



TWO AND THREE DIMENSIONAL SCATTERING FROM TOPOGRAPHICAL STRUCTURES AND ALLUVIAL VALLEYS AND COMPARISONS WITH ACCELEROMETRIC DATA: THE CASE OF MEXICO CITY

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ABSTRACT

A direct boundary element method for calculating two and three-dimensional scattering of seismic waves from irregular topographies and buried valleys due to P-, S- and Rayleigh waves has been used to obtain preliminary and qualitative results of the modelling of the valley of Mexico. The method was formulated with isoparametric quadratic boundary elements and has shown to be accurate and efficient. A comparison with the one-dimensional model and the observed amplification using accelerometric data from the city's network is presented. It is shown that the one-dimensional theory can explain most of the observed amplification, but two- and three-dimensional models are needed to explain the amplification behaviour at some sites.

KEY WORDS

Topography, valley, scatter, model, Mexico City, seismic waves, boundary element, transfer function.

INTRODUCTION

Since 1985, the accelerometric network of Mexico City has grown considerably and now provides strong motion data to study the amplification patterns of the valley. Since then, numerical models have been used in order to understand its seismic response. Most of these works compare modelling results with observed data (Bard *et al.*, 1988; Campillo *et al.*, 1988; Sánchez-Sesma *et al.*, 1988, 1993; Kawase and Aki, 1989; Singh and Ordaz, 1993; Fäh *et al.*, 1994; Chávez-García and Bard, 1994). Although the one-dimensional (1D) theory can explain most of the observed amplification in the valley, two- (2D) and three-dimensional (3D) models are needed to explain the amplification behaviour at the edges of the valley and at the sites where the clay deposit is deeper (Reinoso, 1994).

The method employed in this work is the direct boundary element method based on integral equation formulations of the continuum mechanics (Domínguez, 1993). The unknowns and boundary conditions are displacements and tractions which are approximated over the elements from its values at the nodes using interpolation functions. The method is attractive because the discretization is done only on the boundary, yielding smaller meshes and systems of equations, but the system of equations is non-symmetric and fully-populated, leading to longer computer times. This method efficiently represents the outgoing waves through infinite domains, which again is very useful when dealing with scattered waves.

THE BOUNDARY ELEMENT METHOD

The domain under consideration is the three-dimensional half-space below an infinite traction-free boundary with an irregularity (topography or valley). The medium is assumed to be homogeneous, linearly elastic and

isotropic, where the propagation of harmonic P-, S- and Rayleigh waves is described by the Navier-Cauchy integral equations. Total displacements and tractions are obtained with the contribution of the scattered wave due to the presence of the irregularity and the so-called free-field displacement.

Using the method of collocation taking the boundary nodes as collocation points, the Navier-Cauchy equation is discretized and solved. This numerical solution is carried out using Gauss quadrature rules. However, when the integration node is in the same element as the source point the solution become singular and special integration is needed (Domínguez, 1994; Reinoso, 1994). In this work, quadratic triangular and quadrilateral elements have been used. The stress components at any point on the boundary produce surface force intensities or tractions. At the interface between the valley and the half-space, equal displacements and tractions need to be satisfied.

The method was tested against results of analytical and other numerical methods and for a wide range of problems and it proved to be efficient and accurate (Reinoso, 1994). With a few quadratic boundary elements in the irregularity and in the surface of the half-space, the method can be used to obtain reliable results as long as there are at least 4 quadratic elements per wave length. The free-surface was discretized with a finite number of boundary elements introducing a truncation and, therefore, an error that could be minimized if an appropriate length of discretization is chosen. A parametric study showed that the length of discretization is not relevant if the solution is required at the surface of the irregularity or at a site just outside it. Because of this relatively short length needed for the discretization, together with the difficulty to formulate and solve the Green's functions for dynamics for the half-space, no image solution for elastodynamics was employed. For 2D SH-scattering, the Green's functions for the half-space were used.

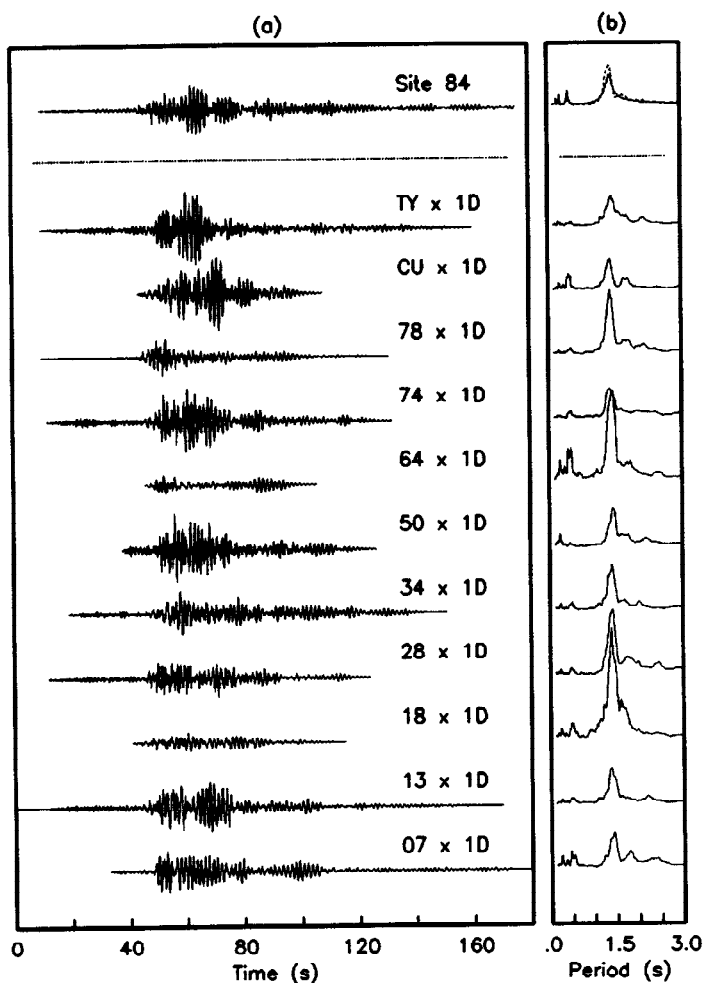


Fig. 1 (a) Record at site 84 (top) and 1D time-responses for 11 hill-zone sites; (b) Average spectral ratio and 1D transfer function (top) and spectral ratios

MODELLING THE RESPONSE OF THE MEXICO CITY VALLEY

One-dimensional model

Because the soft alluvial layer in Mexico City is relatively flat and shallow, it is easy to agree that the one-dimensional theory is enough to reproduce the amplification patterns. Moreover, the high contrast of S-waves between the clays and the basement rock also favours the use of simple, 1D wave propagation models. This model has been the only analytical method actually employed to predict the amplification of the seismic motion at lake-zone sites. This was the case of the design spectra included in the 1987 seismic code that was obtained with 1D results (Rosenblueth *et al.*, 1988) and it has been successfully used to reproduce average ground motion characteristics for Mexico City (Romo and Seed, 1986; Seed *et al.*, 1988).

As a typical example of 1D behaviour we have chosen site 84. With a simple 1D geometry (Reinoso, 1994), the time-response at site 84 was computed with data of the east-west component of motion recorded during the 1989 earthquake using all eleven hill-zone accelerometric stations as reference sites. The comparison with the recorded accelerogram at site 84 is shown in Fig. 1a, while the right part (Fig. 1b) shows the spectral ratio between the respective hill zone sites and site 84; at the top of Fig. 1b, the average spectral ratio (dotted line) and the 1D transfer function (solid line) are also shown. Although important differences can be seen for each

reference station in both frequency and time domains, site 84 exhibits 1D response because the average ratio is practically identical to the 1D transfer function. All the differences observed for these hill-zone sites could be attributed to site effects and to random or noise contributions which have been difficult to evaluate and that disappear when calculating the average amplification. These results help also to illustrate the different amplification that one can get by choosing any reference site; results may vary significantly and comparisons and conclusions have to be presented carefully. From the previous results, it is clear that, in average, the 1D model can reproduce with good accuracy the amplification patterns observed at site 84 for a moderate earthquake. But it is also clear that, if we isolate some of these results, they can be used to argue against the efficiency of the 1D theory just by stressing the observed differences. Comparisons with data from other earthquakes (Reinoso, 1994) show that the azimuth does not affect the response at this site, an amplification pattern which is valid only, in general, for 1D behaviour.

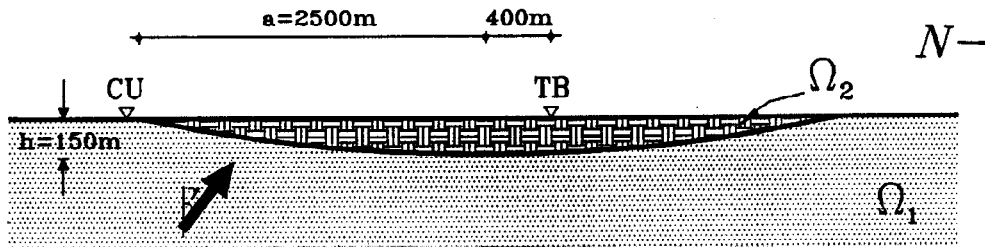


Fig. 2 Two-dimensional parabolic valley used to model site TB

Two-dimensional modelling

Although most accelerometric sites exhibit an amplification pattern rather similar to that predicted by the 1D theory, there are some localized zones where topographic effects seem to be important (Reinoso, 1991; 1994). This can be observed at some edges of the lake-bed zone and at those zones where the clay deposit is thicker. Site TB was chosen as an example of possible 2D effects because of the irregular amplification observed there (spectral ratios with respect to the reference sites are very different so the computation of the average ratio presents large standard deviations), and also because it was easy to propose there a 2D geometry with rough characteristics of the actual valley.

Site TB is at the centre of the alluvial deposit over a stratum of more than 100m of soft clay. The geometry used to represent it was a 5 km wide and 150 m deep parabolic valley (Fig. 2). This geometry was chosen after trying with several 2D flat valleys with different slopes at the borders that yielded responses at the centre very similar to the 1D response. The velocity and mass density ratios between the half-space and the valley were 13 and 2, respectively, the damping and Poisson's ratio of the material inside the valley were 2% and 0.495 whereas damping, Poisson's ratio and S-wave velocity of the bed-rock were 1%, 1/3 and 1300 m/s^2 , respectively. Damping or viscous decay in the scattering wave was considered using complex wavenumbers. The 1D response of this site was computed using the same properties considering a flat valley 145 m deep.

Reinoso *et al.* (1993), obtained transfer functions from this model for incident SH-waves. In order to compare the numerical results with the recorded data, site CU was chosen as reference site. Although accurate results in the time-domain were not expected, this site was selected because the shape of the spectral ratio was the closest one to the 1D response at TB. It was shown (Fig. 3) that the 2D results can reproduce some of the amplification patterns that the 1D theory is not able to predict. These results show clearly that the predictions of a rough 2D model were far closer to the observations than those of the 1D model. At least for the geometry studied, the angle of incident SH-waves did not seem to be a relevant factor in the response of the valley. It was also concluded that time domain simulations for sites in Mexico City with large dominant periods ($T > 2.0$ s) and therefore long responses, are still difficult to match with observations owing to the short duration registered at the reference sites. It is believed that two-dimensional (and three-dimensional) effects contribute significantly to the large duration of the records. But it is also true, as was pointed out by Singh and Ordaz (1992), that the most important part of the duration can be explained by the 1D theory, provided that the register at the reference site is as long as that at the lake-zone site (Fig. 1).

Using the 2D boundary element formulation for elastodynamics, the same configuration was modelled but now focused on the NS component of motion (Reinoso, 1994). Transfer functions for incident P-, SV- and Rayleigh waves were obtained on the surface of the valley at the observation point TB. Figures 4 show, from left to right, these transfer functions for 30° incidence of P- and SV-waves and for Rayleigh waves, the spectral ratio between TB and CU (obtained with the north-south components of the 1989 earthquake) and

the 1D response for vertically incident SV-waves of the site TB computed with the same properties of the valley at the corresponding depth of 149m.

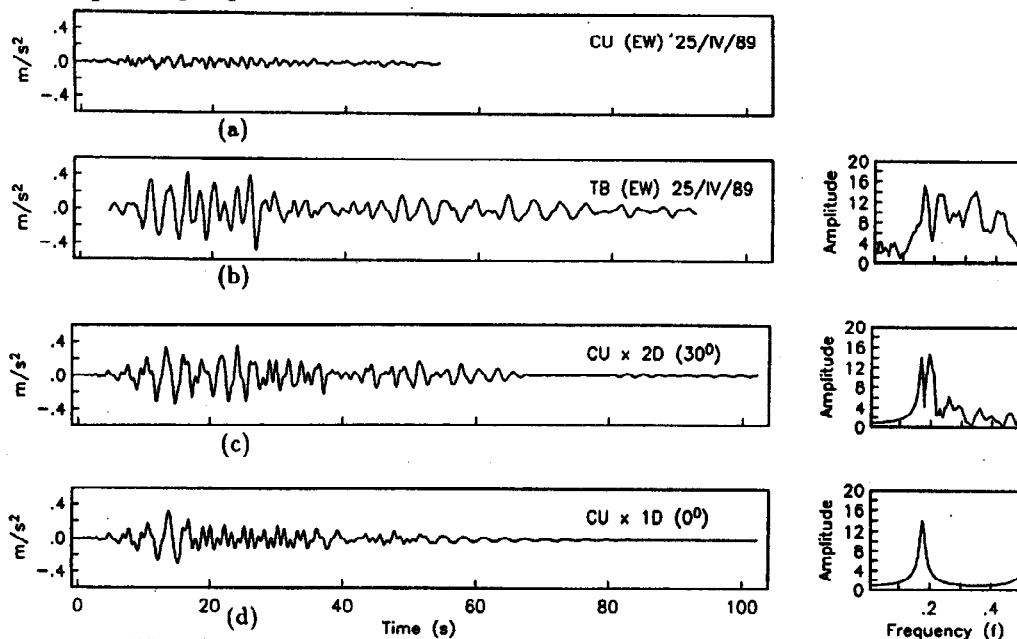


Fig. 3 Observed motion at sites CU (a) and TB (b) during the 1989 earthquake and 2D (c) and 1D (d) responses. The right part of the figure shows the spectral ratio between TB and CU and the 2D and 1D transfer functions (taken from Reinoso *et al.*, 1993)

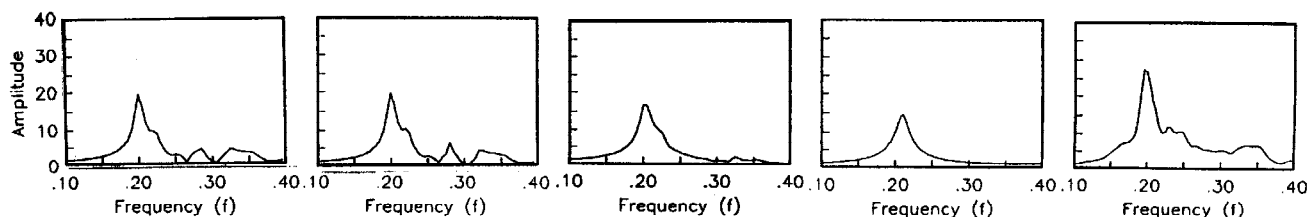


Fig. 4 From left to right: transfer functions for P-, SV- and Rayleigh waves, spectral ratio between TB and CU and 1D transfer function

As can be appreciated in these figures, 2D response for P- and SV-waves is closer to the spectral ratio than the 1D response; however, the ratio has larger amplitude than the 2D responses. The response due to incident Rayleigh wave is very similar to the 1D response. It is possible to say that the proposed geometry of the valley yields a response that is close to the one observed at site TB during the 1989 earthquake using CU as a reference site. However, these results have mainly a qualitative significance, and show that site TB has been exhibiting 2D, and possibly, 3D effects. Time-response using a Ricker's wavelet shown in Fig. 5a corresponds to a P-wavelet with 5s period and 75° incidence, while Fig. 5b corresponds to a SV-wavelet of 5s and 30° incidence. From Fig. 5 it can be observed that the amplification pattern is quite similar for both incident waves. The reason for this could be that this amplification is governed by the same P-S conversion for all types of waves and incidences, which implies that the valley, or at least the geometry studied, is not sensitive to these changes. 1D time-response (Reinoso, 1994) predicts large amplification in both amplitude and duration, although 2D responses show latter arrivals that make the coda slightly larger than the 1D results.

Preliminary 3D modelling

Considering the actual dimensions and properties of the Mexico City Valley, the number of nodes and elements required is still out of the capacity of most of available computers. If we are to obtain reliable results for frequencies as high as 1.0 Hz, with 4 isoparametric elements per wave-length, at least 2 million nodes and 500 thousand boundary elements are needed. Some supercomputers with parallel procesors are capable of solving this problem, but they are expensive and special programming is needed.

Qualitative results for a 3D model of the valley are presented. An elliptical shape in the horizontal and in both vertical planes has been adopted. The material properties of the model are the same as those used previously. The valley is 500 m wide, in the x-direction, and 50 m deep. Four dimensions in the z-direction were used: 2500, 1500, 1000 and 500 m, the last one corresponding to a circular valley in the x-z plane. The frequency

chosen for the computations was 0.5 Hz. The results show that for P-, SV- and Rayleigh waves, the response does not vary significantly for the types of valleys employed. But for the SH-waves the variation is remarkable. The maximum amplitudes (Figure 6) at the centre of the valley are 12, 24, 34 and 48 for the 4 valleys chosen, but the 2D solution yields a maximum amplitude of 83 (continuous line). This means that for this geometry and frequency, 3D effects are significant only for SH-waves. Only one incidence is shown because no difference was noticed for several incident angles.

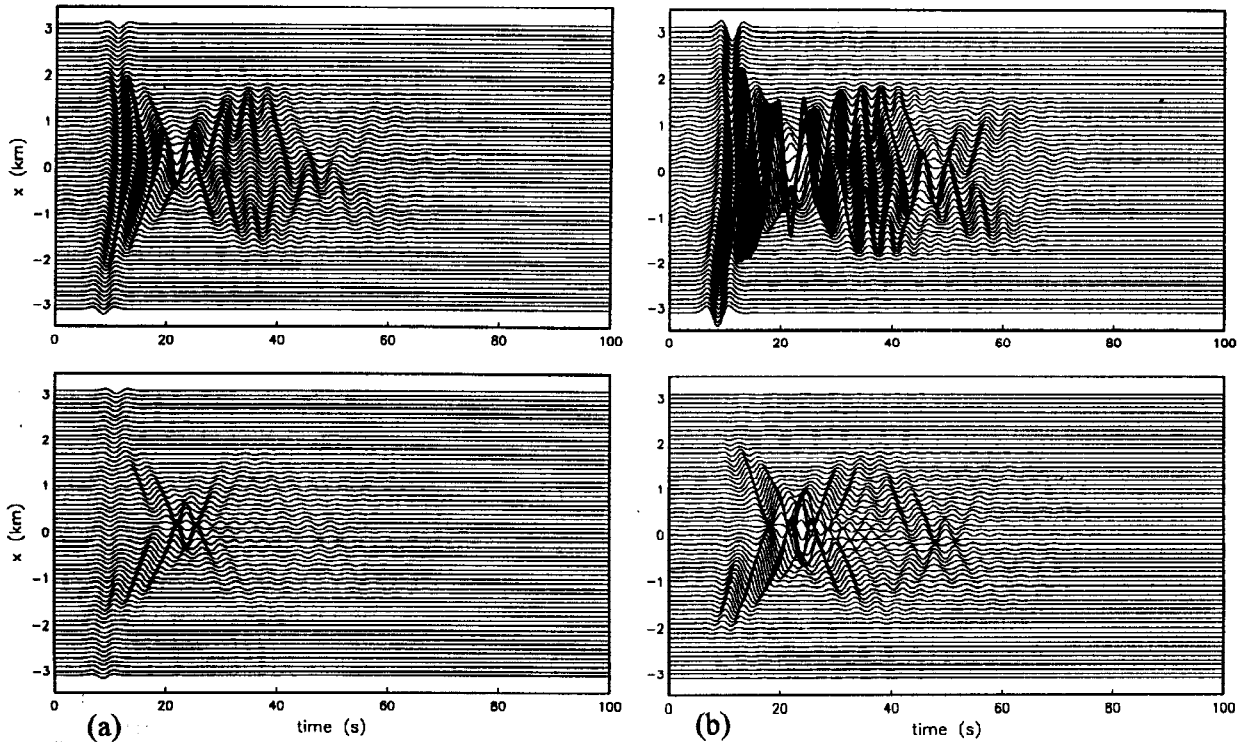


Fig. 5 Time-responses using a Ricker's wavelet for (a) P- and (b) SV-waves

SCATTERING BY MOUNTAINS

In order to obtain qualitative results of the amplification at hill zone sites, two types of topographies were used. Type I topography is a dome-shaped mountain, with a circular base of radius equal to 250 m and an elevation of 200 m. Type II topography is a horse-shoe-shaped mountain with similar dimensions as the dome but with a smooth contact between the irregularity and the half-space. Approximately 100 quadratic boundary elements and 400 nodes were used for each discretization, and the properties of the material were chosen as follows: S-wave velocity of 1500 m/s, density of $1.5 T/m^3$ and 0.4 for Poisson's ratio.

Figure 7 shows the displacements obtained for different points at the surface of the Type I topography for vertical SV-wave incidence. At low frequency, displacements obtained far from the irregularity tend to be the same as those obtained for the free-field motion ($u=2.1$, $v=0$, $w=0.4$). For large frequencies, the presence of the irregularity affects the response at almost every point, being more dramatic at the edges of the dome (sites 11 and 28). For most cases, the amplification at the center is twice larger while at the edge it is more than three times the free-field motion; v -displacements produced by the 3D scattering from the walls could be as large as u -displacements.

Figure 8 shows the response at 2.5 Hz for the three components of displacement due to incident Rayleigh waves (from left to right, u , v and w , respectively). Two- and three-dimensional plots are shown in order to compare and locate where the amplifications and deamplifications occur. In the 2D plots, the shape of the dome is shown with thick line. Amplitudes for the 3D plots have been normalized for all figures in order to compare qualitatively the results. Figure 8 show the time-response of the dome to a Ricker's wavelet with a

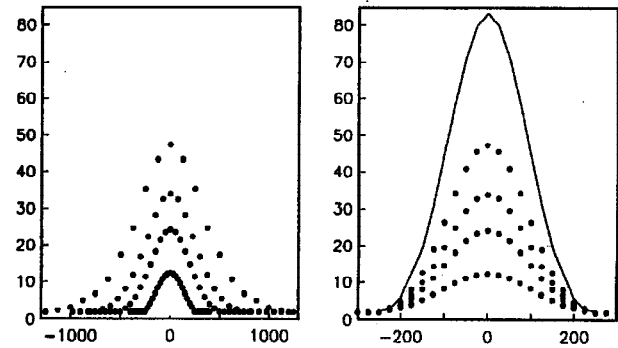


Fig. 6 3D SH-response of a valley 500 m wide and 50 m deep. Results are presented for the z - (left) and x -axis (right)

frequency of 2.0 Hz for the u -displacements. Comparing with the size of the incident wavelet shown in the figure, the amplification caused by the dome is very large. The size and complexity of the computed wavelet is an evidence of the important amplification patterns of this topography. Type II topography is a full 3D irregularity. Because the edge of the irregularity is smooth, large amplifications were located inside the mountain and not at the edges as for type I topography. For vertical incidence of SH-waves, Fig. 9 shows that the sites with maximum but little amplification are those at the centre of the mountain (15, 16, 21 and 22) and that important generation of displacements in the u - and v -components can be observed at sites 10 and 15-17.

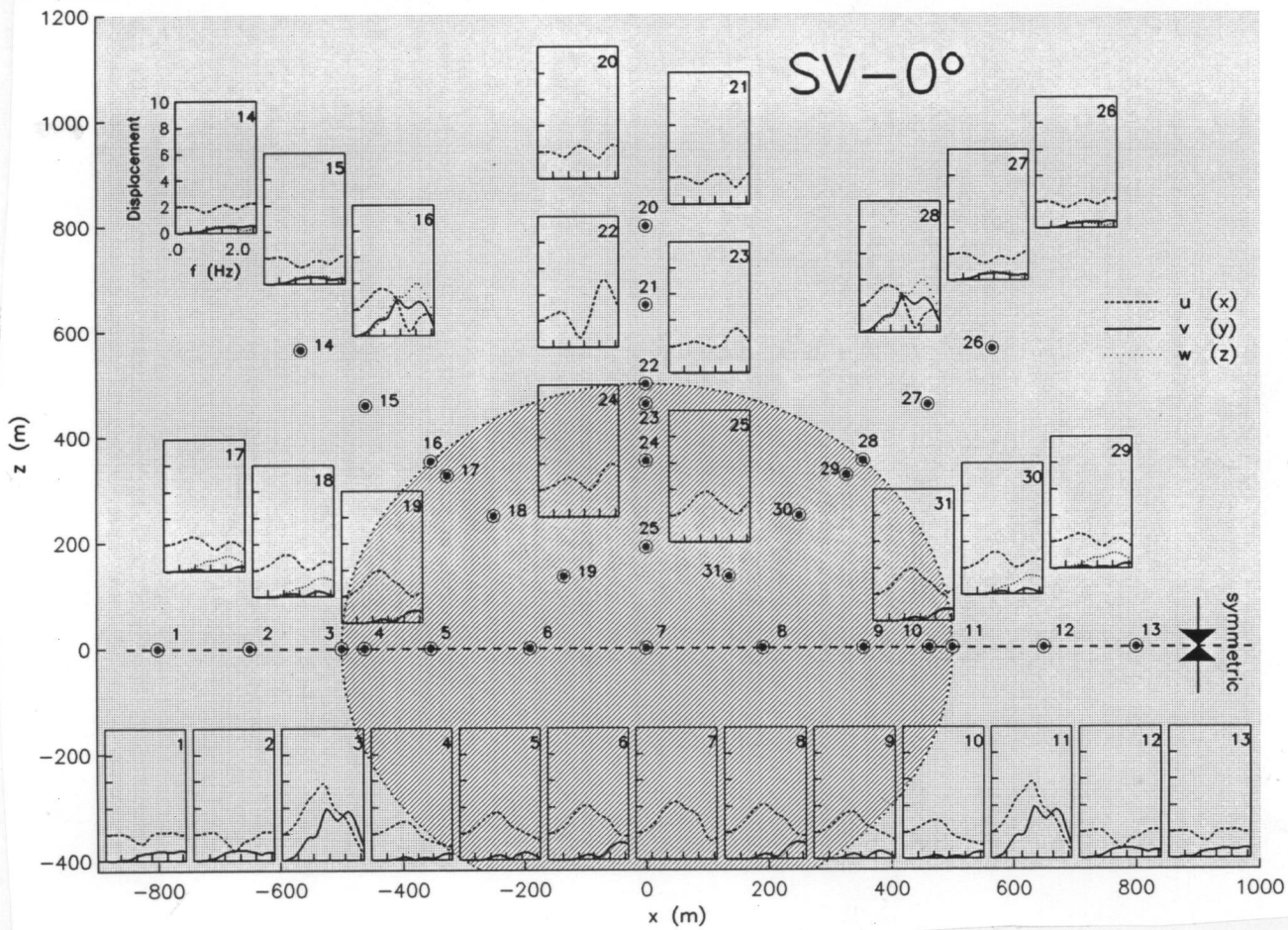


Fig. 7 Displacements at the surface of a 3D dome-shaped mountain for vertical incidence of SV-waves

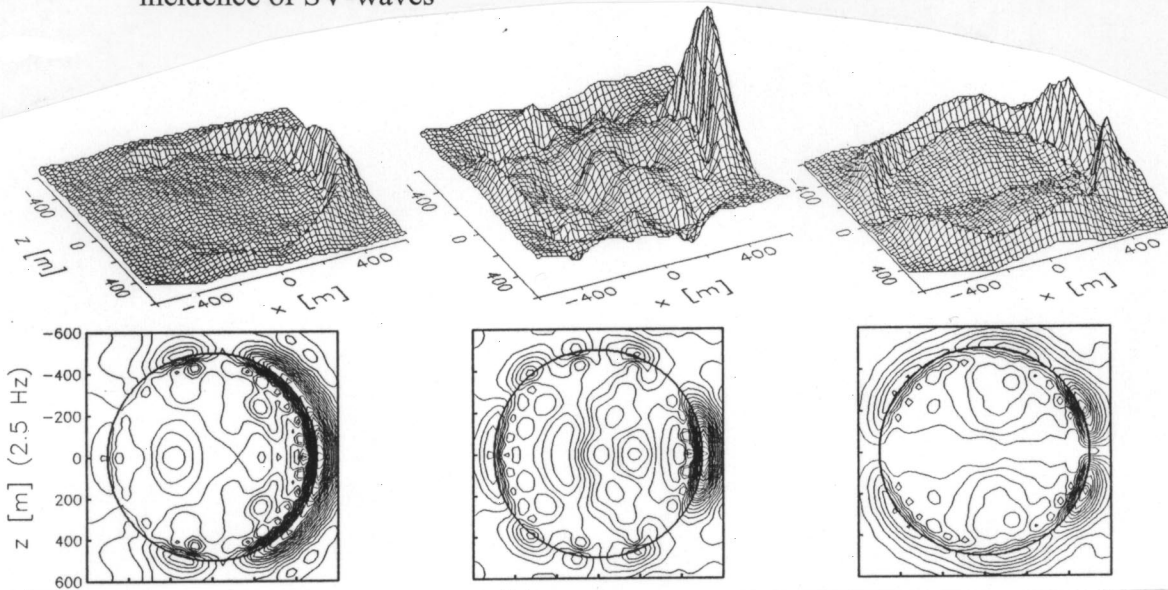


Fig. 8 Response at 2.5 Hz for the three components of displacement due to incident Rayleigh waves (from left to right, u , v and w , respectively)

Figure 11 shows smoothed Fourier spectra for eleven hill zone sites in Mexico City during the 1989 earthquake. It is clear that the shape of all these spectra is similar, but there are important differences for some sites. As has been shown in Figs. 7-10 some of these differences could be due to topography effects.

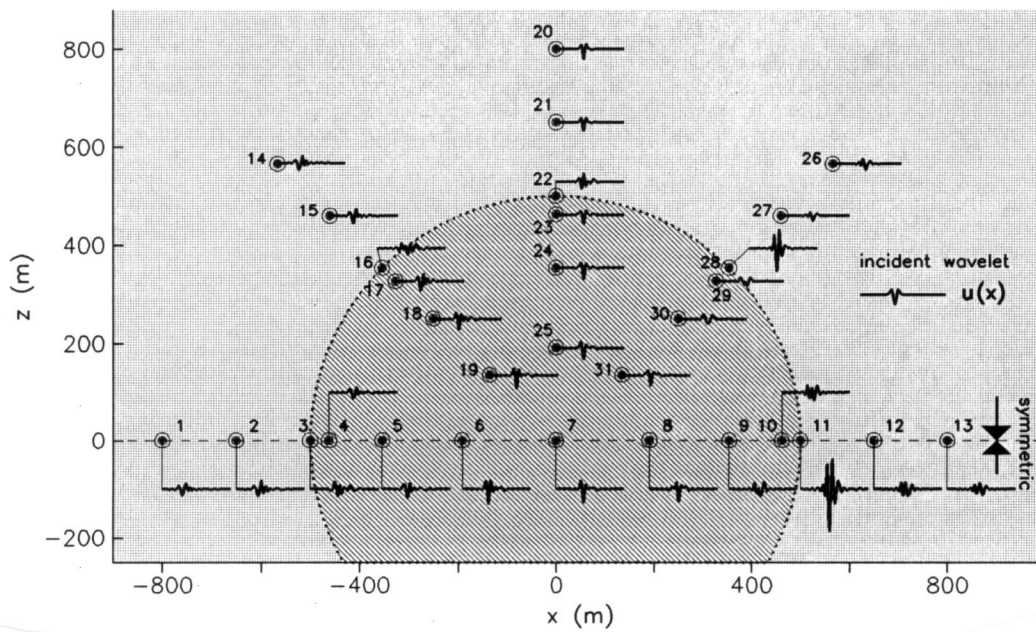


Fig. 9 Time-response of the dome to a wavelet of 2.0 Hz (u -displacements)

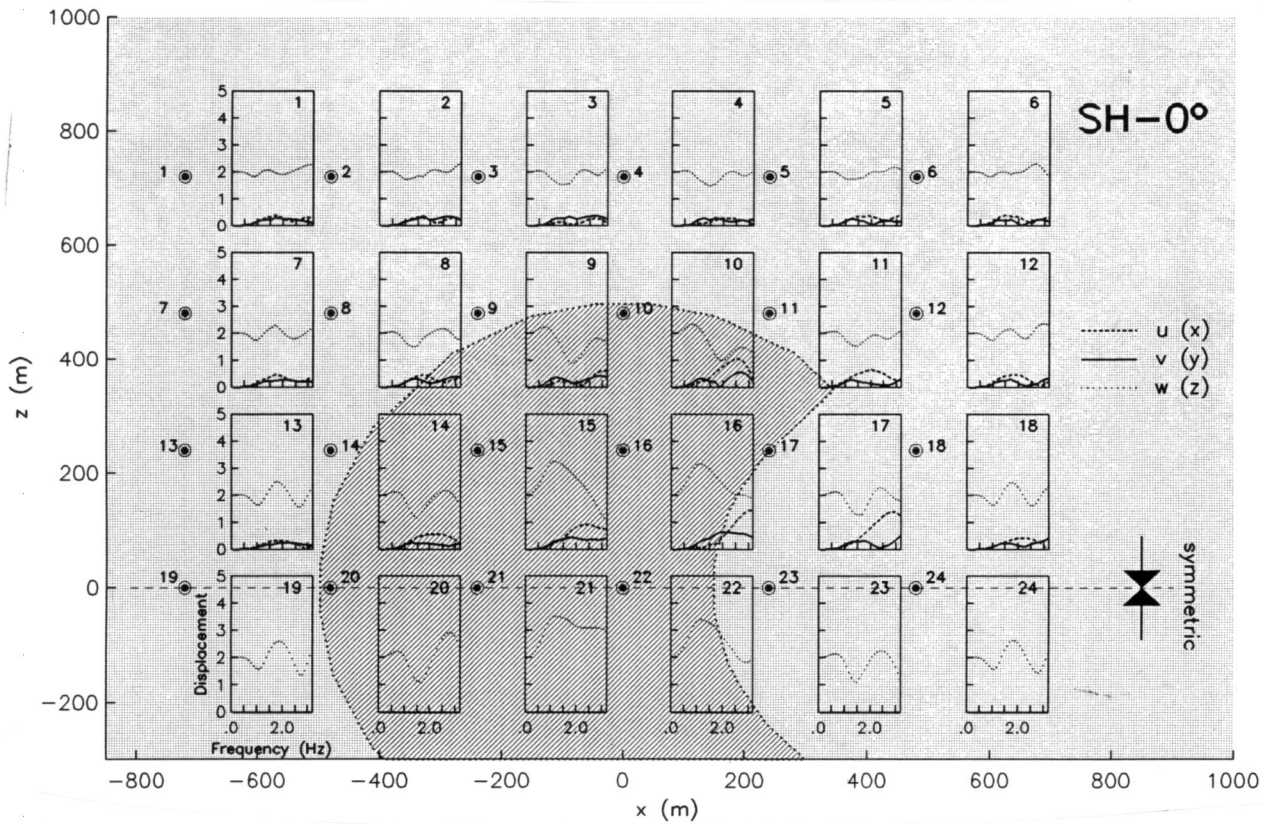


Fig. 10 Displacements at the surface of a 3D horse shoe-shaped mountain for vertical incidence of SH-waves

CONCLUSIONS

A qualitative comparison with the observed amplification at the valley of Mexico using accelerometric data from the city's network was presented. It was shown that the 1D theory, in average, can explain the observed amplification at some sites in the city, but two- and three-dimensional models are needed to explain the amplification behaviour at the borders and at those sites where the clay deposit is thicker.

A 2D direct boundary element method was used to model a north-south section of the valley where irregular response has been recorded during earthquakes. 2D results are closer to the observations than the 1D results. The amplification pattern observed for P-, SV- and Rayleigh waves was fairly similar, showing that probably the same mechanism of amplification is presented for all waves. The 3D method was used to model two types of topographies with dimensions and material properties similar to real mountains. Comparing both results,

the displacements computed for the topography with vertical walls are larger and located in or near the edges, but the response of the type II topography is far more complex. The amplification corresponding to topographies with vertical walls could be as high as 20 for the vertical component and 4 for any horizontal component while in mountains with smooth slopes the amplification can reach up to three times the motion observed at the free-field, but just for few incidences and frequencies.

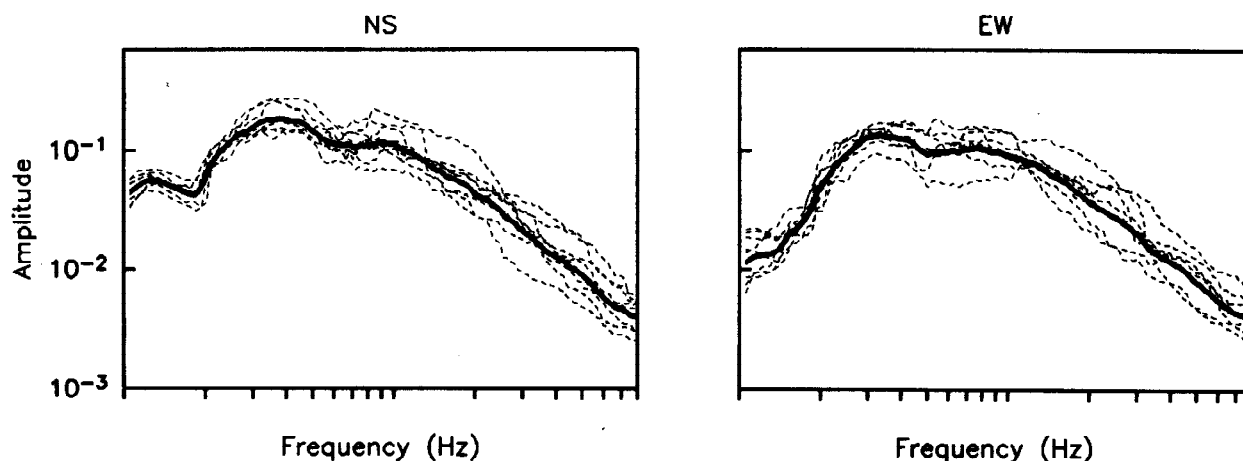


Fig. 11 Fourier spectra at hill zone sites during the 1989 earthquake

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