cross section
- M(x) : Variable bending moment with x
- P(x) : Variable seismic force with x
- Q(x) : Variable shear force with x
- R : Tank radius
- ν : Poisson's ratio for the tank material
- $\rho\omega$, ρ : Mass densities for the tank material and liquid

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