Paper No. 1529. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

ROBUST CONTROL OF A TOWER-LIKE STRUCTURE SUBJECTED TO SEISMIC EXCITATION TAKING ACCOUNT OF PARAMETER VARIATIONS AND RESIDUAL DYNAMICS.

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ABSTRACT

This paper discusses about robust control of structural vibrations taking account of parameter variations and residual dynamics of a flexible structure. The structure has four transverse and four torsional degrees-of-freedom. They may possess transverse-torsional coupled modes due to parameter variations. In this study, a controller design method is proposed by taking account of the parameter variations and the residual dynamics. The $H\infty/\mu$ control theory is applied to synthesize the controller. To examine the capability of the obtained controller, numerical calculations are carried out. An ordinary LQG controller is also designed for the comparison. It is shown that the performance of the $H\infty$ controller taking account of the variations and residual dynamics keeps enough robustness, although the performance of the ordinary LQG controller without the considerations for the perturbations is degraded by the transverse-torsional coupling. Moreover, the robust performance is enhanced by applying μ-synthesis to the proposed method. Computer simulations using external excitation wave records are also performed. The μ-synthesised controller achieved good robust performance even for seismic excitations, while the $H\infty$ controller possessed larger peak responses.

KEY WORDS

Vibration control, Robust control, Active dynamic vibration absorber, Flexible structure, Seismic excitation, $H\infty/\mu$ control, Transverse-torsional coupled vibration

INTRODUCTION

Vibration control of flexible structure is one of the major interests in mechanical, civil and aerospace engineerings. In the design of active vibration control system, the robustness of the controller is an important issue. There are two typical perturbation for the control system, one is the effect of the residual dynamics of the structure, and the other is parameter variations. The effect of residual dynamics is often called spillover. Many design methods are proposed to obtain a robust controller for spillover. Balas (1978) suggested to apply low-pass filter for control input. Sesak and Likins (1979) proposed model error sensitivity suppression (MESS). Yoshida (1989) formulated spillover as a control constraints problem and showed that the technique of output feedback control is applicable. Recently, the applications of robust control theory like $H\infty$ control theory are developed (Balas *et al.* , 1988, Nishimura *et al.* , 1992).

Meanwhile, parameter variations (or parameter uncertainties) are also an important issue among civil and space structures. Wang *et al.* (1992), Adams and Banda (1992) and Wie *et al.* (1992) discussed such problems on "benchmark problem". Watanabe and Yoshida (1994) applied μ-synthesis and showed that the D-scaling matrix can be represented as a modification of decomposition of parameter variations.

This paper discusses on the vibration control of a flexible structure. The aim of the control is to reduce vibrations of the lowest two modes with robustness. A controller design methods proposed by taking account of the parameter variations and the residual dynamics. The H∞/μ control theory is applied to synthesize the controller. To examine the capability of the obtained controller, computer simulations are carried out. An ordinary LQG controller is also designed for the purpose of comparison. Theoretical calculations are carried out to examine the effectiveness of the proposed method.

CONTROL OBJECT

Structure Model

The object of vibration control is a four-storied tower-like structure. The schematic diagram of the theoretical model of the structure is shown in figure 1. Each story is modeled so as to have one degree-of-freedom in transverse direction (in the direction of excitation) and another in angle of torsion around the centeroid of the story. Therefore, theoretical model of the whole structure has eight degrees-of-freedom.

The matrix equation of motion of the structure can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_O + \boldsymbol{C}\dot{\boldsymbol{x}}_O + \boldsymbol{K}\boldsymbol{x}_O + \boldsymbol{d}\ddot{z} + \boldsymbol{b}\boldsymbol{f} = \boldsymbol{0} \tag{1}$$

$$\boldsymbol{x}_O^T = \left[x_1\ x_2\ x_3\ x_4\ \theta_1\ \theta_2\ \theta_3\ \theta_4 \right] \tag{2}$$

$$\boldsymbol{M} = diag\lfloor m_1\ m_2\ m_3\ m_4\ I_1\ I_2\ I_3\ I_4 \rfloor \tag{3}$$

$$\boldsymbol{K} = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & \overline{k_1l_1}+\overline{k_2l_1} & -\overline{k_2l_2} & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & -\overline{k_2l_1} & \overline{k_2l_2}+\overline{k_3l_2} & -\overline{k_3l_3} & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 & -\overline{k_3l_3} & \overline{k_3l_3}+\overline{k_4l_3} & -\overline{k_4l_4} \\ 0 & 0 & -k_4 & k_4 & 0 & 0 & -\overline{k_4l_4} & \overline{k_4l_4} \\ \overline{k_1l_1}+\overline{k_2l_1} & -\overline{k_2l_1} & 0 & 0 & k_1l_1^2+k_2l_1^2 & -k_2l_1l_2 & 0 & 0 \\ -\overline{k_2l_1} & \overline{k_2l_2}+\overline{k_3l_2} & -\overline{k_3l_2} & 0 & -k_2l_1l_2 & k_2l_2^2+k_3l_2^2 & -k_3l_2l_3 & 0 \\ 0 & -\overline{k_3l_2} & \overline{k_3l_3}+\overline{k_4l_3} & -\overline{k_4l_3} & 0 & -k_3l_2l_3 & k_3l_3^2+k_4l_3^2 & -k_4l_3l_4 \\ 0 & 0 & -\overline{k_4l_3} & \overline{k_4l_4} & 0 & 0 & -k_4l_3l_4 & k_4l_4^2 \end{bmatrix} \tag{4}$$

$$\boldsymbol{d}^T = \lfloor m_1\ m_2\ m_3\ m_4\ 0\ 0\ 0\ 0 \rfloor \tag{5}$$

$$\boldsymbol{b}^T = \begin{bmatrix} 0\,0\,0\,1\,0\,0\,0 & l_{fR} \\ 0\,0\,0\,1\,0\,0\,0 & -l_{fL} \end{bmatrix}, \boldsymbol{f} = \begin{bmatrix} f_R \\ f_L \end{bmatrix} \tag{6}$$

$\boldsymbol{x}_o$ is the state vector, and $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the inertia, damping and stiffness matrices of the structure, respectively. $\boldsymbol{d}$ is the noise matrix for excitation acceleration z, and $\boldsymbol{b}$ is the input matrix for the control forces $\boldsymbol{f}$. x_i and θ_i are transverse and $\boldsymbol{x_o}$ is the state vector, and $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the inertia, damping and stiffness matrices of the structure, respectively. $\boldsymbol{d}$ is the noise matrix for excitation acceleration z, and $\boldsymbol{b}$ is the input matrix for the control forces $\boldsymbol{f}$. x_i and θ_i are transverse and torsional displacements of the ith story, m_i and I_i are the mass and the moment of inertia, K_{Ri} and K_{Li} are the spring constants of right and left side of the ith story, l_{Ri} and l_{Li} are the distances from the centroid to the spring of right and left side of the ith story, l_{fR} and l_{fL} are the distances from the centeroid of the 4th story to the right and left actuators, respectively. And the term $k_i l_j$ in the matrix $\boldsymbol{K}$ denotes the cross term between x_i and θ_i, that is

$$\overline{k_i l_i} = k_{Ri} l_{Ri} - k_{Li} l_{Li} \tag{7}$$

The mass distribution of each story is homogeneous and the stiffnesses of four columns are supposed to be the same in the direction of the excitation at all stories. In this condition, l_{Ri} and l_{Li} are all equal and all the cross terms are equal to zero. In this study, however, it is assumed that a lumped load is mounted on the third story and its position is variable between the right end and the left end of the story. Therefore, l_{R3} and l_{L3} is equal to zero and transverse and torsional vibrations are independent as long as the load is located in the middle of the story. If not, the cross terms have certain values and the structure possesses transverse-torsional coupled vibration modes. The parameters of the structure are shown in table 1.

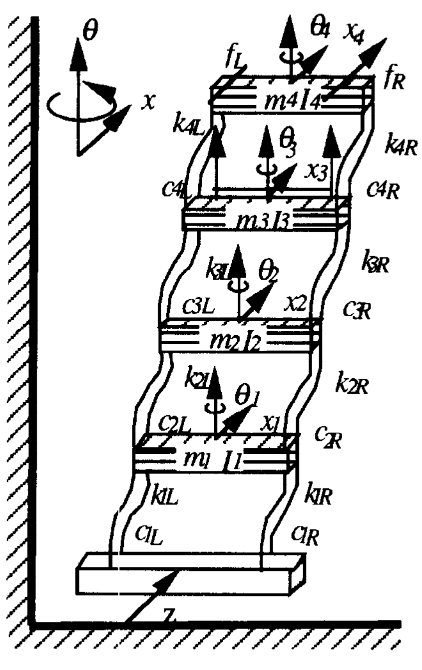

Fig.1 Schematic diagram of structure

Table.1 Parameters of structure
(lumped load centered)

rr a s s	Side	Mass [kg]	Moment of inertia [10 -2 kg•m]	Stiff-ness [kN/m]	Length from centroid to stiff. [cm]
4	R	2.51	3.10	16.0	19.3
	L			16.0	19.3
3	R	2.46	1.40	4.45	14.0
	L			4.45	14.0
2	R	2.42	1.30	4.45	13.8
	L			4.45	13.8
1	R	1.75	1..30	5.93	13.8
	L			5.93	13.8

Active Dynamic Vibration Absorber Model

In this study, two absorbers are mounted on the top of the structure at both edges in parallel. Each absorbers is composed of an auxiliary mass, a moving coil to drive the mass and a linear bearing to support them. The strokes of both absorbers are measured by non-contact gap sensors. Two accelerometers are also mounted on the side of each absorber to detect the accelerations of the structure at both edges. The equations of the circuit is described as

$$L\dot{i} + Ri + E\dot{x}_a = e\ , \ f = Ti \tag{8}$$

where i is current, L is inductance, R is reactance, E is induced voltage coefficient, e is input voltage, f is force and T is thrust constant of each circuits. In addition, the equation of motion of the absorbers is written

$$m_a\left(\ddot{x}_a + \ddot{x}_4 \pm l_f\ddot{\theta}_4 + \ddot{z}\right) = f \ (+\text{: right side ADVA}, \ -\text{: left side ADVA}) \tag{9}$$

where m_a is the auxiliary mass, x_a is the stroke.

State Equation of Control Object

Combining the above-mentioned matrix equations, the state equation of the control object can be written as

$$\dot{\boldsymbol{x}}_f = \boldsymbol{A}_f\boldsymbol{x}_f + \boldsymbol{B}_f\boldsymbol{e} + \boldsymbol{D}_f\ddot{z} \tag{10}$$

where $\boldsymbol{x}_f$ is the state vector of the whole control object, $\boldsymbol{A}_f$ is the system matrix, $\boldsymbol{B}_f$ is the input matrix, $\boldsymbol{D}_f$ is the noise matrix and $\boldsymbol{e}$ is the control voltage matrix.

All the state variables included in the state vector (13) are strictly observable. When applying the ADVA system to an actual structure, however, it is impractical to measure all the variables since it needs so many sensors. In this study, the available outputs of the structure are limited to be the strokes of ADVAs and the accelerations of the structure at the both edges. Using the output matrix y_f, the output matrix equation of the control object is written as follows.

$$\mathbf{y}_f = \mathbf{C}_f\mathbf{x}_f = \left[x_{aR} \ \ x_{aL} \ \ \ddot{y}_{4R} \ \ \ddot{y}_{4L}\ \right] \tag{11}$$

where y_{4R}, and y_{4L} are absolute displacements of the 4th story at right and left edge.

CONTROLLER DESIGN

Description of Parameter Variations

As presented in the former sections, the structure has parameter variations due to uncertain location of the lumped mass. Choosing the case that the mass is located at the middle of the third story as a "nominal" model, the system and output matrix equations with parameter variation can be written as follows:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= (\boldsymbol{A} + \Delta\boldsymbol{A})\boldsymbol{x}(t) + (\boldsymbol{B} + \Delta\boldsymbol{B})\boldsymbol{u}(t) + \boldsymbol{H}\ddot{z} \\ \boldsymbol{y}(t) &= (\boldsymbol{C} + \Delta\boldsymbol{C})\boldsymbol{x}(t) + (\boldsymbol{E} + \Delta\boldsymbol{E})\boldsymbol{u}(t) \end{aligned} \tag{12}$$

$\Delta\boldsymbol{A}$, $\Delta\boldsymbol{B}$, $\Delta\boldsymbol{C}$ and $\Delta\boldsymbol{E}$ denote the maximum fluctuation of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{E}$ matrices when the location of lumped mass is changed, respectively. In this case, however, $\Delta\boldsymbol{B}$ and $\Delta\boldsymbol{E}$ are equal to zero because the lumped mass is located on the third story and its change in position does not effect on $\boldsymbol{B}$ and $\boldsymbol{E}$ in this problem. In general, the control object has certain residual (uncontrolled or unmodeled) dynamics. In this study, we take straightforward approach to represent these uncertainty in the framework of H^{∞} control, and compensate roughness of modeling by applying scaling in the process of μ-synthesis, later. First, we represent the uncertainties as follows:

$$\Delta A = I_a \cdot \Delta_a \cdot \Delta A, \quad \Delta C = I_c \cdot \Delta_c \cdot \Delta C \tag{13}$$

where I_i are unit matrices, Δ_i are diagonal matrices which denote the range of variations.

Description of Uncontrolled Dynamics

In flexible structures, it is quite difficult to identify parameters for higher vibration modes exactly. So the aim of control is usually limited to lowest few modes. Nevertheless, it often causes spillover if the controller is designed without taking account of it. Therefore, in the ordinary H^∞ controller design, weighting functions are applied for control inputs to reduce feedback gains for such higher vibration modes. In this study, we also apply high pass filter as weighting functions for control forces u_r and u_l. In a sense, this filter denotes variations of B matrix. So this method is applicable even when ΔB exists.

In this study, the weighting function is realized by using 4th order high-pass filter

$$W_H(s) = G_H \frac{\left(s^2 + 2\varsigma_\gamma \omega_\gamma + \omega_\gamma^2\right)\left(s^2 + 2\varsigma_\delta \omega_\delta + \omega_\delta^2\right)}{\left(s^2 + 2\varsigma_\alpha \omega_\alpha + \omega_\alpha^2\right)\left(s^2 + 2\varsigma_\beta \omega_\beta + \omega_\beta^2\right)} \tag{14}$$

where s is the Laplace operator. The filters are designed on s-domain and converted to state- space formulae to describe generalized plant in state-space.

H∞/μ Controller Design

According to these forms, the generalized plant is formulated to the following expression.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 w(t) + B_2 u(t) \\ z(t) &= C_1 x(t) \qquad\qquad\quad + D_{12} u(t) \\ y(t) &= C_2 x(t) + D_{21} w(t) + D_{22} u(t) \end{aligned} \tag{15}$$

The next step is to compute H^∞ controller. We employed computer-aided control system design tool "MATLAB" in this study.

Moreover, μ-analysis and synthesis are also adopted. It is already the description of parameter variations are modified by applying μ-synthesis and the robustness is enhanced (Watanabe and Yoshida, 1994).

COMPARISON OF CONTROL PERFORMANCE

Comparisons of Stationary Performances (Frequency Responses)

To examine the property of proposed design method, theoretical calculations are carried out first. A H^∞ con troller and μ-controller are obtained throughout numerical calculations using MATLAB. Moreover, a LQG controller using Kalman-filter as an observer is also calculated to examine the effect of parameter variations

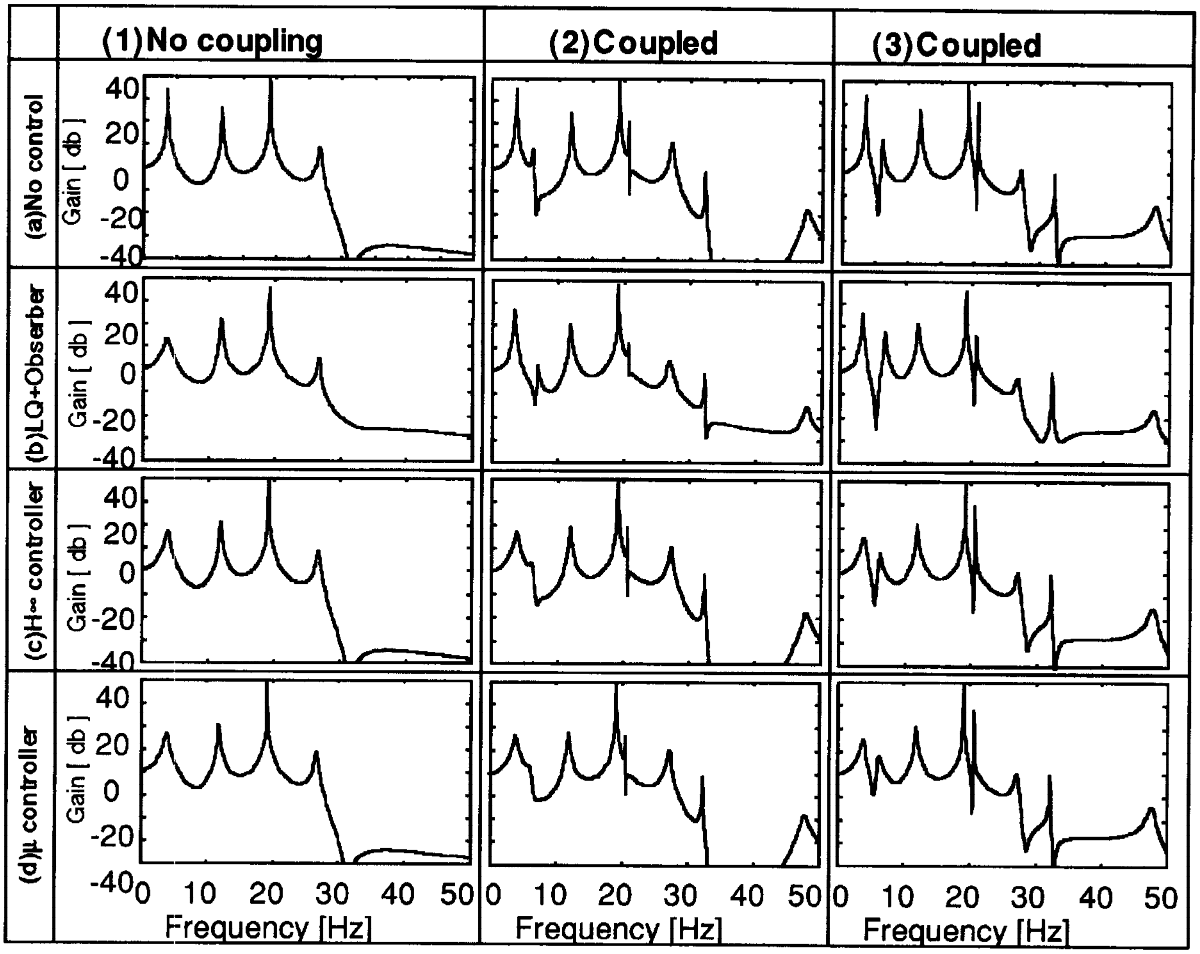

Fig.2 Frequency responses of the accelerations of the structure subjected to random excitation

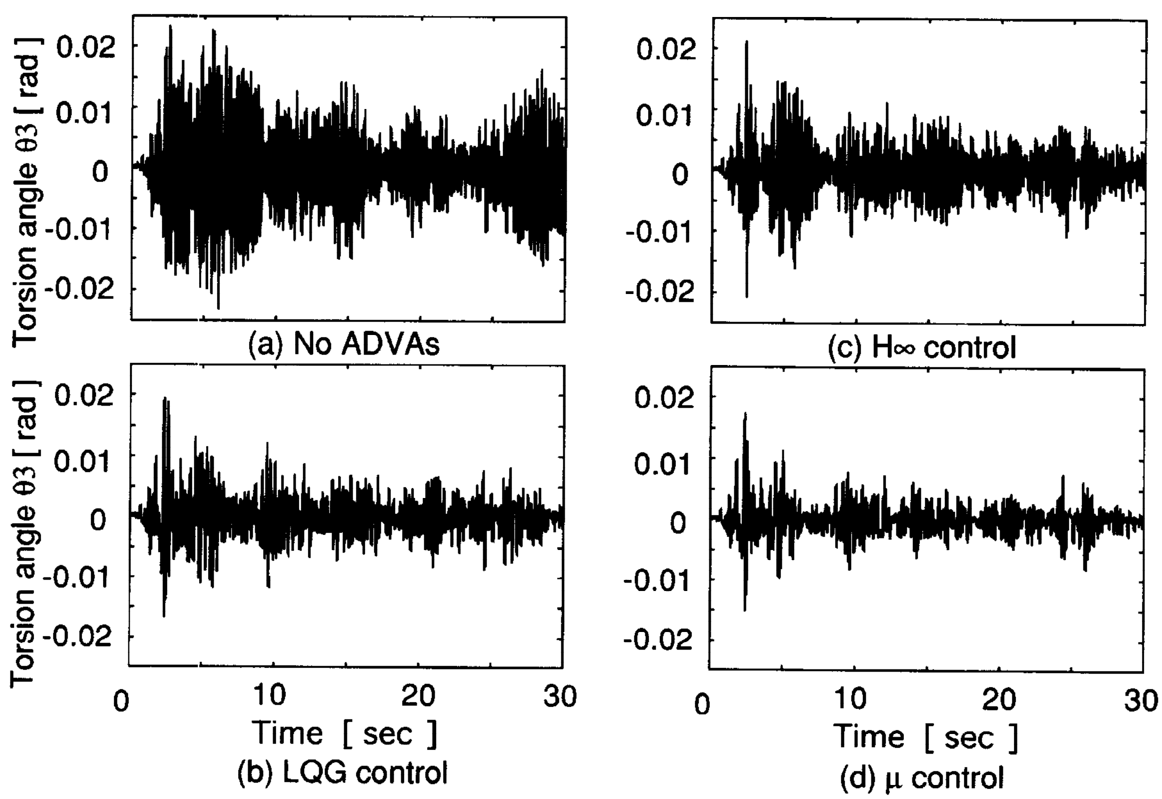

Fig.3 Time history of torsional angle of the 3rd floor subjected to El-Centro excitation (coupled mode)

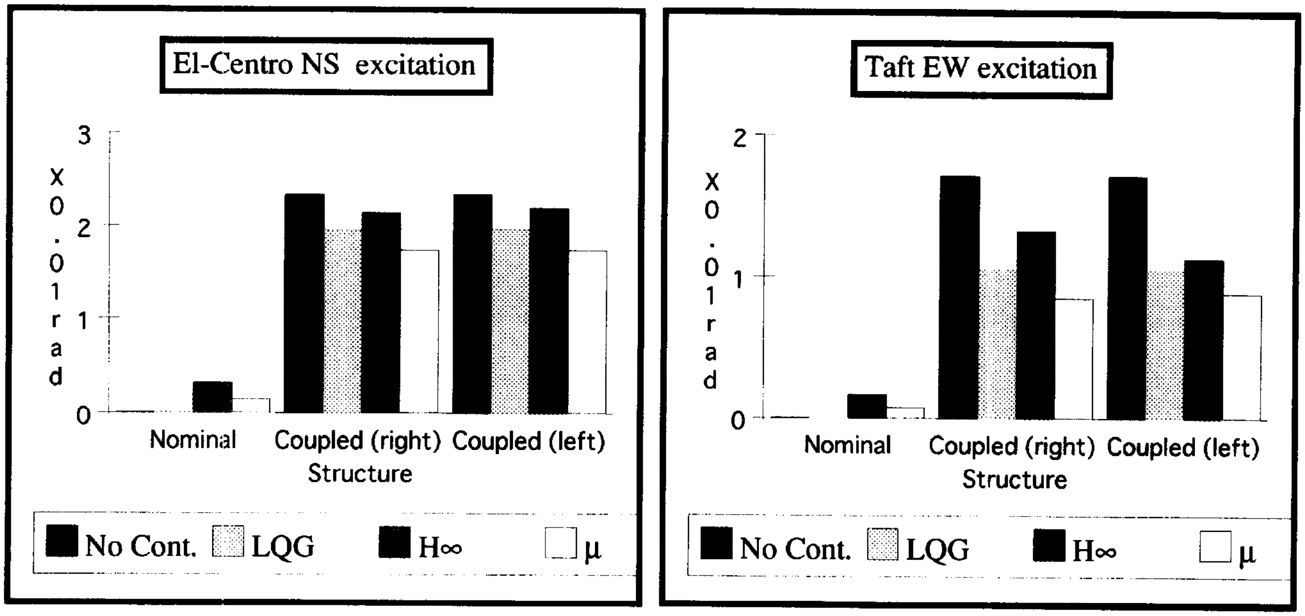

Fig.4 Maximum torsional angles of the structure subjected to El-Centro NS of Taft EW Excitation

on the ordinarily designed controller. The results are shown in fig. 2. These figures denote the frequency responses of the acceleration at the top of the structure on right edge to the excitation acceleration. In the Fig.2, (a) denotes the case without control, (b) shows the case with the controller obtained according to the ordinary LQG design, (c) denotes the case with H^{∞} controller obtained according to the procedure shown in former section and (d) is the case with μ-controller. Meanwhile, (1), (2) and (3) denote the nominal location, the case when the lumped mass is located to the right, and the case to the left, respectively. All controllers achieve good control performance in the case (1). In the case (2) and (3), however, the performance of LQG controller is greatly degraded due to the effect of transverse-torsional coupling. On the contrary, the H^{∞} controller possesses good robustness in spite of the existence of parameter variations. Moreover, the robustness of the H^{∞} controller is enhanced by applying μ–synthesis. In the case (3), the control performance of H^{∞} controller possesses a little fluctuation on 7Hz (1st torsional mode), while the variation of μ controller is a little reduced. Compared with the case without the dynamics of ADVAs and frequency-shaped weighting functions for control inputs (Watanabe and Yoshida, 1995), the control performances of the H^{∞} controller and the μ–controller in the higher frequency domain are degraded, while their robustness are is still conserved.

Comparisons of Time-domain Responses (Earthquake Excitation Responses)

To investigate the control performances of the controllers, time history of the systems subjected to seismic excitations are carried out. El-Centro and Taft acceleration wave records are adopted for comparisons. Figure 3 shows the angular displacements of the 3rd story of the structure with transverse-torsional coupled vibration modes subjected to El-Centro NS excitation. The structure shows great torsional responses without controllers. The LQG controller and the H^{∞} controller are not so effective to reduce torsional displacement, while the μ–controller reduces the maximum and the average responses clearly. Figure 4 shows a graph of the maximum responses of the torsional displacements of the 3rd story subjected to El-Centro NS or Taft EW excitation. They show that the LQG controller and the H∞ controller has poor robust performance, while the μ controller achieves the highest robust performance. Compared with the case without the dynamics of ADVAs and frequency-shaped weighting functions, the robustness of the H^{∞}

controller are degraded, while that of the μ controller are is still conserved even in terms of the maximum responses to the earthquake excitations.

CONCLUSIONS

In this paper, the robustness of vibration control system for a flexible structure using multiple active dynamic vibration absorbers is discussed. The design method of controller was proposed, which is based on H^{∞}/μ control theory adopting structured uncertainty model for parameter variations and weighting functions for uncontrolled dynamics. Theoretical calculations and computer simulations using earthquake excitation wave record were carried out and the conclusions were obtained as follows.

(1) Parameter variations strongly affects on the stationary performance of the ordinary LQG output feedback controller. The H^{∞} controller obtained by using proposed design method can achieve robust performance. Moreover, the robustness of H^{∞} controller is enhanced by applying μ-synthesis.

(2) To the earthquake excitation, the structure controlled by the H^{∞} controller possess the maximum torsional responses as large as those by the LQG controller, while the μ-controller achieves less maximum responses. That is , the μ-synthesis is effective to enhance robust performance for non-stational input.

(3) According to these results, the usefulness of the proposed design method was verified even in the case with the dynamics of ADVAs and frequency-shaped weighting functions.

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