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LIQUEFACTION PREDICTION WITH ENERGY-RELATED PARAMETERS OF SOIL AND GROUND MOTION

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ABSTRACT

An explicit formula to evaluate the factor of safety against liquefaction due to random excitation is derived from energy-related parameters of soil and ground motion. The parameters are determined from moments of the power spectral density function (PSDF) of ground motion and non-linear regression coefficients of the liquefaction strength curve (LSC) of soil. This procedure is applied to the liquefaction of the Port Island in 1995 Kobe earthquake using the ground acceleration measured in a vertical array.

KEYWORDS

liquefaction prediction; power spectrum; liquefaction strength curve; energy dissipation; vertical array; Kobe earthquake;

ENERGY METHOD

The energy-based factor of safety against liquefaction F_{le} is defined as the ratio of the strain energy increment in the pore water corresponding to liquefaction ΔE_l to that due to the earthquake ground motion ΔE_e.

$$F_{le} := \sqrt{\frac{\Delta E_l}{\Delta E_e}} \tag{1}$$

where the symbol := implies definition and the strain energy in the pore water in the unit volume of soil

$$E := \frac{1}{2} nCp^2 \tag{2}$$

is defined by the porosity n, the compressibility C and the pressure p of the pore water. If we define that the liquefaction is the state where the pore pressure increases from the initial value p_0 as much as the initial effective overburden stress σ_{v0}', then the strain energy increment corresponding to liquefaction ΔE_l is evaluated

$$\Delta E_l = \frac{1}{2} nC(1 + 2\beta)\sigma_{v0}'^2, \ \beta := p_0 / \sigma_{v0}' \tag{3}$$

The strain energy increment in the pore water ΔE due to cyclic shear deformation is assumed to correlate with the dissipated energy D through the internal friction of the soil, with a factor η, namely the energy absorption ratio.

$$\Delta E = \eta D \tag{4}$$

And D is assumed to be governed by the energy related parameters of the loading.

$$D = 4 f_\alpha UN \exp\left(-\frac{U_c}{U}\right) \tag{5}$$

where U and N are the mean energy and the number of cycles of the external loading, respectively. U_c is the critical energy that characterizes the damping ratio of the soil, and f_α is a factor depending only on the bandwidth of the loading and is assumed to be unity for the simple sinusoidal loading. From Eqs. (4) and (5), the strain energy increment in the pore water due to random loading is written

$$\Delta E = 4\eta \exp\left(-\frac{U_c}{U}\right)\cdot f_\alpha UN \tag{6}$$

Eq. (6) has a pair of material constants, namely the energy absorption ratio η and the critical energy Uc. The former is the strain energy increment in the pore water due to the unit energy dissipation. The above set of equations is a kind of a constitutive law written in terms of energy.

The mean energy of cyclic sinusoidal loading with the shear stress amplitude τ_d is written with the shear modulus G.

$$U = \frac{1}{2G}\tau_{rms}^2 = \frac{1}{4G}\tau_d^2 \tag{7}$$

The critical shear stress amplitude is defied by the critical energy Uc, and is assumed to depend linearly on the initial effective mean stress σ_{m0}' with a friction angle ϕ_c

$$\tau_c := 2\sqrt{GU_c} = \sigma_{m0}'\tan\phi_c \tag{8}$$

The stress ratio of the loading R and the critical stress ratio R_c is defined:

$$R := \tau_d/\sigma_{v0}', \quad R_c := \tau_c/\sigma_{v0}' \tag{9}$$

Substituting Eqs. (7) through (9) into Eqs (6), (1) and (3), putting $F_{le}=1$, solving for N and dividing by $\sigma_{v0}'^2$, an analytical form of the liquefaction strength curve, i.e. the relation between the number of sinusoidal loading cycles N and the stress ratio R just to cause liquefaction, is obtained.

$$N = \frac{\varepsilon^2}{R^2}\exp\left(\frac{R_c^2}{R^2}\right) \tag{10}$$

where ε^2 is the normalized value of the liquefaction energy increment ΔE_l .

$$\varepsilon^2 := \frac{n}{2\eta}(1+2\beta)CG \tag{11}$$

Eq. (11) can be linearized by the following transformation

$$X := 1/R^2, \quad Y := \ln NR^2 \tag{12}$$

into

$$Y = R_c^2 X + \ln\varepsilon^2 \tag{13}$$

Using this expression, the constants R_c and ε^2 are determined from results of cyclic loading tests as follows. 1) Transform the data points (R, N) into (X, Y) by Eq. (12). 2) Regress Y against X and find the Y intercept *b* and the slope *a* of the regression line. 3) Calculate R_c=sqrt(*a*), and ε=sqrt(exp(*b*)).

Usually, the liquefaction of a soil specimen is defined for the maximum strain amplitude reaching a threshold value γ_{max}, such as 5% double amplitude, i.e. γ_{max}=2.5%. Therefore, if N=1/4, then the sample yields the threshold strain amplitude γ_{max} in a monotonic loading. The mean shear modulus for this monotonic loading is

$$G_m = \frac{\tau_m}{\gamma_{max}} = \frac{R_m\sigma_{v0}'}{\gamma_{max}} \tag{14}$$

where the subscript m stands for the monotonic loading. Substituting, in Eq. (10), N=1/4 and neglecting the exponential term by assuming that R is large compared to R_c, we find

$$\varepsilon = \frac{1}{2} R_m \tag{15}$$

From Eqs (14) and (15), the mean shear modulus for monotonic loading G_m is evaluated.

$$G_m = \frac{2\varepsilon\sigma'_{v0}}{\gamma_{max}} \tag{16}$$

Figs. 1 and 2 illustrate a liquefaction strength curve and a monotonic loading curve. Usually, the former is monotonically decreasing. The latter is monotonically increasing. If the stress ratio is less than R_m, then the number of cycles to reach the threshold strain amplitude γ_{max} is larger than 1/4. The first cyclic shear modulus G_1 is larger than G_m and the last one G_l is smaller than G_m as illustrated in Fig. 2. Therefore, we adopt G_m for the estimate of the mean shear modulus for all N and R.

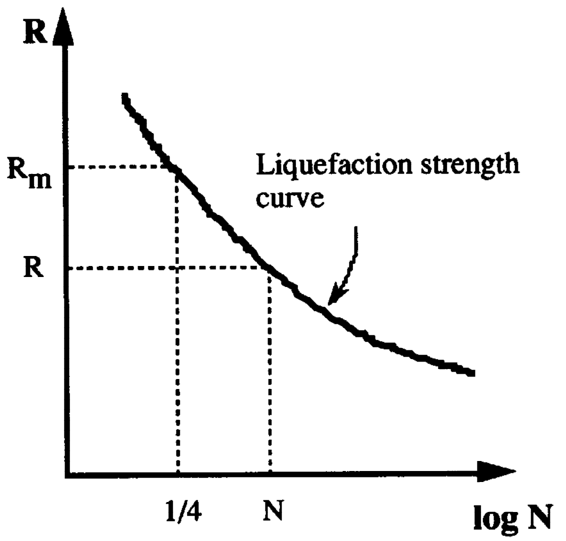

Fig. 1 R_m and R on the liquefaction strength curve

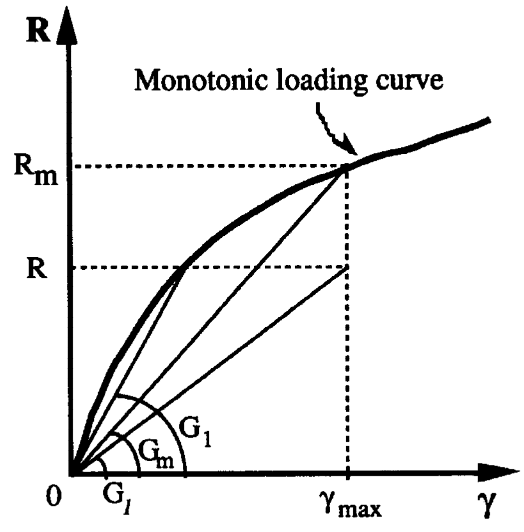

Fig. 2 G_m is between G_1 and G_l

Using G_m for G in Eq. (11), the coefficient η is expressed by the known quantities

$$\eta = \frac{nC}{\gamma_{max}\varepsilon}(1+2\beta)\sigma'_{v0} \tag{17}$$

Substituting Eqs. (17), (4), (6), (7) and (3) into Eq. (1), an explicit formula to evaluate the factor of safety

$$F_{le} = \frac{\varepsilon\sigma'_{v0}}{\tau_{rms}}\sqrt{\frac{1}{2 f_{\alpha} N}\exp\left(\frac{\sigma'^{2}_{m0}\tan^2\phi_c}{2\tau_{rms}^{2}}\right)} \tag{18}$$

is obtained. The strength of the soil is expressed by a pair of coefficients ε and ϕ_c, which are determined from the result of the cyclic loading test. The effect of the ground motion is represented by three quantities, namely root mean square shear stress τ_{rms}, the number of cycles N, and the factor of bandwidth f_α.

The shear stress τ acting at H m deep in a ground is approximately evaluated from the acceleration a(t) at the surface.

$$\tau = \rho H f_H\, a(t) \tag{19}$$

where ρ is the average density, f_H is a factor to express the deviation from the constant distribution along the depth H. Therefore, the number of cycles, RMS shear stress are evaluated from the acceleration time history a(t) of the ground surface. It is simpler to use moments of the power spectral density function $S(\omega)$.

$$\lambda_i := \int_{-\infty}^{\infty} S(\omega)\omega^i d\omega, \quad S(\omega) := \frac{1}{2\pi s_0}\left| \int_0^T a(t)e^{-i\omega t}dt \right|^2 \tag{20}$$

where i is an integer, s_0 is a strong motion duration. The number of cycles N is calculated from the central frequency ω_a of the ground acceleration.

$$N = \frac{s_0\omega_a}{2\pi}, \quad \omega_a := \sqrt{\frac{\lambda_2}{\lambda_0}} \tag{21}$$

The root mean square (RMS) shear stress is evaluated from the RMS acceleration.

$$\tau_{rms} = \rho H f_H a_{rms} = \rho H f_H \sqrt{\lambda_0} \tag{22}$$

The bandwidth factor is introduced to account for the difference between the energy dissipation of the sinusoidal loading and a random loading. From an analytical study of the response of the SDOF system with frictional energy dissipation mechanism due to random excitation, the effect of the bandwidth is evaluated by the following factor (Igarashi, 1986)

$$f_\alpha = \alpha + \frac{\pi}{2}\sqrt{1-\alpha^2}, \quad \alpha := \frac{\lambda_0}{\sqrt{\lambda_{-2}\lambda_2}} \tag{23}$$

in which α is called a bandwidth index, being zero for the white noise, and being unity for the sinusoid. f_α becomes unity for α=1 as assumed previously, and ranges between 1 and 1.862.

EQUIVALENT STRESS RATIO AND NUMBER OF CYCLES

By comparing the expression of the energy-based factor of safety (Eq. (18)) with the analytical form of the liquefaction strength curve (Eq. (10)), the equivalent stress ratio and number of cycles of the random loading is obtained

$$R_{eq} := \frac{\sqrt{2}\tau_{rms}}{\sigma'_{m0}}, \quad N_{eq} := \frac{\sigma'^2_{m0}}{\sigma'^2_{v0}} f_\alpha N \tag{24}$$

These values locate a particular ground motion on the R-N plane along with a liquefaction strength curve (LSC). If the point is above the LSC, F_{le} is less than unity. If the point is on the LSC, F_{le} is unity.

THE KOBE GROUND MOTION AND LIQUEFACTION OF MASADO

On the January 17th in 1995, an earthquake of JMA magnitude 7.2 hit Kobe city and caused liquefaction in the Port island which is an artificial island made of soil called Masado. Triaxial dynamic loading tests were conducted for undisturbed samples of masado by Nagase et al. (1995). The resulting 4 pairs of (N, R) data are transformed to (X, Y) by Eq. (12) and plotted in Fig. 4, along with the regression line for these points. From the slope and Y-intercept, ϕ_c and ε are determined using Eqs. (13), (9) and (8), as ε=0.58 and ϕ_c=11.4 degrees. The analytical liquefaction strength curve for these coefficients are drawn in Fig. 3. Although the test data appear not so close to the regression line in the X-Y plane in Fig. 4, they are almost on the analytical line in Fig. 3. The X-Y transformation exaggerates the distance along the R axis in the N-R plane because of the relation $X=1/R^2$.

A vertical array of accelerometers has been operated by the Kobe city at the Port Island. The accelerogram recorded at the ground surface was processed according to Eqs. (20) through (23) and the RMS acceleration

a_{rms}, number of cycles N, and bandwidth index α are determined for the 5% and 95% energy duration, as a_{rms}=1.21m/sec^2, N=12.05, α=0.361. The equivalent stress ratio and number of cycles are calculated as R_{eq}=0.366 and N_{eq}=9.77 respectively. The F_{le} is calculated by Eq. (18) as 0.59. The point (R_{eq}, N_{eq}) is plotted in Fig. 3. The corresponding point is plotted in the X-Y plane in Fig. 4. The intersection of the liquefaction strength curve with the line of R=0.366 is about 3. This means that the Kobe ground motion had at least enough energy to liquefy the sand with this liquefaction strength curve within 3 cycles. This observation coincides with a result of an effective stress analysis using a constitutive law called the stress density model (Cubrinovski, M and Ishihara, K, 1995).

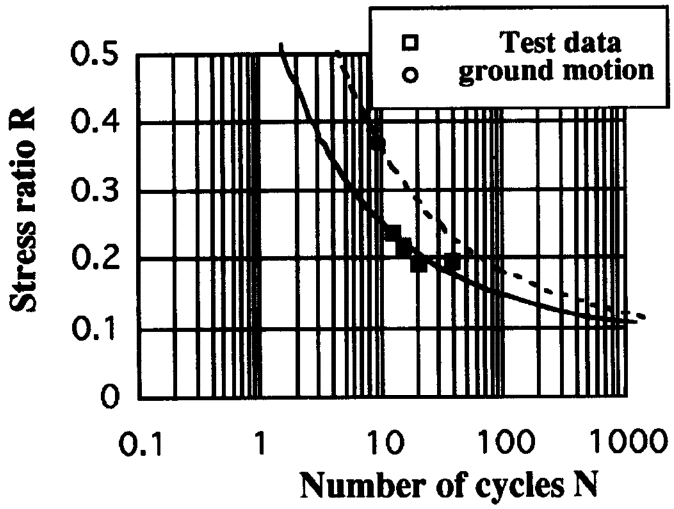

Fig. 3 Liquefaction strength curve and effective energy

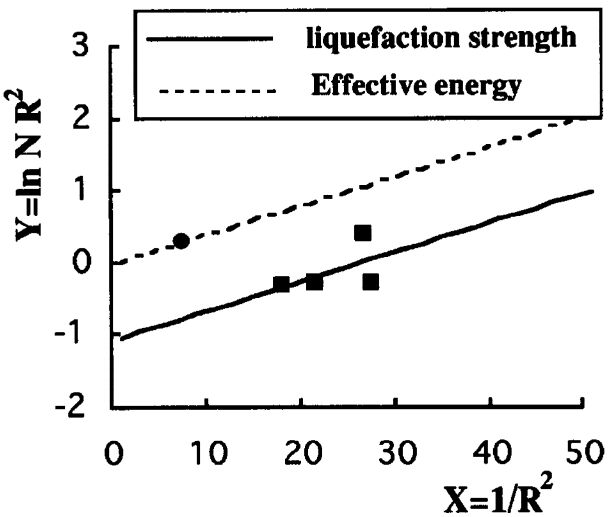

Fig. 4 X-Y plot of Masado and Kobe ground motion

EFFECTIVE ENERGY

Conventional factor of safety against liquefaction F_l is defined

$$F_l := \frac{R_N}{R_e} \tag{25}$$

Where R_e is the stress ratio mobilized by the earthquake, and R_N is the stress ratio on the liquefaction strength curve with a specific number of cycles N, usually N=20. If this number coincides with N_{eq}, i.e. the number of cycles of the shear stress mobilized by the ground motion, then the conventional factor gives a true safety margin. If the number of cycles is smaller than the specified value, the conventional factor underestimates the actual safety margin. Otherwise, it overestimates. On the other hand, the proposed method take both the amplitude and the number of cycles into account by using the energy of loading and the resistance. Graphically, F_{le} is the distance between the liquefaction strength curve and the point (R_{eq}, N_{eq}). Substituting Eq. (24) into Eq. (18), and taking natural logarithm,

$$Y = R_c^{\,2} X + \ln\left(\frac{\varepsilon^2}{F_{le}^2}\right), \quad X := 1/R_{eq}^2, \quad Y := \ln N_{eq} R_{eq}^2 \tag{26}$$

This means that all the ground motions whose N_{eq} and R_{eq} satisfy Eq. (26) yield the same F_{le} for given R_c and ε. The effect of the ground motion with N_{eq} and R_{eq} is expressed by this line in the X-Y plane. Let us call this the effective energy line of a particular ground motion. In Fig. 4, this line for the Kobe ground motion is drawn by a dashed line. This line becomes a curve of a type of Eq. (10) as illustrated in Fig. 3. The distance between these two curves, i.e. the liquefaction strength and the effective energy, is the distance between the two lines along the Y axis in X-Y plane, and is $\ln F_{le}^2$. Note that the effective energy line of a ground motion depends on the choice of R_c and ε, i.e. the strength of the soil. Graphically, the effective energy line crosses the point (N_{eq}, R_{eq}) and is parallel to the liquefaction strength curve.

NECESSARY IMPROVEMENT OF THE SOIL

Figs. 3 and 4 illustrate that if the liquefaction strength curve of a soil is above the dashed line, then the factor of safety against the Kobe ground motion is more than unity, meaning no liquefaction. From Eq. (26), the necessary value of ε is calculated.

$$\varepsilon_{imp} = \frac{\varepsilon}{F_{le}} \tag{27}$$

For F_{le} =0.59, ε=0.58, ε_{imp}=0.98. Figs. 5 and 6 show the dependence of ε and ϕ_c with the SPT N value for 2 different types of sand (Igarashi, 1995). They are computed from the cyclic loading test results of undisturbed samples. The Urayasu sand contains 0% to 30% fines(Taya, et al., 1994). The Niigata sand is a clean sand with D_{50} of 0.25 to 0.3mm and contains less than 2% fine (Yoshimi, 1989). In Fig. 5, ε appears to have a linear or parabolic relationship with the SPT N value for each type of sand. On the contrary, data points of ϕ_c have little correlation with SPT N and distribute around ϕ_c =15 degrees, ranging from 10 to 20 degrees except for one data of Niigata sand with largest N value. The ϕ_c of the previous example is 11.4 degrees and is similar to the values of the Urayasu sand. Judging from Fig. 5, a sand similar to the Urayasu sand with SPT N of more than 16 is predicted to avoid liquefaction due to the ground motion measured in the ground surface of the Port Island.

The shape of the liquefaction strength curve is expressed by both ϕ_c and ε. Therefore both coefficients should be considered to predict liquefaction. Above example assumes that the dynamic friction angle ϕ_c remains in the same value while ε increases as the SPT N value. Fig. 6 shows that ϕ_c of Urayasu sand has little correlation with SPT N but deviates from 10 to 15 degrees. The effect of ϕ_c on the liquefaction strength depends on the stress ratio R, and increases as R becomes smaller (Eq. (10)). Therefore, for ground motions like the Kobe wave, with large R_{eq} and small N_{eq}, the change of ϕ_c has little effect on F_{le}.

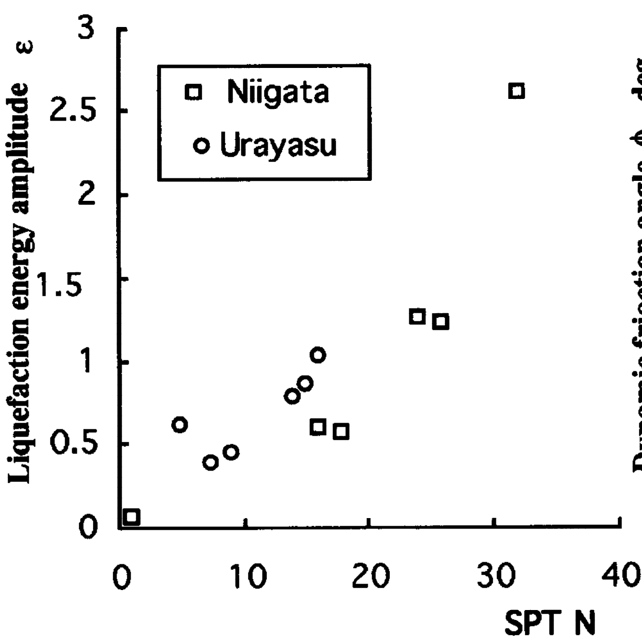

Fig. 5 Liquefaction energy amplitude ε and SPT N value

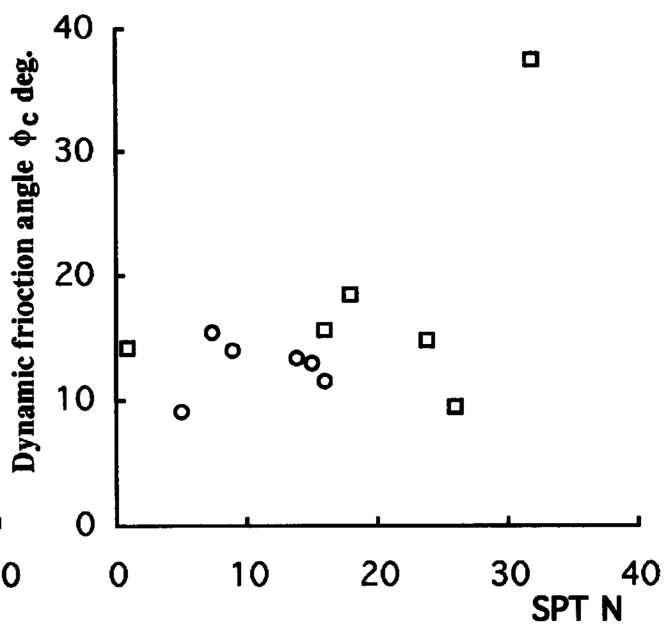

Fig. 6 Dynamic friction angle and SPT N value

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