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OPTIMAL DESIGN OF SUPPLEMENTAL DAMPERS FOR CONTROL OF STRUCTURES

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ABSTRACT

This paper presents a method for the design of supplemental dampers and uses experimental results with viscous dampers and friction devices to design equivalent friction dampers. Optimal control theory is used to design linear passive devices dependent on velocity. The design is aimed at minimizing a performance cost function which produces a configuration of devices that maximizes their effect. One optimal strategy is proposed so as to enable the design engineer to determine the best configuration of practical dampers for a wide range of excitations.

KEYWORDS

Passive Energy Dissipators (PED); Viscous and friction dampers; Optimal control theory; Autonomous optimal design;

INTRODUCTION

Active control theory provides a suitable framework for design of control systems in which forces are introduced in structures to reduce the unwanted effects of vibrations. The control theories assume that each force generating device has the capability to process simultaneously information from all observable sensors and generate compatible forces. This control can be obtained using either active or semi-active operating systems. Passive devices (Hanson et al., 1993; Constantinou et al., 1994) produce forces depending either on their elongation or internal velocity, or both, dictated by the structure movement. The parameters that govern such behavior are fixed by design. For example, viscoelastic type damping devices develop forces which can be approximated by $F_d = k_d x(t) + c_d \dot{x}(t)$ in which k_d and c_d are constant parameters for unique frequency input and constant temperature. The optimal linear control approach is used in this paper to determine the constant coefficients for the damping devices. Inaudi et al., (1993) used a stochastic linearization along with a similar optimization procedure to determine initial design values for damping devices. The procedure can be used with some approximation for design of other force delivery devices with passive characteristics (friction or hysteretic), (Constantinou et al., 1994). The process is illustrated by a design example for a small model structure used for laboratory experiments.

OPTIMAL CONTROL THEORY

For a frame structure braced by devices that control its vibration the equation of motion may be written as:

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=\mathbf{E}\mathbf{f}(t)+\mathbf{D}\mathbf{u}(t) \tag{1}$$

in which, matrices **M**, **C**, **K** characterize mass, structural damping and stiffness related to the deformations **x**(t) at various degrees of freedom. The brace forces are included in the system as control forces **u**(t) placed according to the location matrix **D** designed to reduce the response due to excitation forces, **f**(t) at locations indicated by **E**. The equation of motion can be easily compacted to a state space formulation (Soong 1990):

$$\dot{\mathbf{z}}(t)=\mathbf{A}\mathbf{z}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{H}\mathbf{f}(t) \tag{2}$$

where, $\mathbf{z}(t)=\{\mathbf{x}(t),\dot{\mathbf{x}}(t)\}^{\mathrm{T}}$, and the parameter matrices for the system, **A**, for the control location, **B**, and for force operation, **H**, are: $\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{1}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}\end{bmatrix}$, $\mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{M}^{-1}\mathbf{D}\end{bmatrix}$, $\mathbf{H}=\begin{bmatrix}\mathbf{0}\\ -\mathbf{M}^{-1}\mathbf{E}\end{bmatrix}$. Assuming for the sake of simplicity that the control forces are linearly defined as:

$$\mathbf{u}(t)=\mathbf{G}\mathbf{z}(t)=[\mathbf{G}_{\mathbf{x}}\vdots\mathbf{G}_{\dot{\mathbf{x}}}]\mathbf{z}(t)=\mathbf{G}_{\mathbf{x}}\mathbf{x}(t)+\mathbf{G}_{\dot{\mathbf{x}}}\dot{\mathbf{x}}(t) \tag{3}$$

in which, the gain matrix, **G** includes the constant coefficients, $\mathbf{G}_{\mathbf{x}}$, $\mathbf{G}_{\dot{\mathbf{x}}}$ for the structural control devices. The equation of motion reduces to:

$$\dot{\mathbf{z}}(t)=\mathbf{A}_{\mathbf{c}}\mathbf{z}(t)+\mathbf{H}\mathbf{f}(t) \tag{4}$$

in which, the matrix of the controlled system, $\mathbf{A}_{\mathbf{c}}$, is: $\mathbf{A}_{\mathbf{c}}=\mathbf{A}+\mathbf{B}\mathbf{G}$. The gain matrix **G**, (the matrix of coefficients) can be determined by minimizing a quadratic performance index:

$$\mathbf{J}=\int_{0}^{t_f}\left(\mathbf{z}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{z}(t)+\mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\right)dt \tag{5}$$

constrained by the equilibrium equation, Eq. 4. The matrices **Q** and **R** are weighting matrices of factors for the optimization. The selection of matrices **Q** and **R** will enable solutions within the structural or resources limitations as illustrated further in the numerical examples. The gain matrix **G** is obtained from the minimization of the performance index, **J**, as:

$$\mathbf{G}=-0.5\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P} \tag{6}$$

in which, **P** is the solution of the algebraic Ricatti equation:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}+\mathbf{P}\mathbf{A}-0.5\mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{P}+2\mathbf{Q}=\mathbf{0} \tag{7}$$

If the control forces are supplied by passive viscous dampers in a structure (attached to the chevron brace in Fig. 1) then

$$\mathbf{u}^{*}(t)=\mathbf{C}_{\dot{\mathbf{x}}}\dot{\mathbf{x}}(t) \tag{8}$$

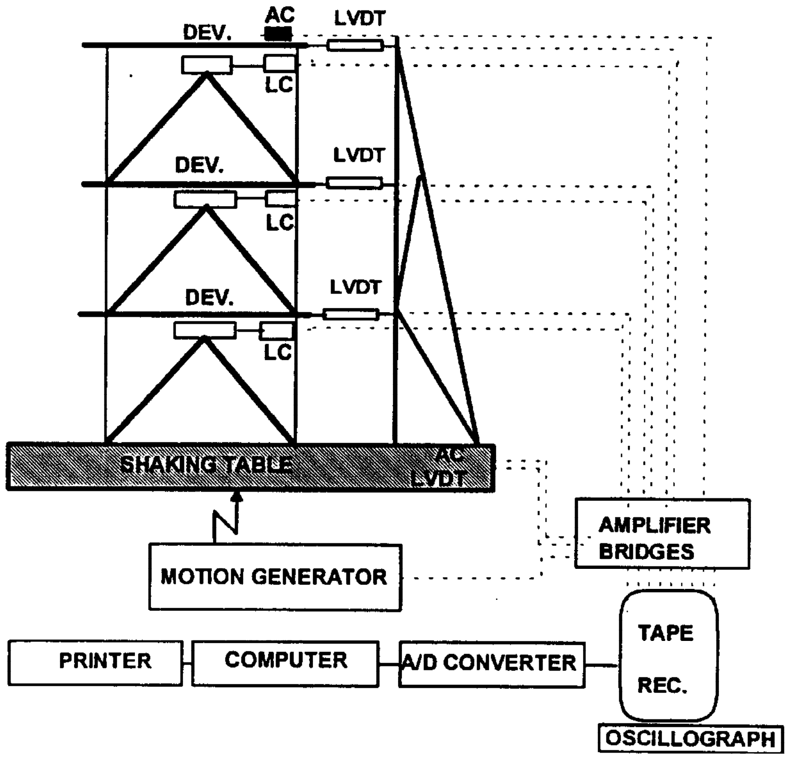

Fig 1. Schematic of an experimental passively controlled steel frame

The damping coefficients are derived below from the, now known, gain coefficients. Equations. 3 and 8 are now transformed using story-drift formulation obtained from the linear transformation $\mathbf{x}(t) = \mathbf{T}\mathbf{d}(t)$ where

$\mathbf{T} = \begin{bmatrix} 1 & & & & 0 \\ 1 & 1 & & & \\ 1 & 1 & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$. The forces in the braces, thus, become: $\mathbf{v}(t) = \mathbf{T}^{\mathrm{T}}\mathbf{u}(t)$ where, $\mathbf{v}(t)$ are the control forces in terms of story drifts and drift velocities obtained from Eq. 3 are given as:

$$\mathbf{v}(t) = \mathbf{G}_{\mathbf{d}}\mathbf{d}(t) + \mathbf{G}_{\dot{\mathbf{d}}}\dot{\mathbf{d}}(t) \tag{9}$$

in which $\mathbf{G}_{\mathbf{d}} = \mathbf{T}^{\mathrm{T}}\mathbf{G}_{\mathbf{x}}\mathbf{T}$, and $\mathbf{G}_{\dot{\mathbf{d}}} = \mathbf{T}^{\mathrm{T}}\mathbf{G}_{\dot{\mathbf{x}}}\mathbf{T}$. Using the same transformation the brace forces of Eq. 8 can be written as:

$$\mathbf{v}^*(t) = \mathbf{C}_{\mathbf{d}}\dot{\mathbf{d}}(t) \tag{10}$$

in which: $\mathbf{C}_{\mathbf{d}} = \mathbf{T}^{\mathrm{T}}\mathbf{C}_{\dot{\mathbf{x}}}\mathbf{T} = \mathrm{diag}(\Delta c_i)$. Also $\mathbf{K}_{\mathbf{d}} = \mathbf{T}^{\mathrm{T}}\mathbf{K}_{\mathbf{x}}\mathbf{T} = \mathrm{diag}\,(\Delta k_i)$ and $\mathbf{M}_{\mathbf{d}} = \mathbf{T}^{\mathrm{T}}\mathbf{M}\mathbf{T}$ where, Δk_i, Δc_i are supplemental stiffness (not considered in this paper) and damping properties from each brace in the structure at level i.

Applying the least squares approximation to the difference between the formulations of Eq. 9 (2nd term only) and Eq. 10 leads to:

$$\frac{d}{d\dot{d}_k(t)}\left\{\int_0^{t_f}\sum_j\left[g_{kj,d}\dot{d}_j(t)-\Delta c_k\dot{d}_k(t)\right]^2 dt\right\}=0 \tag{11}$$

to yield:

$$\Delta c_k = \frac{\int_0^{t_f}\sum_j g_{kj,d}\dot{d}_j(t)dt}{\int_0^{t_f}\dot{d}_k(t)dt} \tag{12}$$

where t_f is the total time for the event considered. Equation 12 is further simplified below using a single mode.

The Single Mode Approach

Assume that the velocity can be obtained from the modes using the square root of the sum of the squares as $\dot{d}_{ji}=\left[\sum_i\left(\phi_{ji}P_iS_{vi}\right)^2\right]^{\frac{1}{2}}$ where $\dot{d}_{ji}$ = velocity in mode i at d.o.f. j; ϕ_{ji} = mass normalized modal shapes; P_i = modal participation factor ($=\sum_j m_j\phi_{ji}$); S_{vi} = spectral velocity of mode i. Equation 12 becomes:

$$\Delta c_k = \frac{\sum_i g_{kj,\dot{d}}\left[\sum_i\left(\phi_{ji}P_iS_{vi}\right)^2\right]^{\frac{1}{2}}}{\left[\sum_i\left(\phi_{ki}P_iS_{vi}\right)^2\right]^{\frac{1}{2}}} \tag{13}$$

The modal spectral formulation presented above includes the influence of all or several modes of vibration. In applications involving buildings under earthquake excitations only one mode of vibration is often relevant. In that case Eq. 13 reduces to:

$$\Delta c_k = \frac{\sum_i g_{kj,\dot{d}}\phi_{jm}}{\phi_{km}} = \sum_i g_{kj,\dot{d}}\phi_{jm}^k \tag{14}$$

where, $\phi_{jm}^k = \phi_{jm}/\phi_{km}$ is the modal shape normalized to unit at degree-of-freedom k. The damping coefficients are no longer dependent on the history of the event, but on the characteristics of the structure.

The optimal solution given by Eq. 9 can be implemented only by active means, which can provide the combination of information from all degrees-of-freedom through the gain matrix, **G**, whereas the use of passive braces with viscous added damping or friction dampers will provide an *autonomous optimal design* for protection of the structure against any earthquake.

NUMERICAL EXAMPLES

To illustrate the procedures outlined above and the performance of optimally, or near optimally designed dampers, a 1:5 scale three-story model structure having a span of *0.8 m* and a floor height of *0.6 m* made of two flexible shear frames with rigid floors is considered (see Fig. 1). The complete mass, stiffness and structural damping of the structure without dampers, as obtained from structural identification are:

$$\mathbf{M} = \begin{bmatrix} 200.40 & 0 & 0 \\ 0 & 200.40 & 0 \\ 0 & 0 & 178.00 \end{bmatrix} [kg]; \quad \mathbf{K} = \begin{bmatrix} 238{,}932 & -119{,}466 & 0 \\ -119{,}466 & 238{,}932 & -119{,}466 \\ 0 & -119{,}466 & 119{,}466 \end{bmatrix} [N/m];$$

$$\mathbf{C} = \begin{bmatrix} 264.99 & -78.09 & -16.08 \\ -78.09 & 2246.89 & -92.15 \\ -16.08 & -92.15 & 162.02 \end{bmatrix} [N \cdot sec/m].$$

The natural frequencies of the structure are *1.78 Hz*, *4.96 Hz*, and *7.05 Hz* for the first three modes in the direction under consideration, respectively. Fluid viscous dampers with linear functions of velocity within the expected range of operations and without stiffening characteristics ($k_d = 0$) are considered in this paper. The autonomous optimal design of the dampers was developed using the procedure outlined above for weighting matrices, **R** and **Q** (in Eq. 5) of: $\mathbf{R} = 10^{-p}\mathbf{I}_{3\times3}$; $\mathbf{Q} = \mathbf{I}_{6\times6}$; in which, $\mathbf{I}_{n\times n}$ indicates a unit diagonal matrix of size n × n, and p is a variable parameter used to adjust the solution's weights toward the practical range. The optimal solution was obtained by solving the algebraic Ricatti equation (Eq. 7) using MATLAB™ package. The gain matrix was determined using Eq. 6. The gain matrices were obtained for p varying between *2* and *9*. By increasing the parameter p, the demand for damping increases and the response decreases. Therefore by increasing p one can increase the size of dampers up to the limit of "shelf" availability. Following are results for $p=6$. The system matrix **A**, the controller matrix **B** and the Riccati matrix **P** are given below. The gain matrix is obtained from Eq. 6. This matrix is a *3*-row by *6*-column matrix. It is the last three columns that are of interest since they constitute matrix $\mathbf{G}_{\dot{\mathbf{x}}}$. The control forces in terms of story drifts require the matrix $\mathbf{G}_{\dot{\mathbf{d}}}$ which was defined as $\mathbf{G}_{\dot{\mathbf{d}}} = \mathbf{T}^{\mathrm{T}}\mathbf{G}_{\dot{\mathbf{x}}}\mathbf{T}$. Finally the supplemental damping properties Δc_k are obtained using Eq. 14 and the 1st mode, $\mathbf{\Phi}_1$, which comes from the eigenvalue problem when taking $\mathbf{K_d}$ and $\mathbf{M_d}$ instead of **K** and **M**.

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1.0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.0 \\ -1192.3 & 596.14 & 0 & -1.3223 & 0.38968 & 0.08026 \\ 596.14 & -1192.3 & 596.14 & 0.38968 & -1.232 & 0.45985 \\ 0 & 671.16 & -671.16 & 0.090365 & 0.51772 & -0.91023 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.005 & -0 & 0 \\ 0 & 0.005 & 0 \\ 0 & -0 & 0.00562 \end{bmatrix};$$

$$\mathbf{P} = \begin{bmatrix} 357.49 & -160.73 & -10.655 & 0.0132 & 0.0247 & -0.0089 \\ -160.73 & 345.98 & -170.41 & -0.0163 & 0.0136 & 0.0371 \\ -10.655 & -170.41 & 184.73 & 0.0081 & -0.0184 & -0.0148 \\ 0.0132 & -0.0163 & 0.0081 & 0.3121 & 0.0247 & 0.006 \\ 0.0247 & 0.0136 & -0.0184 & 0.0247 & 0.3193 & 0.0298 \\ -0.0089 & 0.0371 & -0.0148 & 0.006 & 0.0298 & 0.3018 \end{bmatrix};$$

$$\mathbf{G}_{\dot{\mathbf{x}}} = \begin{bmatrix} 778.78 & 61.66 & 15.00 \\ 61.66 & 796.56 & 74.37 \\ 16.88 & 83.73 & 847.85 \end{bmatrix}; \quad \mathbf{G}_{\dot{\mathbf{d}}} = \begin{bmatrix} 2736.5 & 1879.2 & 937.2 \\ 1881.1 & 1802.5 & 922.2 \\ 948.5 & 931.6 & 847.9 \end{bmatrix}; \quad \mathbf{\Phi}_1 = \begin{Bmatrix} 0.7469 \\ 0.5895 \\ 0.3078 \end{Bmatrix}; \quad \mathbf{\Delta c} = \begin{bmatrix} 4606 & 0 & 0 \\ 0 & 4667 & 0 \\ 0 & 0 & 4934 \end{bmatrix}.$$

The above design was evaluated for the earthquake at El Centro (N-S 1940 accelogram with peak ground acceleration, (PGA) of 0.34g) using step-by-step dynamic analysis based on Newmark-beta scheme. The drift at the first floor of the structure is shown in Fig. 2. The peak response is shown in Table 1.

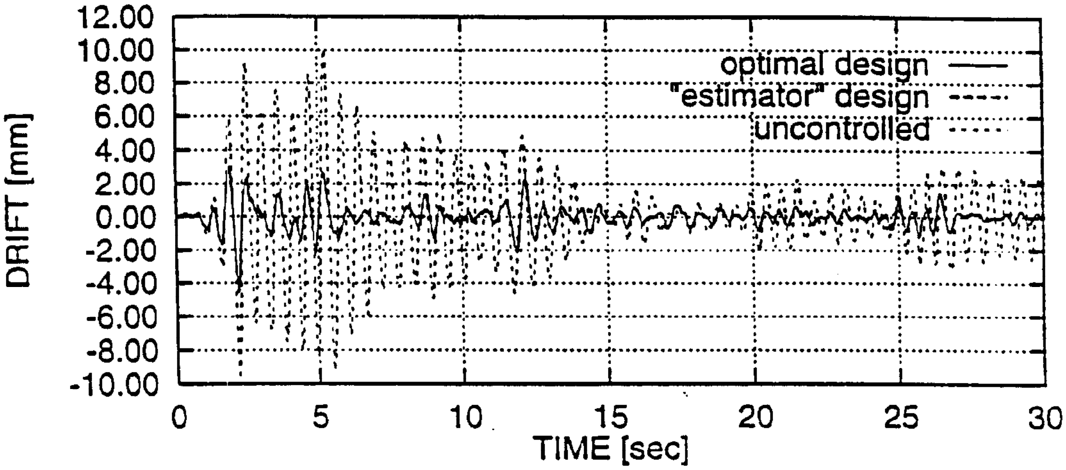

Fig. 2. First story drift - El Centro 1940

Table 1. - Peak Response of Top Floor Subjected to Ground Motion

	El Centro 1940		
	Displacement [mm]	Velocity [m/s]	Acceleration [m/s^2]
Uncontrolled	22.5	0.25	3.10
Optimal Design	9.20	0.10	1.55
Single Mode Design	9.30	0.10	1.52

The structure without dampers has the largest response ("uncontrolled"), while the response using the design based on single-mode approach is almost identical to that for the exact optimal solution (which can be obtained only by active means and considers the gain matrix). The single-mode approach used the first mode of the structure, which is dominant in earthquake response of tall buildings. If there is uncertainty as to which modes might be dominant, then the response spectrum design can be used.

FRICTION DAMPING EQUIVALENT

This section describes a procedure for deducing the friction damping characteristics (as defined by Reinhorn *et al*, 1995) that will produce an equivalent response to that of the viscous dampers. Equivalence is achieved as follows. The structural model in the experimental setup of Fig. 1 is excited by five earthquakes and the first floor force-displacement hysteresis recorded. The equivalent friction hysteresis is required to have the same area of the maximum displacement curve cycle (maximum energy dissipation cycle) as the viscous hysteresis for a statistical value of the mean + one standard deviation (SD). Dividing the area by four times the maximum displacement will yield the slippage force and, thus, define the friction damping characteristic.

Table 2 shows the controlled first level peak displacement as a percentage of the uncontrolled first level peak displacement for two experimental setups. In the first setup the structure was equipped with viscous dampers whereas in the second it was equipped with closely equivalent friction dampers. The mean+SD shows an equivalence of *1.176* i.e. that the viscous hysteresis area divided by *1.176* should be taken for comparison so that both the friction and the viscous dampers will exhibit the same response reduction of *15.87%*.

Table 3. shows the viscous maximum energy dissipation cycle from which the equivalent friction force is determined for each earthquake considered resulting in a mean+SD force of *225N*. Figures of the like of Fig. 3 for all the earthquakes were used to evaluate the entries. According to Table 2 the friction force at first level should be *225/1.176= 191.33N*. The force, as a matter of fact, was *200N* which is clearly seen in the friction damping hysteresis in Fig 4.

Table 2. First level control effect

Earthquake	Influence	
	Viscous	Friction
El Centro	46%	50%
Taft	63%	74%
Parkfield	30%	28%
Pacoima Dam	75%	86%
Mexico	58%	69%
	mean+SD=71.55%	mean+SD=84.13%
	E=84.13/71.55=1.176	

Table 3. First level viscous maximum energy dissipation cycle

Earthquake	Viscous dissipating energy [*N·cm*]	Force for equivalent friction [*N*]	Mean+SD [*N*]
El Centro	334	155	
Taft	318	162	
Parkfield	185	132	225
Pacoima Dam	1053	267	
Mexico	414	135	

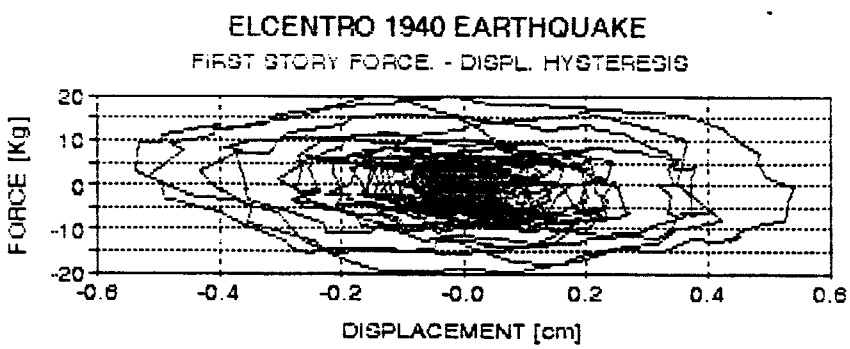

Fig. 3. Experimental viscous damping hysteresis

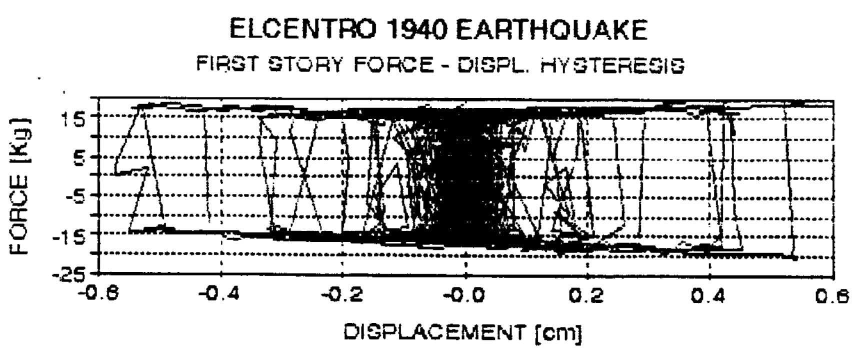

Fig. 4. Experimental viscous damping hysteresis

CONCLUSIONS

The paper presents a method for the design of supplemental passive damping devices based on optimal linear control theory. The design presented can be used to size viscous, or added damping devices, for a preset location. The design considered herein is based on the optimal control gains, which can actually be implemented only by an active system. The numerical example shows that passive devices are effective if, however, the structures are dominated by only one mode of vibration. The design solution for the passive devices is always stable, as provided implicitly by the formulation based on the Riccati equation. It is free of time delays (implicitly avoided by all the reactive passive systems) and reliable.

The procedure outlined in the paper can be used to size also friction dampers for which the reaction force is dependent on an adjustable normal force to the surfaces of friction. This was successfully done (4.3%) by extracting the appropriate slippage force in the friction device from the maximal experimental hysteretic response in viscous fluid dampers.

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