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A SERIES SOLUTION FOR WAVE DIFFRACTION BY A SEMI-CYLINDRICAL ALLUVIAL VALLEY

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ABSTRACT

This paper presents the analysis of two dimensional responses of the semi-cylindrical alluvial valley subjected to incident plane waves. This problem is decomposed into the problems for interior and exterior regions. In the exterior region the diffracted wave fields are constructed with linear combinations of two independent families of Lamb's singular solutions, which are the integral solutions for surface vertical and horizontal line loads with their derivatives up to the order n, are used to represent this diffracted wave field. The wave fields of interior region are constructed by the similar series but bounded at the origin. Continuity conditions along the interface between exterior and interior region are satisfied in the least-square sense.

KEYWORDS

Alluvial valley, wave diffraction, singular solution, series solution

INTRODUCTION

Local geological conditions can generate large amplifications and spatial variations of ground motion. The seismic response of geological inhomogeneities is essential for the aseismic design of important facilities. Detail understanding of the effects of wave scattering and diffraction through geological inhomogeneities is of importance for earthquake engineering and strong motion seismology.

The problem of the two-dimensional scattering and diffraction of plane waves by an alluvial valley in an elastic half-plane has been studied by many investigators. Using an exact series solution, Trifunac(1971) has studied the surface motion in and around a semi-cylindrical alluvial valley excited by plane SH-waves. Wong and Trifunac(1974) have solved a similar problem involving alluvial valley of semi-elliptical shape. Sanchez-Sesma and Esquival(1979) have used the source method to investigate the scattering and diffraction of SH-waves by an arbitrarily shaped alluvial valley. Dravinski(1983) has applied the indirect boundary integral method to study the scattering of plane SH-waves by dipping layers of arbitrary shape. For the problem of incident P and SV waves, the solution becomes more complicated because of the mixed mode conversion involved. To solve this problem many techniques have been proposed. Dravinski(1982) has used the source method to analyze the alluvial valley of arbitrary shape subjected to P waves. Bravo and Sanchez-Sesma(1990) have solved the same problem

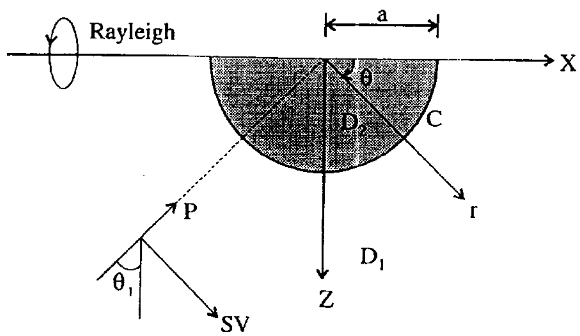

Fig. 1 Semi-cylindrical alluvial valley contained in an elastic half-plane

by applying Treffetz's method, they construct the diffracted and refracted fields by wave functions satisfying the wave equation. Mossessian and Dravinski(1987) have used an indirect boundary integral equation approach to investigate the diffraction of plane P, SV, and Rayleigh waves by dipping layers of arbitrary shape. Yeh *et al.*(1995a,1995b) have used boundary element method to solve the cases of obliquely incident P, SV, and Rayleigh waves on an alluvial valley of arbitrary shape. Fishman and Ahmad(1995) have adopted boundary element method to study the influence of valley depth, frequency, impedance ratio, and incident angle on surface motion of an alluvial valley.

In this work, the method of solution which has been applied by Maunsell(1936) to analyze a classical edge notched problem in elastostatic will be adpoted and extented to treat dynamic wave propagation. Analogous to Maunsell approach, two linear independent sets of functions which are n-th order Lamb's singular solutions in integral form, which automatically satisfy boundary conditions at horizontal ground surface are developed to represent the diffracted fields. The incoming, outgoing and standing waves are considered as the same series with the integrals which are integrated along different paths so that the exterior and interior diffracted fields can be described. This method has been applied to analyze P, SV, and Rayleigh waves diffraction about a semi-cylindrical canyon by Yeh *et al.*(1995c). The numerical results for wave diffraction of a semi-cylindrical alluvial valley subjected to different types of incident waves with various frequencies and angle will be considered.

STATEMENT OF THE PROBLEM

The geometry of the problem is depicted in Fig. 1. It consists of a homogeneous, isotropic, linearly elastic semi-cylindrical alluvial valley with domain D_2 and perfectly bonded to an elastic half-plane D_1. For each region the Lamé constant λ and μ, and mass density ρ should be specified. The half-plane is excited by incident P, SV, and Rayleigh waves with harmonic time dependence of the type $e^{i\omega t}$, where ω is the circular frequency. For simplicity, the factor $e^{i\omega t}$ will be dropped from all expressions. In this two dimensional model, the displacement field $\mathbf{u}$ is related to the displacement potential in each region according to

$$\mathbf{u}_i = \nabla\phi_i + \nabla \times (0, \psi_i, 0) \quad ; \quad i = 1,2 \tag{1}$$

where ϕ_i and ψ_i are P and SV wave potential, respectively, which are governed by

$$\nabla^2\phi_i + k_{pi}^2\phi_i = 0 \tag{2}$$

$$\nabla^2\psi_i + k_{si}^2\psi_i = 0 \quad ; \quad i = 1,2 \tag{3}$$

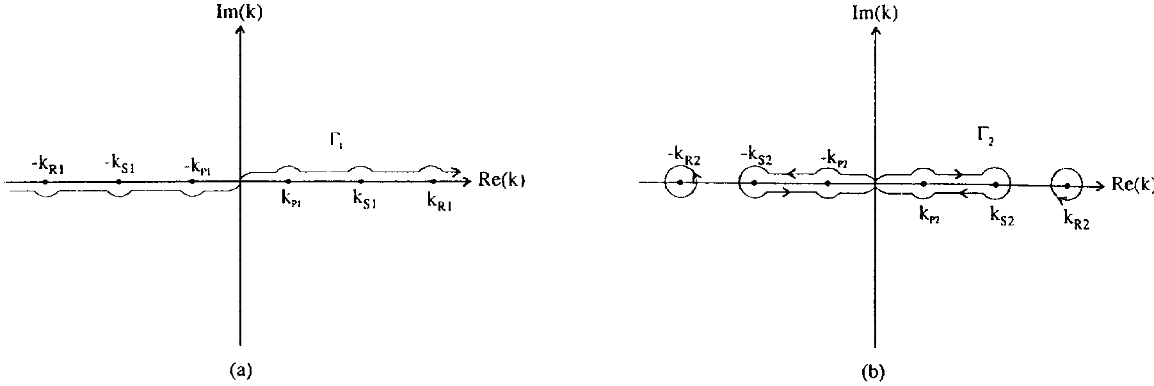

Fig. 2 Path of integration in the k-plane, (a) exterior region, (b) interior region

where $k_{pi} = \omega/c_{pi}$ and $k_{si} = \omega/c_{si}$ denote the longitudinal and the transverse wave number, respectively. c_{pi} and c_{si} denote the longitudinal and the transverse wave speed, respectively, and are defined as

$$c_{pi} = \sqrt{\frac{\lambda_i + 2\mu_i}{\rho_i}} \tag{4}$$

$$c_{si} = \sqrt{\frac{\mu_i}{\rho_i}} \quad ; \quad i = 1,2 \tag{5}$$

METHOD OF SOLUTION

As the incident wave impinge on the alluvial valley, it is partly reflected back into the half-space and partly transmitted into the alluival valley through the interface C, hence the total displacement in the exterior region $\mathbf{u}^1$, and in the interior region $\mathbf{u}^2$, are given by

$$\mathbf{u}^1 = \mathbf{u}^f + \mathbf{u}^{s_1} \quad ; \quad r \in D_1 \tag{6}$$

$$\mathbf{u}^2 = \mathbf{u}^{s_2} \quad ; \quad r \in D_2 \tag{7}$$

where the superscripts f denote the free wave field, and s_j, j=1,2, denote the scattered wave field. With the geometry of this problem in mind, a suitable set of functions representing the scattered field of exterior region must satisfy the free surface conditions and radiation conditions at infinity. Then the Lamb's solution for surface vertical and horizontal line loading and their derivatives with respect to horizontal coordinate x up to order n (n is arbitrary positive integer) satisfy the required conditions mentioned above, thus, the scattered displacement field for exterior region D_1 can be written as

$$\mathbf{u}^{s_1} = \sum_{n=0}^{N} A_n^1 \mathbf{u}_{(n)}^{v_1} + \sum_{n=0}^{N} B_n^1 \mathbf{u}_{(n)}^{h_1} \tag{8}$$

where $\mathbf{u}_{(n)}^{v_1}$ and $\mathbf{u}_{(n)}^{h_1}$ are the n-th order derivatives of Lamb's singular solution due to vertical surface line loading and horizontal surface line loading, respectively. A_n^1 and B_n^1 are the unknown coefficients which can be determined from interface conditions, and N is the order of approximations.

The integral representation of the displacement of Lamb's singular solution due to surface line loading with magnitude Q has the form

$$u_{x(n)}^{s_1} = \frac{Q}{2\pi\mu_1} \int_{\Gamma_1} (ikC_n^{s_1} e^{-\nu_1 z} + \nu_1' D_n^{s_1} e^{-\nu_1' z}) e^{-ikx} \, dk \tag{9}$$

$$u_{z(n)}^{s_1} = \frac{Q}{2\pi\mu_1} \int_{\Gamma_1} (\nu_1 C_n^{s_1} e^{-\nu_1 z} - ikD_n^{s_1} e^{-\nu_1' z}) e^{-ikx} \, dk \tag{10}$$

where $s_1 = v_1$ or h_1 and $\nu_1 = \sqrt{k^2 - k_{p1}^2}$, $\nu_1' = \sqrt{k^2 - k_{s1}^2}$.
For $s_1 = v_1$ (vertical surface line load)

$$C_n^{s_1} = (-ik)^n \frac{2k^2 - k_{s1}^2}{F_1(k)} \tag{11}$$

$$D_n^{s_1} = (-ik)^n \frac{-2ik\nu_1}{F_1(k)} \tag{12}$$

$$F_1(k) = (2k^2 - k_{s1}^2)^2 - 4k^2\nu_1\nu_1'$$

and for $s_1 = h_1$ (horizontal surface line load)

$$C_n^{s_1} = (-ik)^n \frac{2ik\nu_1'}{F_1(k)} \tag{13}$$

$$D_n^{s_1} = (-ik)^n \frac{2k^2 - k_{s1}^2}{F_1(k)} \tag{14}$$

The integral of eqn(9) and (10) are integrated along the path Γ_1 in the k-plane and shown in Fig. 2(a). The scattered field of interior region must satisfy the free surface conditions and bounded at the origin, thus, the integral representation of scattered displacement field for the interior region has the similar integrand as those for the exterior region, but integrated along different path, that is

$$\mathbf{u}^{s_2} = \sum_{n=0}^{N} A_n^2 \mathbf{u}_{(n)}^{v_2} + \sum_{n=0}^{N} B_n^2 \mathbf{u}_{(n)}^{h_2} \tag{15}$$

with the components form

$$u_{x(n)}^{s_2} = \frac{Q}{2\pi\mu_2} \int_{\Gamma_2} (ikC_n^{s_2} e^{-\nu_2 z} + \nu_2' D_n^{s_2} e^{-\nu_2' z}) e^{-ikx}\, dk \tag{16}$$

$$u_{z(n)}^{s_2} = \frac{Q}{2\pi\mu_2} \int_{\Gamma_2} (\nu_2 C_n^{s_2} e^{-\nu_2 z} - ikD_n^{s_2} e^{-\nu_2' z}) e^{-ikx}\, dk \tag{17}$$

where Γ_2 is the integrated contour as shown in Fig. 2(b).

Substituting eqn(9)-(17) into the stress-displacement relations, we have total stress expressed in terms of the stresses of the free field and the scattered field as

$$\left\{ \begin{array}{c} \sigma_{rr}^1 \\ \sigma_{r\theta}^1 \end{array} \right\} = \left\{ \begin{array}{c} \sigma_{rr}^f \\ \sigma_{r\theta}^f \end{array} \right\} + \sum_{n=0}^{N} A_n^1 \left\{ \begin{array}{c} \sigma_{rr(n)}^{v_1} \\ \sigma_{r\theta(n)}^{v_1} \end{array} \right\} + \sum_{n=0}^{N} B_n^1 \left\{ \begin{array}{c} \sigma_{rr(n)}^{h_1} \\ \sigma_{r\theta(n)}^{h_1} \end{array} \right\} \quad ; \quad r \in D_1 \tag{18}$$

$$\left\{ \begin{array}{c} \sigma_{rr}^2 \\ \sigma_{r\theta}^2 \end{array} \right\} = \sum_{n=0}^{N} A_n^2 \left\{ \begin{array}{c} \sigma_{rr(n)}^{v_2} \\ \sigma_{r\theta(n)}^{v_2} \end{array} \right\} + \sum_{n=0}^{N} B_n^2 \left\{ \begin{array}{c} \sigma_{rr(n)}^{h_2} \\ \sigma_{r\theta(n)}^{h_2} \end{array} \right\} \quad ; \quad r \in D_2 \tag{19}$$

where σ_{rr}^i, $i = 1, 2$, is the radial normal stress and $\sigma_{r\theta}^i$ is the shearing stress. The incident, reflected and scattered wave fields automatically satisfy the traction free conditions at the half-space surface. Therefore, the remaining boundary conditions to be satisfied are:

$$\begin{aligned} \sigma_{rr}^1(r = a, \theta) &= \sigma_{rr}^2(r = a, \theta) \\ \sigma_{r\theta}^1(r = a, \theta) &= \sigma_{r\theta}^2(r = a, \theta) \\ u_x^1(r = a, \theta) &= u_x^2(r = a, \theta) \\ u_z^1(r = a, \theta) &= u_z^2(r = a, \theta) \quad ; \quad 0 \le \theta \le \pi \end{aligned} \tag{20}$$

By imposing these boundary conditions at L points along the interface C, we have a system of linear equations of the form

$$[G]\{c\} = \{f\} \tag{21}$$

where vector $\{c\}$ contains the complex coefficients A_n^i and B_n^i, $i = 1, 2$, vector $\{f\}$ corresponds to the stress and displacement fields of the free field along the interface C, and matrix $[G]$ contains the singular solutions. The size of matrix $[G]$ is $(4L) \times (4N + 4)$, where $4L > (4N + 4)$ is the order for solving eqn(21) in the least square sense. Once the complex coefficients are determined, the displacement fields can be evaluated throughout the elastic medium according to eqn(6) and (7).

EXCITATION: INCIDENT P, SV, AND RAYLEIGH WAVES

The incident P wave can be represented by the potential

$$\phi^i = A_1 e^{-i(\xi x - \alpha z - \omega t)} \tag{22}$$

where

$$\xi = k_p \sin\theta_1 \quad , \quad \alpha = k_p \cos\theta_1$$

θ_1 is the incident angle as shown in Fig. 1, and A_1 is the amplitude. Far from the alluvial valley, the incident wave is reflected from the free surface. A reflected P and SV waves with amplitude A_2 and B_2 will be given by

$$\phi^r = A_2 e^{-i(\xi x + \alpha z - \omega t)} \tag{23}$$

$$\psi^r = B_2 e^{-i(\xi x + \beta z - \omega t)} \tag{24}$$

where $\beta = k_s \cos\theta_2$, θ_2 is the angle of reflection of SV wave, and

$$\frac{A_2}{A_1} = \frac{\sin 2\theta_1 \sin 2\theta_2 - \kappa^2 \cos^2 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + \kappa^2 \cos^2 2\theta_2} \tag{25}$$

$$\frac{B_2}{A_1} = \frac{-2\sin 2\theta_1 \cos 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + \kappa^2 \cos^2 2\theta_2} \tag{26}$$

in which $\kappa = c_p/c_s$ is the ratio of the wave speeds

$$\kappa = \sqrt{\frac{2(1-\sigma)}{1-2\sigma}} \tag{27}$$

with σ being the Poisson ratio of the half-space.

For the incidence of SV wave with amplitude B_1

$$\psi^i = B_1 e^{-i(\xi x - \beta z - \omega t)} \tag{28}$$

When the incident SV waves are reflected from the free surface, two different reflected P waves will result, depending on the incident angle $\theta_1 > \theta_{cr}$(critical angle) or $\theta_1 < \theta_{cr}$. The case where the incident angle large than the critical angle is not considered here because the free field solution becomes complex and causes attenuation in the z-direction. The critical angle, θ_{cr}, is defined by

$$\theta_{cr} = \sin^{-1}\left(\frac{c_s}{c_p}\right) \tag{29}$$

For the case of $\theta_1 < \theta_{cr}$, the reflected waves with potential take the form

$$\phi^r = A_2 e^{-i(\xi x + \alpha z - \omega t)} \tag{30}$$

$$\psi^r = B_2 e^{-i(\xi x + \beta z - \omega t)} \tag{31}$$

with

$$\frac{B_2}{B_1} = \frac{\sin 2\theta_1 \sin 2\theta_2 - \kappa^2 \cos^2 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + \kappa^2 \cos^2 2\theta_2} \tag{32}$$

$$\frac{A_2}{B_1} = \frac{2\kappa^2 \sin 2\theta_2 \cos 2\theta_2}{\sin 2\theta_1 \sin 2\theta_2 + \kappa^2 \cos^2 2\theta_2} \tag{33}$$

For the incidence of Rayleigh wave, the displacements generated by these waves decrease with increasing z and tend to zero as z increases beyond bounds. The corresponding potentials are assumed to be of the form

$$\phi_R = A_3 e^{-b_1 z} e^{-i(k_R x - \omega t)} \tag{34}$$

$$\psi_R = B_3 e^{-b_2 z} e^{-i(k_R x - \omega t)} \tag{35}$$

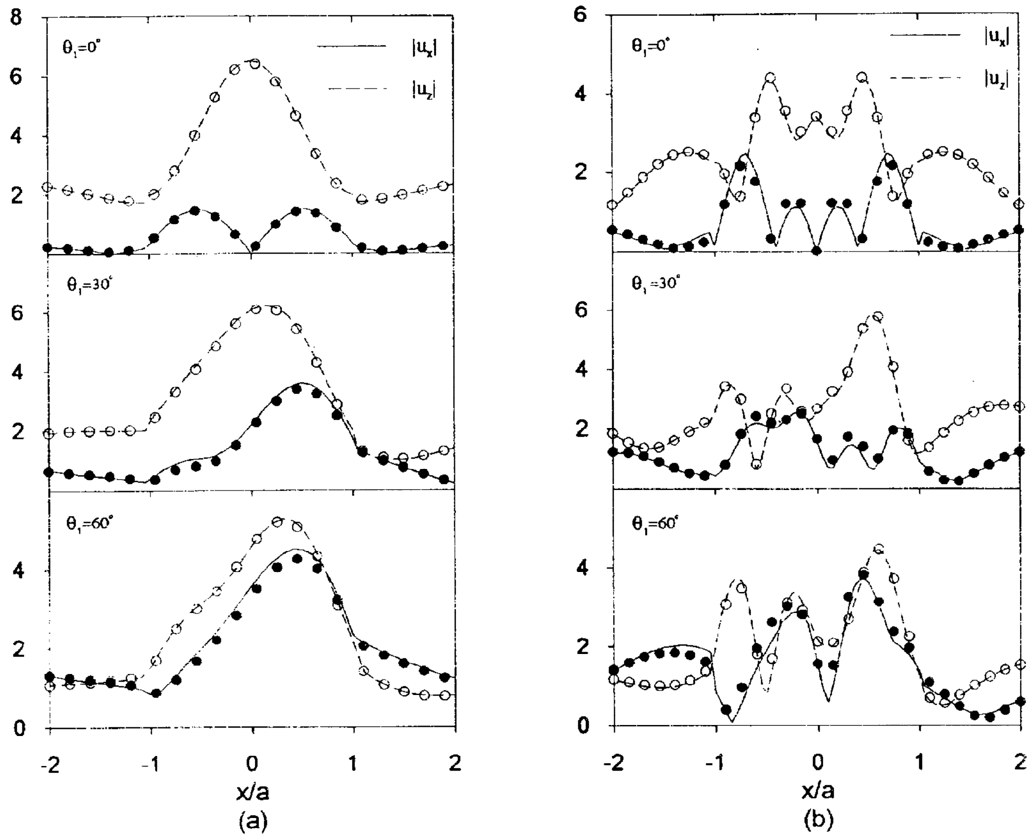

Fig. 3 Amplitudes of the surface displacements generated by incident P wave for η=(a) 0.5, (b) 1.0. Circles represent the results by BEM.

where

$$b_1 = k_R\sqrt{1 - c_R^2/c_p^2} \tag{36}$$

$$b_2 = k_R\sqrt{1 - c_R^2/c_s^2} \tag{37}$$

$k_R = \omega/c_R$ is the Rayleigh wave number, c_R is the Rayleigh wave speed. By imposing the traction free boundary conditions at the free surface of the half-space, one can find the Rayleigh wave speed $c_R = 0.46626c_p$ for Poisson ratio $\sigma = 1/3$ and the ratio of A_3 and B_3 is

$$\frac{B_3}{A_3} = \frac{2b_1 ik_R}{b_2^2 + k_R^2} \tag{38}$$

NUMERICAL RESULTS

The surface displacements are calculated on and near the alluvial valley with different dimensionless frequency $\eta = \omega a/\pi c_{s1}$, which is defined as the ratio of the width of alluvial valley to the incident wave length of the half-plane shear wave. The Poisson's ratio for both regions is taken to be 1/3, and $\mu_1/\mu_2 = 6$, $\rho_1/\rho_2 = 1.5$ for all cases studied here.

The x and z displacements component u_x and u_z can be calculated by eqn(7) for the interior region and by eqn(6) for the exterior region. The corresponding amplitudes of displacements are defined as

$$|u_x| = \sqrt{[Re(u_x)]^2 + [Im(u_x)]^2} \tag{39}$$

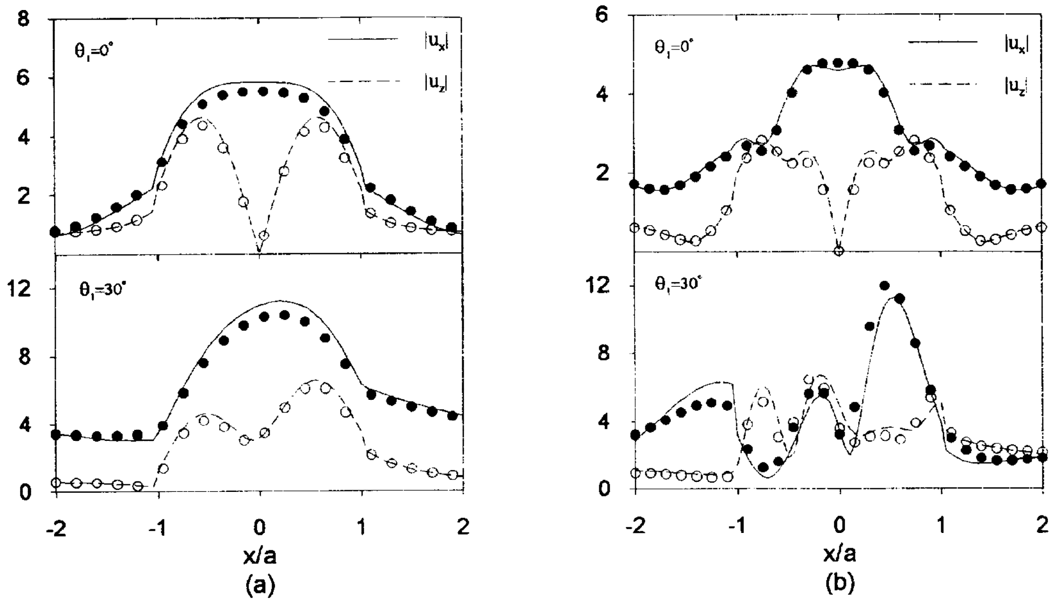

Fig. 4 Amplitudes of the surface displacements generated by incident SV wave for η=(a) 0.5, (b) 1.0. Circles represent the results by BEM.

$$|u_z| \quad = \quad \sqrt{[Re(u_z)]^2 + [Im(u_z)]^2} \qquad (19)$$

The order of the expansions and the number of the collocation points depend on the excitation frequency, more collocation points and higher order of expansion are required at higher frequencies in this method. For a semi-cylindrical alluvial valley subjected to incident P waves with different incident angle and frequency, the displacement amplitude $|u_x|$ and $|u_z|$ versus the dimensionless distance x/a are depicted by Fig. 3, and compared to the results obtained by boundary element method provided by Yeh *et al.*(1995a). Fig. 4 and 5 shows the diffraction on a semi-circular alluvial valley for incident SV and Rayleigh waves, respectively.

CONCLUDING REMARKS

Some results are presented for the surface displacement generated by incidence of P, SV, and Rayleigh waves upon a semi-cylindrical alluvial valley on and near the surface of an elastic half-plane. These results are obtained by means of Maunsell's method using n-th order Lamb's singular solution as series expansion functions for the interior and exterior regions. Boundary conditions at the interface C itself are satisfied in the least square sense. Excellent agreements with the solution by BEM are shown. This fact and the simplicity of the procedure presented confirm that the method can be used with advantages in many problems of earthquake engineering and seismology.

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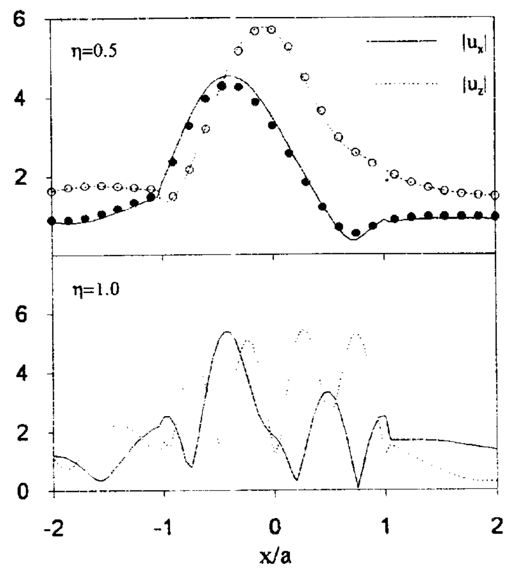

Fig. 5 Amplitudes of the surface displacements generated by incident Rayleigh wave. Circles represent the results by BEM.

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