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CONDITIONAL SIMULATION OF STOCHASTIC WAVES BY USING KALMAN FILTER AND KRIGING TECHNIQUES

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ABSTRACT

We have developed a hybrid method of Kalman filter and kriging techniques with which the stochastic waves in a time-space random field are simulated sequentially by adopting observed waves. Many methods have been proposed for the simulation of stochastic waves involving deterministic time functions. The proposed procedure here simulates waves at arbitrary points in accordance with the observed waves specified at one or more points. The developed method is an on-line hybrid estimation technique which satisfies the given covariance function of both time and space field. We have theoretically shown that the simulated waves satisfy the characteristics of the given random field.

KEYWORDS

Conditional simulation; Kalman filter; kriging; stochastic wave; real time; on-line

INTRODUCTION

Seismic ground motions are a function of time and space. The fundamental step for assessing the scale of disaster or the damage to structures in an area is to estimate the seismic ground motions. The procedure available for determining the spatial distribution of input motions in an area are classified into three methods. The first uses observed records directly as possible invariant realizations in the area. This method is inadequate when the number of observed data are limited. The second is to interpolate the available data and estimate the optimal values by introducing a mathematical model of random field. The third is the conditional simulations, which simulate sample fields conditioned by observed data at discrete points. Many discussions and studies have been published, the results are very useful for the construction of disaster prevention strategy.

Recently major earthquake disasters occurred in large urban areas in the USA and JAPAN. Those are the 1994 Northridge earthquake and the 1995 Southern Hyogo earthquake. Many countermeasures against earthquake disasters were planned and implemented. As the disaster is the outcome in the of human social system into which the earthquake affects, we have to know input ground motions first. A seismic input evaluation system is an example. An adequate number of seismograms in the area are arranged and the spatial distribution of seismic severity is immediately evaluated, the purpose of which is different from the array observations installed so far. This is a subsystem of the earthquake information system for emergency response just after the earthquake.

In Japan, the Japan Meteorological Agency has increased its seismographs in accordance with the change of the definition of seismic intensities. It changed the measuring process from an experimental way to mechanical measurement. The Science Technology Agency has also decided to install a thousand of seismographs and to make the network widespread all over the nation. The information obtained from the network is collected and disseminated by a management center. Although the seismic observation networks are arranged more densely than in the past, the number of installed equipment is not enough when we evaluate the spatial distribution of seismic ground motions especially for small municipalities. In order to cover information of the area with less observations, we have to develop a method to estimate the ground motion at an arbitrary point in the area.

Many methods from a simple interpolation to a probabilistic approach are proposed in order to estimate the ground motion. Conditional simulation, which simulates waves at an arbitrary points in accordance with the observation waves, is one of the probabilistic approaches and a useful tool to provide important information. In this method, the deterministic time functions are involved in simulated waves as realizations of a random field. Kawakami (1989) proposed the method to simulate conditional waves which satisfy the given correlation function by using the double Fourier series expansion and optimization techniques. He also proposed the methods according to the multi-variable stochastic process and multi-variable AR model (1992, 1994). Kameda *et al.* (1992) developed analytical conditional simulation technique based on the conditional density function for the Fourier coefficients in frequency domain. Kriging technique is applied in order to incorporate the observations with the simple manner. Suzuki (1988), Kiyono *et al.* (1989) applied this technique to the spatial interpolation of the soil properties and physical factors of ground motion in the field of soil mechanics and earthquake engineering. Vanmark *et al.* (1991) interpolated the time histories of earthquake array observation by the use of kriging and Monte Carlo simulation techniques. Kiyono *et al.* (1992, 1995a) simulated the waves of inhomogeneous field such as the ground with irregular interfaces. Hoshiya (1993) investigated a conditional simulation method which has a step by step expansion procedure. Maruyama *et al.* (1993) dealt with the wave propagation problem according to above Hoshiya's formulation and simulated the sample waves of the conditional random field. Noda *et al.* (1994) has extended the conditional estimation problem for the Gaussian random field to that for non-Gaussian random field.

The purpose of this study is to simulate the conditional stochastic waves at an arbitrary point which satisfies the characteristics of the given stochastic field. The kriging and Kalman filter techniques are adopted because the estimation error covariances for both techniques are identical. According to the developed procedure, we can carry out the on-line estimation of stochastic waves. The wave at an arbitrary point can be calculated as soon as the waves are observed at the other fixed points. The Kalman filter technique has a time update algorithm and a measurement update algorithm, but the kriging technique does not have the time update algorithm. We here developed a hybrid method in which the kriging equation is incorporated into the state transition process of the Kalman filter. The correlation function is derived from a linear differential equation and has an exponential form. The important aspect of the developed procedure is that the real-time estimations of the waves become possible because the estimates are obtained sequentially as the algorithm is executed in every time step.

KALMAN FILTER AND KRIGING

Kalman Filter

Kalman filter technique, a method that updates the estimated value at each time step by incorporating new observation data, is used in this procedure. As the Kalman filter processes data sequentially, based on orthogonal projection and minimum error criteria, it needs the dynamic characteristics of a linear filter, the stochastic characteristics of noise, a priori information about the initial values, and observed data given sequentially.

Consider the signal generation and observation processes written as a discrete linear system;

$$\mathbf{u}_{k+1} = \mathbf{\Phi}_{k+1|k}\mathbf{u}_k + \mathbf{G}_k\mathbf{w}_k \tag{1}$$

$$\mathbf{y}_k = \mathbf{M}_k\mathbf{u}_k + \mathbf{v}_k \tag{2}$$

in which $\mathbf{u}_k$ is the state vector at $k\Delta t$ (Δt the sampling interval), $\mathbf{y}_k$ the observation vector at k; **F** the state transition matrix; **M** the measurement matrix; and **G** the system noise matrix. **w** and **v** are the Gaussian system noise and observation noise vectors with zero mean values

$$E\left\{\begin{pmatrix} \mathbf{w}_m \\ \mathbf{v}_n \end{pmatrix}\begin{pmatrix} \mathbf{w}_m^T & \mathbf{v}_m^T \end{pmatrix}\right\} = \begin{bmatrix} \mathbf{R}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_n \end{bmatrix}\delta_{mn} \tag{3}$$

in which E represents the expectation and δ_{mn} Kronecker's delta; $\mathbf{R}_m$ and $\mathbf{S}_n$ the symmetric and positive definite matrices. If the governing equations are transformed to Eqs. (1) and (2) by the appropriate modeling, the problem can be solved by use of the Kalman filter algorithm.

Kriging

When we deal with a time-series process (stochastic process), it often is assumed to be stationary and ergodic. In addition, the mean often is assumed to be zero, and measurements are numerous and equi-spaced in time. If the spatial random field is assumed to be stationary, we can estimate its value at an arbitrary point using the mean and the spatial covariance function.

The realization of random field, z(x), is idealized as the sum of two basic contributions; a deterministic component representing the expectation of z(x), m(x), and a stochastic component with zero mean, u(x).

$$z(x) = m(x) + u(x) \tag{4}$$

in which x is a position vector. In case of considering a problem for the seismic ground motion, the expectation of z(x) is assumed to be zero. Thus the observed value at a fixed point x_i ($i=1, \cdots, n$) is $u(x_i)$. When the observed value, $u(x_i)$, is given, the linear estimator, $u^*(x_s)$, at a fixed but arbitrary point, x_s, is considered to be a weighted average of n available data.

$$u^*(x) = \sum_{i=1}^{n} \lambda_i u(x_i) \tag{5}$$

in which λ_i ($i=1,2,\ldots,n$) is the weighted coefficient which satisfies the equation, $\Sigma\lambda_i=1$.

Calculating λ_i by minimizing the estimation error covariance and substituting it into Eq.(5), we get a linear optimal estimation, $u^*(x_s)$. The estimation error is written as

$$\sigma^2(x_s) = E[u^2(x_s)] - \sum_{i=1}^{n} \lambda_i C[u(x_s), u(x_i)] \tag{6}$$

In the procedure, the estimation error becomes zero when the estimated point, x_s, coincides with the sample point, x_i. The optimal estimator, therefore, exactly coincides with the observed value.

Kalman Filter and Kriging

Consider a state vector, **u**, without a trend component (m(x) in Eq.[4])

$$\mathbf{u} = \{\mathbf{u}^T(x_i), \mathbf{u}^T(x_s)\}^T \tag{7}$$

in which $\mathbf{u}(x_i)$ is the state vector at an observation point, x_i, and $\mathbf{u}(x_s)$ the state vector of an estimated point, x_s. In the kriging technique used here, the mean value of which is assumed to be known (=0). The estimation error covariance matrix is

$$E[\{\mathbf{u}-\mathbf{u}^*\}\{\mathbf{u}-\mathbf{u}^*\}^T] = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{bmatrix} \tag{8}$$

in which

$$\mathbf{P} = \mathbf{Q}_{ss} - \mathbf{Q}_{si}\mathbf{Q}_{ii}^{-1}\mathbf{Q}_{is} \tag{9}$$

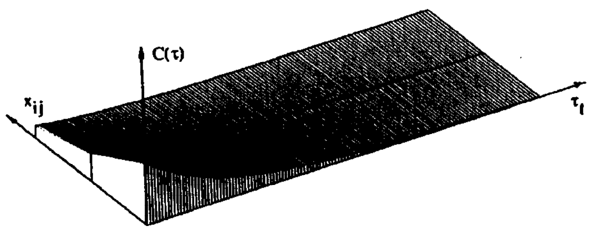


Fig. 1. Covariance function assumed. X-axis is the time difference and y-axis the distance between two points.

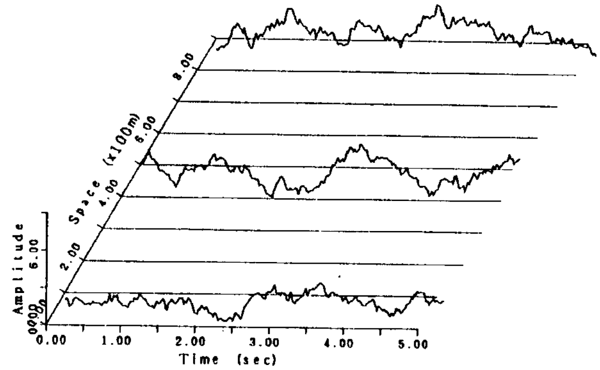


Fig. 2. Assumed observed waves which satisfy the given covariance function.

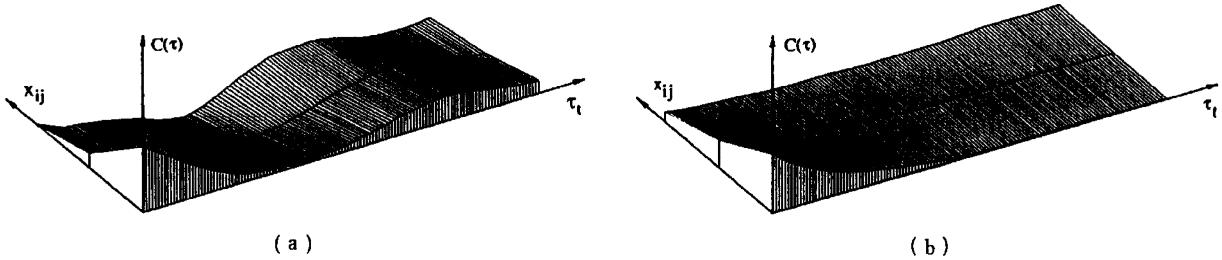


Fig. 3. (a) Covariance function of a sample wave
(b) Ensemble average of covariance function (20 waves)

and where $\mathbf{Q}_{ii}$, $\mathbf{Q}_{is}$, $\mathbf{Q}_{si}$ and $\mathbf{Q}_{ss}$ are the covariance matrices for $\mathbf{u}(x_i)$ and $\mathbf{u}(x_s)$. This covariance matrix coincides with that of measurement update algorithm in Kalman filter.

$$
\begin{aligned}
\mathbf{Q}_{ss} &= \mathbf{P}_{k+1|k} \\
\mathbf{Q}_{si} &= \mathbf{P}_{k+1|k}\mathbf{M}_{k+1}^T \\
\mathbf{Q}_{is} &= \mathbf{M}_{k+1}\mathbf{P}_{k+1|k} \\
\mathbf{Q}_{ii} &= \mathbf{M}_{k+1}\mathbf{P}_{k+1|k}\mathbf{M}_{k+1}^T + \mathbf{S}_{k+1}
\end{aligned} \tag{10}
$$

This means, if we incorporate the estimated unknown values into the state vector in Eq.(7) and calculate estimation error covariance, the calculated error covariance coincides with the kriging error covariance.

CONDITIONAL SIMULATION OF STOCHASTIC FIELD

Stochastic Waves

Suppose the stochastic differential equation;

$$\dot{u} = au + b\gamma \tag{11}$$

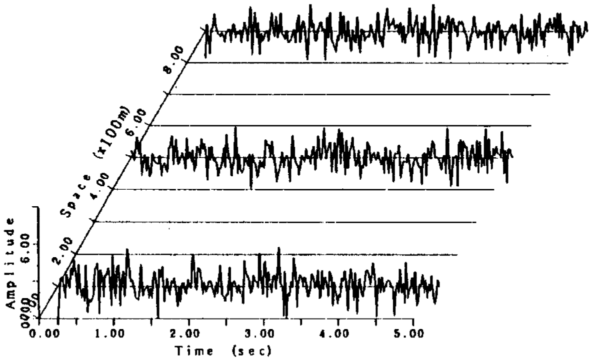


Fig. 4. Estimated input white noise

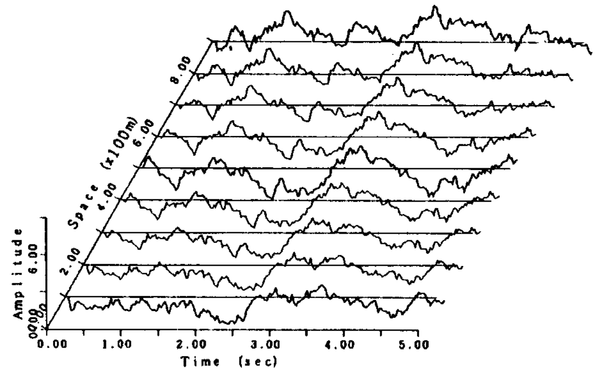


Fig. 5. Interpolated waves by kriging technique.

in which u is a state variable that is driven by the white noise, γ, coefficients a and b are the parameters which govern the stochastic field. The discretized equation of Eq. (11) is

$$u_k = \Phi u_{k-1} + b\gamma_{k-1}\sqrt{\Delta t} \tag{12}$$

The state variable which satisfies the above equation has the following covariance function

$$C(\tau) = -\frac{b^2}{2a} e^{a\tau} \tag{13}$$

As we consider the covariance function which decreases as the time difference and the distance between two points increase, the corresponding exponential function is given as

$$\tau = \tau_t + \tau_{ij} \tag{14}$$

$$\tau_{ij} = \frac{x_{ij}}{v} \tag{15}$$

in which v_0 is the velocity and its value is assumed 1000m/sec. In this study, the parameters, a and b in Eq. (11), which govern the stochastic field is known, and their values are -2 and 2 respectively. The covariance function for Eq.(13) is shown in Fig. 1 X-axis denotes the time lag, τ_t, and y-axis the distance between the point i and j, x_{ij}. The covariance functions are plotted for a time lag range of $0\text{sec} \le \tau \le 5\text{sec}$ and a distance range of $0\text{m} \le x_0 \le 800\text{m}$. The function is decreasing exponentially along both axes. Sample waves that satisfy the given time-space correlation (Eqs. [13], [14] and [15]) is simulated by using Eq. (12) (Fig. 2). We assume these sample waves as observed waves for the simulation of stochastic waves. In Fig. 2, the observed waves are given in three points (100, 500 and 900m) and the others (200, 300, 400, 600, 700 and 800m) are of the estimated points.

The covariance function for sample waves and the ensemble average of covariance function for 20 sample waves are shown in Fig. 3 (a) and (b). The ensemble average approximates the assumed covariance function. The input white noise, γ, in Eq.(11) is generally unknown. But if the parameters which control a random field are known, we can identify the input motion. We calculated the input white noise by using the Kalman error filter (Toki *et al.*, 1992). The result is shown in Fig. 4

Hybrid Method of Kalman Filter and Kriging Techniques

Interpolated waves by the kriging technique are shown in Fig. 5. The value at t+1 for the estimated point is computed by the linear combinations of the values of the observation points at t. As the estimator is provided

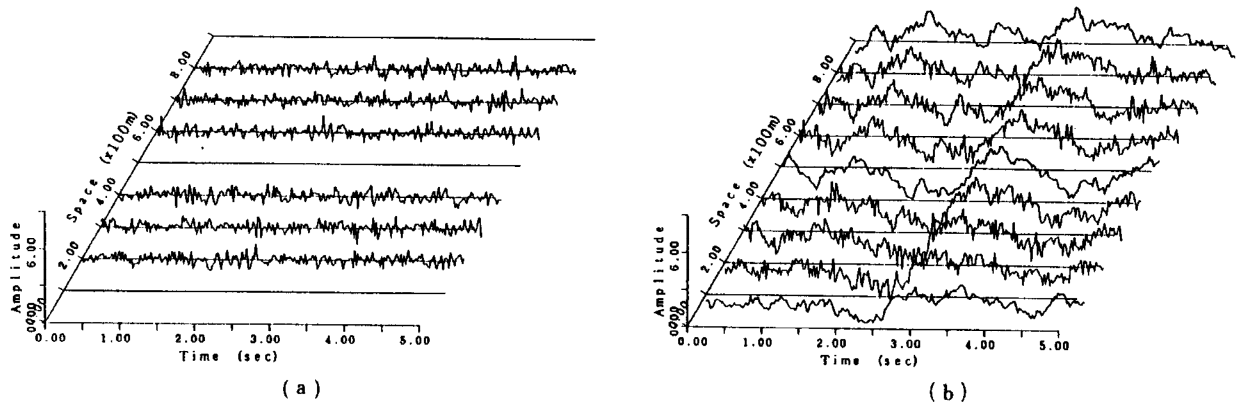


Fig. 6. (a) Waves obtained by factorizing the estimation error covariance
(b) Synthesized waves by adding the waves in (a) to the kriged waves

according to the criteria of minimizing the estimation variance, the kriged waves do not always have the characteristics of the given stochastic field.

Consider the state and the measurement equations of Kalman filter

$$\mathbf{u}_k = \begin{bmatrix}\Phi & \mathbf{0}\\ \Lambda & \mathbf{0}\end{bmatrix}\mathbf{u}_{k-1} + \begin{Bmatrix}\mathbf{B}\\ \mathbf{0}\end{Bmatrix}\gamma_{k-1} \tag{16}$$

$$\mathbf{y}_k = [\,\mathbf{I}\ 0\,]\mathbf{u}_k + \mathbf{v}_k \tag{17}$$

in which $\mathbf{u}$ consists of known and unknown state variables shown in Eq. (7); Λ the matrix of which components are weighted kriging coefficients, λ_i; $\mathbf{v}_k$ the measurement noise but does not exist in this study.

The estimation error covariance for Kalman filter algorithm coincides with that for kriging technique. The calculated estimation error covariance matrix, $\mathbf{P}$, which is updated at every measurement, is factorized as follows. The generated wave, $\mathbf{u}^c$, that satisfies the estimation error variance is added to the kriged wave, $\mathbf{u}_k$.

$$\mathbf{P} = \mathbf{L}\cdot\mathbf{L}^T \tag{18}$$

$$\mathbf{u}_k^c = \mathbf{L}\psi_k \tag{19}$$

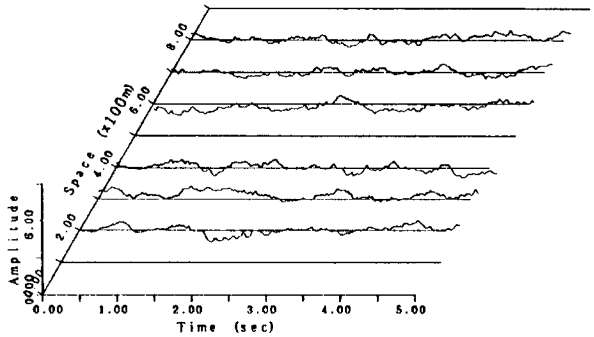
$$\mathbf{u}_k'(x_s) = \mathbf{u}_k(x_s) + \mathbf{u}_k^c(x_s) \tag{20}$$

in which ψ is a sample noise vector generated by the standard normal distribution density function and $\mathbf{u}'_k$ is a synthesized wave. The time histories obtained from the factorized $\mathbf{P}$, and the synthesized waves by adding the kriged waves and the waves obtained from the factorized $\mathbf{P}$ are shown in Fig. 6 (a) and(b), respectively. As the estimation error covariance matrix is factorized at every time step independently, the simulated waves satisfy spatial correlations but do not satisfy the correlations in respect to the time. To overcome these defects, we input the waves shown in Fig. 6 (a) into the governing equation (Eq. [12]) again and add the obtained waves, $\mathbf{u}^p$, to the kriged waves. Considering the diagonal components of $\mathbf{P}$, or auto-covariance, p, the following equation can be derived.

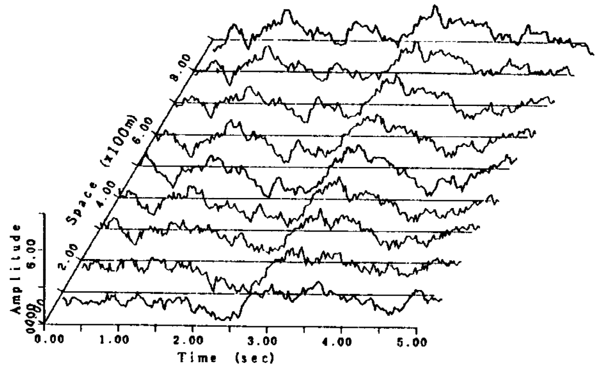
$$E[u^p u^p] = C(0)E[u^c u^c] = C(0)\,p \tag{21}$$

in which scalar u^p is a component of $\mathbf{u}^p$. Therefore when the additional waves are attached, they must be multiplied by $C(0)^{-1/2}$. According to the above procedure, Eq. (17) is replaced as

$$\mathbf{u}_k''(x_s) = \mathbf{u}_k(x_s) + \mathbf{u}_k^p(x_s) \tag{22}$$

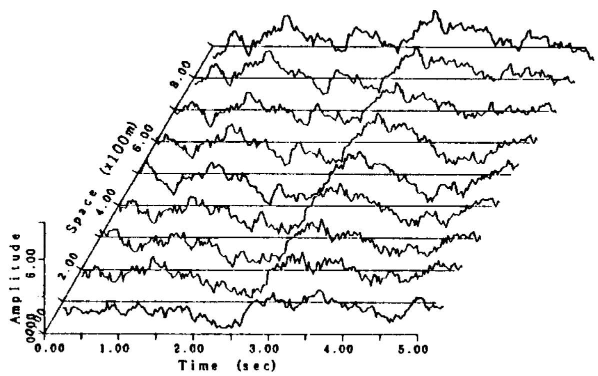


(a)

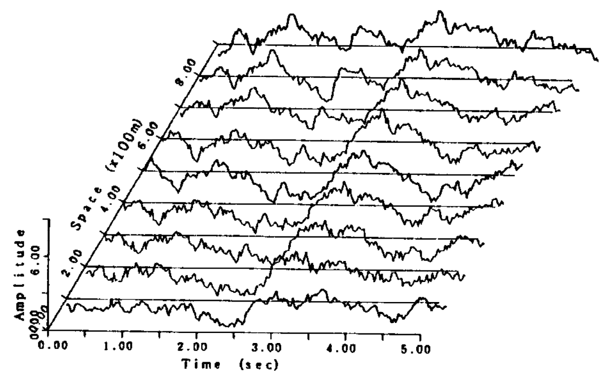


(b)

Fig. 7. (a) waves obtained by passing the waves in Fig. 6 (a) through the governing equation
(b) Synthesized waves by adding the waves in (a) to the kriged waves (sample wave 1)



(a)



(b)

Fig. 8. Sample waves simulated by the developed procedure;
(a) sample wave 2, (b) sample wave 3

The obtained additional waves and synthesized waves are shown in Fig. 7 (a) and (b). Two more sets of sample waves are shown in Fig. 8 (a) and (b). The fact that the correlation in time axis obeys the function in Eq.(13) is proved analytically (Kiyono *et al.*, 1995b). Hence, the covariance for the simulated waves coincide with the given function in Eq. (13).

CONCLUSIONS

Our proposed method provides a practical means for simulating stochastic waves by using Kalman filter and kriging techniques. The procedure developed and conclusions obtained in this study are the following:

1) The estimation error covariance for the kriging technique coincides with that obtained from the Kalman filter algorithm. Taking this relation into account, the algorithm of conditional simulation is developed by combining the Kalman filter and kriging techniques with which the stochastic waves in a time-space random field are simulated sequentially by adopting observed waves.
2) After determining the weighted coefficients by the kriging, these coefficient are incorporated in the state equation of the Kalman filter and carried out the optimum interpolation of random waves.
3) In the above procedure, we input the waves which satisfy the spatial correlation into the governing equation again and added the output waves to the kriged waves. The obtained waves satisfied the given time-space correlation.
4) On the basis of the conditional simulation, we could generate random stochastic waves which satisfy the given properties of a stochastic random field. By using the developed algorithm, the waves at arbitrary points can be simulated sequentially as soon as the waves at one or more points are observed.

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