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Paper No. 1616. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

NONLINEAR DYNAMICS OF TRANSMISSION LINES

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ABSTRACT

Seismic behaviour of transmission lines is determined from the dynamic analysis of towers and cables when the system is subjected to earthquake ground motion. The objective of this paper is to present a simple yet accurate method to determine the response of transmission lines when subjected to seismic ground motion. The towers are modelled by three-dimensional truss elements while the cables are modelled by two node elements that account for their geometric nonlinearity. The nonlinear analysis shows that the displacement of the cable when subjected to transverse and vertical ground motions can be substantial specially for earthquakes with relatively small acceleration to velocity (A/V) ratio.

KEYWORDS

Transmission lines, seismic response, towers, cables, dynamic analysis, nonlinear.

INTRODUCTION

Transmission lines are generally designed for wind and ice loads in the transverse direction. However, seismic loads may be important in some cases where the transmission line extends in areas of high seismicity. When subjected to ground motion, the transmission line towers and cables may be subjected to higher forces and stresses. However, of major concern in the response of transmission lines during earthquakes is that large cable displacements do not cause the cables to touch each other or any surrounding object, thus causing power failure.

Modelling of different parts of transmission lines has attracted the attention of several researchers. Transmission towers can be modelled by truss or frame elements. The results of a field test were compared to those of a finite element analysis (Kempner *et al.*, 1981). The comparison indicated that tower members can be modelled adequately by truss elements. Cables have geometric nonlinearity because large displacement of the cable changes its stiffness and hence its frequencies of free vibration. Research work was done to develop finite elements that take into account the cable geometric nonlinearity (Henghold and Russell, 1976; Gambhir and Batchelor, 1978). Cables can be modelled by two or three node elements. Gambhir and Batchelor (1978) showed that the frequencies predicted using straight two node elements converge to their true values as the number of elements increases. A limited number of studies were conducted to analyze the seismic behaviour of transmission lines. A method to design transmission towers against earthquakes was proposed by Suzuki *et al.* (1992). The dynamic interaction between towers and cables was investigated by Ozono and Maeda (1992).

MODELLING OF LINE ELEMENTS

The power transmission line system consists of towers and cables. The towers may be of different configurations and spaced at various distances. A typical transmission tower such as the one shown in Fig. 1 was considered in the analysis. The total weight of the tower is about 11 tons. The base points of the tower were assumed fixed.

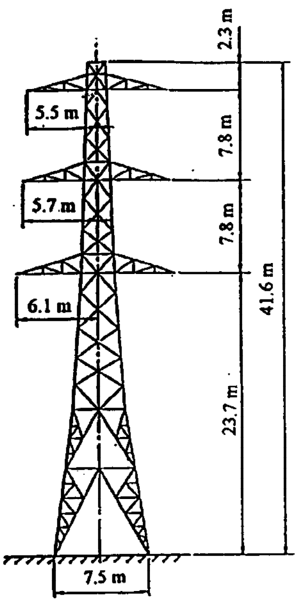


Fig. 1 Example transmission tower

The equation of motion of a sagged cable fixed at both ends under earthquake loading can be written in the following matrix form :

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K_T]\{u\} = -[M]\,\ddot{u}_g \tag{1}$$

where $\ddot{u}_g$ is the ground acceleration; $\{u\}$, $\{\dot{u}\}$ and $\{\ddot{u}\}$ are the nodal displacement, velocity and acceleration, respectively; $[M]$ is the lumped mass matrix; $[C]$ is the damping matrix; and $[K_T]$ is the total stiffness matrix which is defined as :

$$[K_T] = [K_E] + [K_L] \tag{2}$$

where $[K_L]$ is the conventional symmetric stiffness matrix that accounts for the large deflection of the cable and $[K_E]$ is the geometric stiffness matrix. The stiffness matrices $[K_E]$ and $[K_L]$ are defined as follows (Henghold and Russell, 1976) :

$$[K_E] = \frac{1}{2}\int_0^L EA\,(\{U_N\}^T [N']^T [N'] \{U_N\} - 1)\,[N']^T [N']\,ds \tag{3}$$

$$[K_L] = \int_0^L EA\,[N']^T [N'] \{U_N\} \{U_N\}^T [N']^T [N']\,ds \tag{4}$$

where, E is Young's modulus of the cable material; A is the cross sectional area of the cable; L is the element length; $\{U_N\}$ is the nodal coordinate vector; and $[N]$ is the shape function.

A straight two node element is developed to model the cable. On the basis of equations (3) and (4), the stiffness matrix $[K_L]$ is written in the form :

$$[K_L] = \frac{EA}{L^3} \begin{bmatrix} [K] & -[K] \\ -[K] & [K] \end{bmatrix} \tag{5}$$

where,

$$[K] = \begin{bmatrix} (\Delta X)^2 & \Delta X \Delta Y & \Delta X \Delta Y \\ \Delta X \Delta Y & (\Delta Y)^2 & \Delta Y \Delta Z \\ \Delta X \Delta Z & \Delta Y \Delta Z & (\Delta Z)^2 \end{bmatrix} \tag{6}$$

where X_i, Y_i and Z_i are the coordinates of the i'th node at any instant; and $\Delta X = X_{i+1} - X_i$; $\Delta Y = Y_{i+1} - Y_i$ and $\Delta Z = Z_{i+1} - Z_i$. The stiffness matrix $[K_E]$ is given by:

$$[K_E] = \frac{EA\ ((\Delta X)^2 + (\Delta Y)^2 + (\Delta Z)^2 - L^2)}{2\ L^3} \begin{bmatrix} [I] & -[I] \\ -[I] & [I] \end{bmatrix} \tag{7}$$

where [I] is the unit matrix.

DYNAMIC ANALYSIS OF LINE ELEMENTS

Tower Analysis

The tower shown in Fig. 1 was modelled using truss elements. The damping ratio of the tower was assumed to be 0.02 for all modes. A free vibration analysis of a free standing tower was carried out, and the frequencies of free vibration were determined to be 1.8, 8.5, and 14.8 Hz. The tower was subjected to a horizontal sinusoidal ground acceleration of maximum amplitude of 0.28 g which represents the level of excitation for the city of Victoria on Canada's west coast according to NBCC (1995). The maximum displacement at the top level of the tower is plotted against the excitation frequency in Fig. 2. This figure shows that the tower response is governed by the first mode and the contribution of other modes to the tower displacement is negligible.

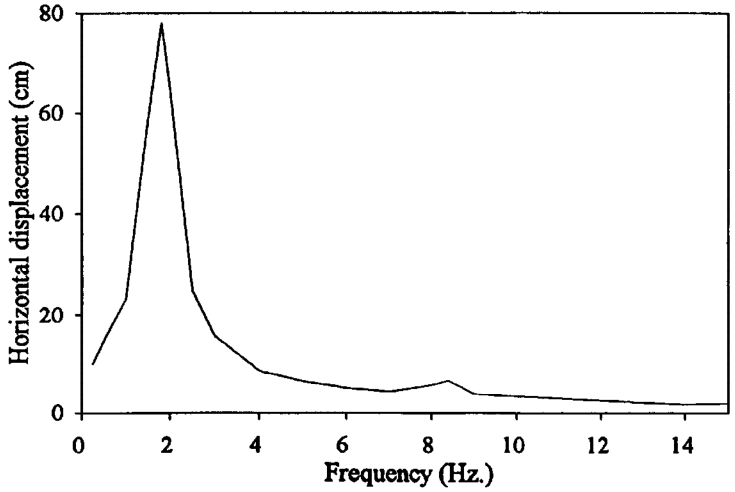


Fig. 2. Frequency response relationship for the transmission tower

Cable Analysis

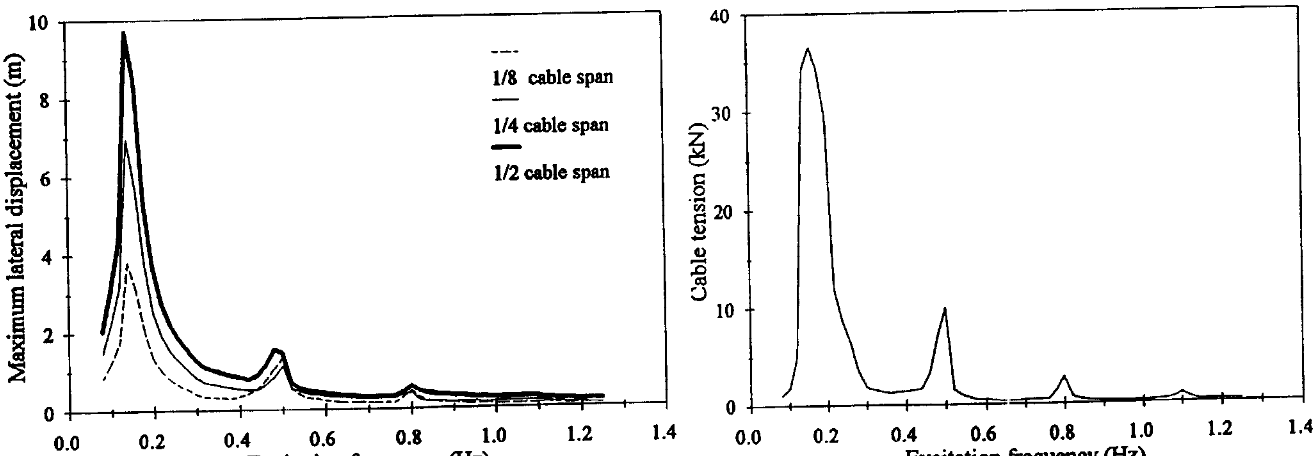
A typical transmission line cable was selected to be used in the analysis. The cable has the following properties:

Span = 400 m Sag to span ratio = 0.03
Young's modulus = 55 x 10^3 MPa Weight = 22.23 N/m
Cross sectional area = 10^{-3} m^2 Damping ratio = 0.01

The developed cable element was incorporated in the PC-ANSR computer program (Maison, 1992). The program uses Newton-Raphson method to achieve equilibrium within each time step and Newmark-β method as a numerical integration scheme in the dynamic analysis. The cable was modelled using 32 elements which represent the dynamic behaviour fairly accurately. The integration time step was taken equal to 0.005 s which was found to be small enough to obtain adequate accuracy and convergence.

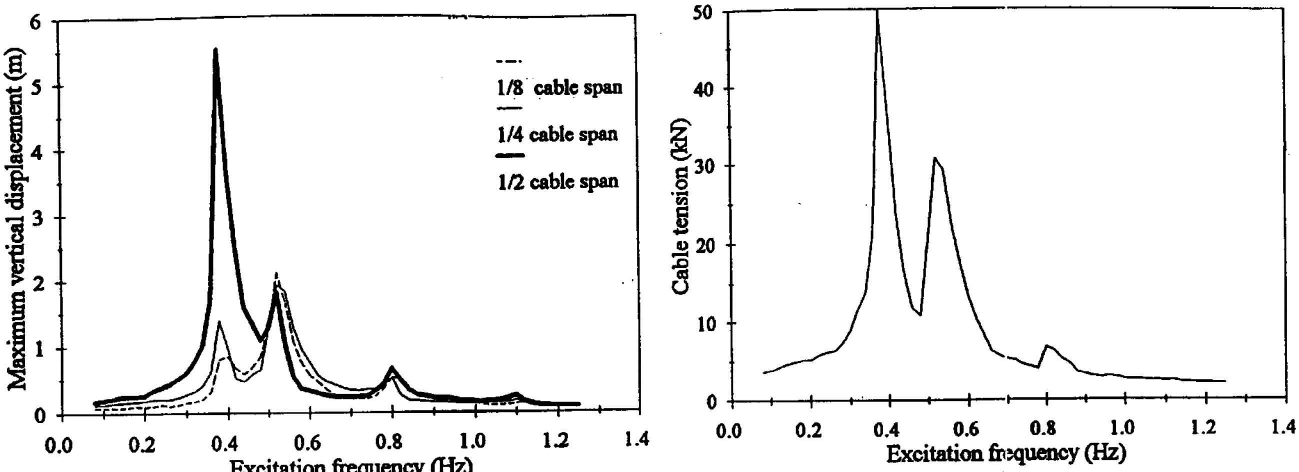
The cable was subjected to a sinusoidal ground acceleration with a peak of 0.1 g in the vertical and transverse directions. The envelopes of maximum displacement of various points along the cable span against the excitation frequency are shown in Figs. 3 and 4 for the transverse and vertical directions, respectively. Figures 3(a) and 4(a) show that the displacement of the cable mid-point is mainly governed by the first mode, specially in the transverse direction, where the contribution of other modes is negligible. Other modes contribute to the displacement of cable points at ¼ and ⅛ the span. The tension in the cable was found to be almost constant through the cable span, and therefore the average value of the tension is shown in Figs. 3(b) and 4(b). Figure 3(b) shows that due to the transverse excitation, the response is mainly governed by the first mode. However the contribution of higher modes may not be negligible for cable tension. Figure 4(b) show that due to the vertical excitation, the response is not governed by the first mode only as other modes have significant contribution to the response.

Figure 3 shows that the first three out-of-plane frequencies of the cable are 0.15, 0.47, and 0.80 Hz which are in good agreement with the equations derived by Irvine (1981) based on his linear theory of cable vibrations. This agreement can be explained by the fact that for sag to span ratio of 0.03, the nonlinear geometry effects in the cable are small. Figure 4 shows that the first three in-plane frequencies of the cable are 0.38, 0.52, and 0.80 Hz which are in a very good agreement with the first three symmetric in-plane modes as reported by Rao and Iyengar (1991).



(a) (b)

Fig. 3 Frequency response relationship for a single cable due to transverse excitation : (a) displacement and (b) tension



(a) (b)

Fig. 4 Frequency response relationship for a single cable due to vertical ground motion : (a) displacement and (b) tension

SEISMIC ANALYSIS

The analysis was carried out using the model shown in Fig. 5 where the transmission line consists of several towers of the type shown in Fig. 1 with cables spanning between them. The typical span was taken to be 400 m. The line is subjected to the transverse and vertical components of the ground motion records.

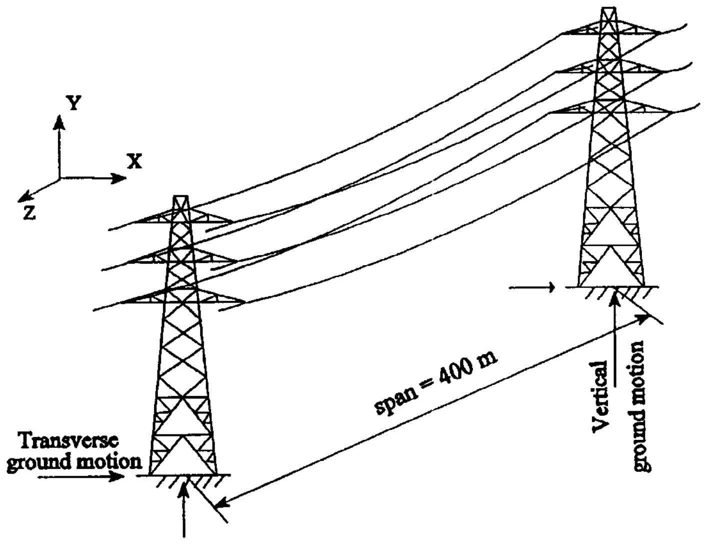


Fig. 5 Transmission line model

Selected Ground Motion Records

Six ground motion records were used in the analysis. The records were selected to have different frequency content as represented by the ratio A/V where A is the peak ground acceleration in g's and V is the peak ground velocity in m/s. The details of the earthquake records are shown in table 1. The horizontal component of ground motion was scaled to a peak value of 0.28 g to represent the city of Victoria in Canada. The vertical component of ground motion was scaled to a peak value of ⅔ that of the corresponding horizontal component.

Table 1 Description of earthquake records

Date & Event	Station	Horizontal Component				Vertical Component		
		Comp.	A (g)	V (m/s)	A/V	A (g)	V (m/s)	A/V
1985 Mexico	Mesa Vibradora	N90W	0.040	0.11	0.36	0.021	0.084	0.24
1971 San Fernando	800 W first St., L.A.	N37E	0.088	0.179	0.49	0.052	0.087	0.71
1979 Monte Negro	Albatros Hotel, Ulcinj	N00E	0.171	0.194	0.88	0.163	0.118	1.38
1940 Imperial Valley	El Centro	S00E	0.348	0.334	1.04	0.21	0.108	1.95
1966 Parkfield	Temblor No. 2	N65W	0.269	0.145	1.86	0.132	0.040	3.31
1970 Lytle Creek	Wrightwood, California	S25W	0.198	0.096	2.06	0.054	0.032	1.69

Results

The transmission line was subjected to the horizontal and vertical components of the six ground motion records given in table 1 normalized for a peak ground acceleration of 0.28 g. The maximum values of cable displacements and tension and the tower internal forces are shown in table 2 for various earthquakes

Table 2 Maximum response of the transmission line

Event	Transverse ground motion			Vertical ground motion		
	Cable tension (kN)	Cable transverse displ. (cm)	Tower force (kN)	Cable tension (kN)	Cable vertical displ. (cm)	Tower force (kN)
Mexico	2.8	209.8	113.4	17.0	150.8	10.7
San Fernando	1.9	97.9	88.8	4.4	31.7	6.7
Monte Negro	1.6	33.8	229.0	2.1	14.9	12.8
Imperial Valley	1.4	47.1	155.4	1.7	8.8	10.1
Parkfield	0.4	10.9	39.7	1.1	7.6	10.8
Lytle Creek	0.6	9.3	85.2	0.7	4.2	11.8

The results of the analysis indicate that for the transverse ground motion, the maximum response in the cable results from the earthquakes with low A/V, i.e. low frequency content. This is expected since the first three frequencies of the cable are less than 0.8 Hz. It is also observed that while the cable displacements can be as high as 210 cm for the Mexico earthquake, the cable tension does not exceed 2.8 kN which is negligible as compared to the tension developed in the cable due its own weight (37.1 kN). The reason for the small values of cable tension is that the cable transverse vibrations are of a swinging type and does not cause change in length and therefore does not produce significant strains and stresses. The forces in the transmission towers are significant and can be as high as 229.0 kN due to the Monte Negro earthquake which has one of its predominant frequencies near 1.82 Hz.

On the other hand, for the vertical ground motion, the records with low A/V caused the maximum response in the cable. The maximum displacement and tension in the cable result from the Mexico earthquake. The maximum displacement equals to about 150 cm and the maximum tension is about 17.0 kN which is almost half the static tension. The internal forces in the tower due to vertical ground motion is insignificant. Comparing the response of the transmission line due to the vertical component of ground motion to the response due to the horizontal component indicates that the maximum displacements were mainly due to the transverse excitation, while the cable tension is mainly due to the vertical excitation. The tower forces due to the vertical excitation are relatively small.

Effect of transmission towers

To investigate the effect of transmission towers on the cable displacements, the previous results were compared to the displacement that occurred in a single cable fixed at both ends. The Mexico and Monte Negro earthquakes were selected for this analysis. The Mexico earthquake was chosen for this analysis because it produces the maximum displacement in the cable, while the Monte Negro earthquake was chosen because its predominant frequency is close to the tower first frequency. The envelopes of the maximum cable displacement for the two models due to Mexico and Monte Negro earthquakes are plotted in Figs. 6 and 7 for the horizontal and vertical components, respectively.

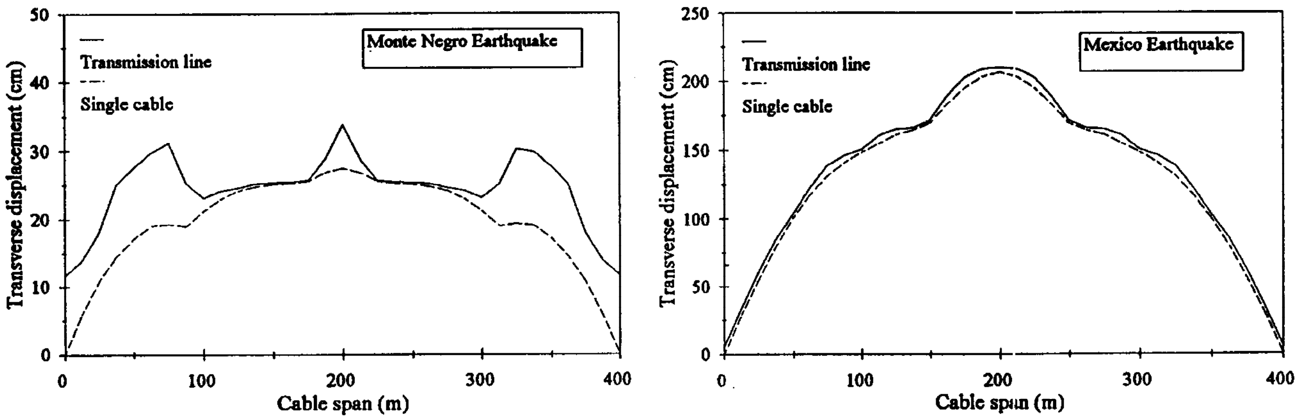


Fig. 6 Envelopes of maximum response to horizontal ground motion

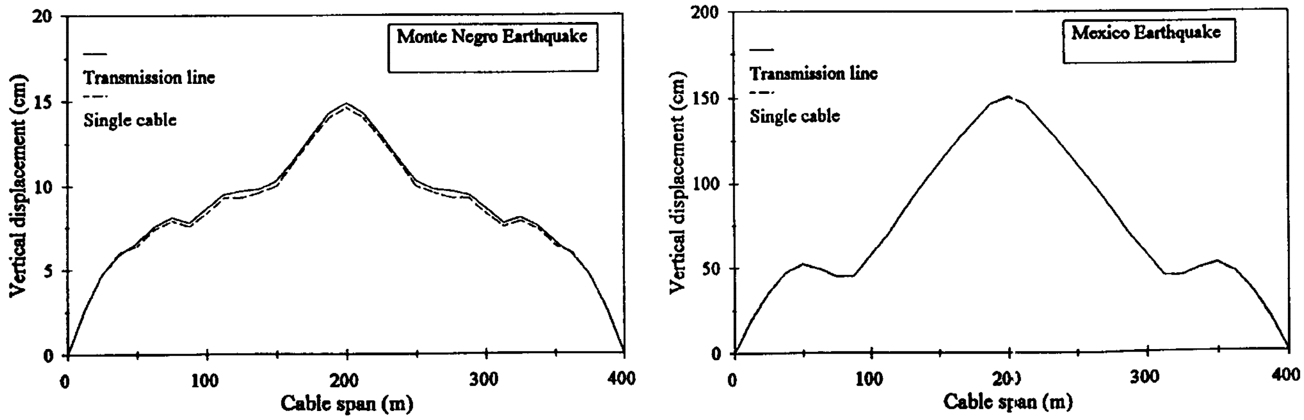


Fig. 7 Envelopes of maximum response to vertical ground motion

The first model represents a cable tower system as shown in Fig. 5, while the second model is a cable fixed at both ends. In the figures, the first model is referred to as "Transmission line", while the second model is referred to as "Single cable". The results indicate that the effect of the tower on cable vibrations due to seismic ground motion is not significant. The earthquakes with low A/V ratio cause large displacements in the cable while the tower is moving almost in a rigid body mode. The reason for this is that the frequency content of the records with low A/V is much lower than the tower natural frequency. The earthquakes that affect the tower such as the Monte Negro earthquake will cause somewhat insignificant cable displacement due to the decoupling of the frequencies of free vibration of the cable and the tower.

CONCLUSIONS

The following conclusions are arrived at from the seismic analysis of transmission lines :

1. The displacement response of transmission towers and cables is mainly dominated by the first mode.
2. The response of the cable to the earthquakes with low A/V ratio is much larger than their response to earthquakes with higher A/V ratio.
3. Significant vertical and transverse displacements may result in cables due to seismic ground motion. These calculated displacements may form the basis to develop criteria for line separation. The tension developed in the cable due to vertical ground motion is always less than approximately half the static tension and the tension developed due to transverse ground motion is negligible.
4. The transverse component of ground motion produces significant forces in the towers, while the effect of the vertical component is insignificant.
5. The contribution of the towers to the seismic performance of transmission lines can be neglected. For simplicity, the line can be modelled by cables fixed at both ends.

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