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A RELATION BETWEEN THE PEAK VALUE AND PERIOD OF STRONG GROUND MOTIONS

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ABSTRACT

A relation between the characteristic periods and peak values of strong ground motions is theoretically presented with the aid of the random-vibration theory and Parseval theorem. The relation shows that the central periods of acceleration, velocity and displacement spectra are dependent only on the peak values of their time histories. Especially the bounds of central periods for spectra are specified by a combination of the peak values. The validity of the theoretical peak-period relation is confirmed in comparison with observed strong-motion records. Following the confirmation of validity, the theoretical relation is further expanded to scale acceleration, velocity and displacement spectra. In scaling spectra, the ω-square model is used along with an empirical attenuation law of strong-motion peaks. The spectra scaled by such a semi-empirical method show a relatively good agreement with observed spectra.

KEYWORDS

Central period; Peak value; Acceleration; Velocity; Displacement; Time history; Spectra; Random-vibration theory; Parseval theorem; Empirical prediction.

INTRODUCTION

Period characteristics of strong ground motions play an important role in causing earthquake damage as well as motion amplitude. As is well known, acceleration, velocity and displacement motions are interrelated by means of differential and/or integral operations through their spectral content. Since the peak values of these motions reflect somewhat their spectral content, the characteristic periods giving peak acceleration, peak velocity and peak displacement might have a close relation with the peak values. In other words, there is possibility that we obtain spectral information of acceleration, velocity and displacement motions merely by investigating the interrelationships of the three kinds of peaks. The objective of this study is to derive a theoretical relation between the strong-motion peaks and their characteristic periods, especially central periods of spectra and to empirically confirm the validity of the relation based on strong-motion records. Furthermore, a unified prediction of spectra for acceleration, velocity and displacement motions are made in cooperation with an empirical attenuation law of strong-motion peaks.

THEORETICAL RELATIONS BETWEEN CENTRAL PERIODS AND PEAK VALUES OF STRONG GROUND MOTIONS

Derivation of a theoretical relation

Now let $f(t)$ and $F(f)$ be a time history of strong ground motions and its Fourier transform, respectively. From the Parseval theorem, we then get

$$rms[f(t)]_T = \sqrt{\frac{1}{T}\int_{-\infty}^{\infty} |F(f)|^2 df} \quad (1)$$

where $rms[\ ]_T$ means the rms value of $f(t)$ for the duration T, $|\ |$ is the absolute value and t and f are time and frequency each.

Eq.(1) leads to the following expressions for acceleration motion $a(t)$ and velocity motion $v(t)$:

$$rms[a(t)]_{T_a} = \sqrt{\frac{1}{T_a}\int_{-\infty}^{\infty} |A(f)|^2 df} \quad (2)$$

$$rms[v(t)]_{T_v} = \sqrt{\frac{1}{T_v}\int_{-\infty}^{\infty} |V(f)|^2 df} \quad (3)$$

In Eqs. (2) and (3), $A(f)$ and $V(f)$ are, respectively, the Fourier transforms of acceleration and velocity motions and T_a and T_v mean each the durations for both motions.

According to the differential or integral operations, $A(f)$ and $V(f)$ are connected with each other as follows:

$$2\pi f\,|V(f)| = |A(f)| \quad (4)$$

In order to consider a characteristic period of the velocity motion, we here deal with the second order moment m_v^2 and zero-th order moment m_v^0 of the energy spectrum $|V(f)|^2$. These values can be defined in the frequency domain and given using Eq.(4) as

$$m_v^2 = 2\pi\int_{-\infty}^{\infty} (2\pi f)^2 |V(f)|^2 df$$
$$= 2\pi\int_{-\infty}^{\infty} |A(f)|^2 df \quad (5)$$

$$m_v^0 = 2\pi\int_{-\infty}^{\infty} |V(f)|^2 df \quad (6)$$

By the use of Eqs. (2) and (3), m_v^2 and m_v^0 become respectively

$$m_v^2 = 2\pi T_a \{rms[a(t)]_{T_a}\}^2 \quad (7)$$

$$m_v^0 = 2\pi T_v \{rms[v(t)]_{T_v}\}^2 \quad (8)$$

We now designate the central frequency of $V(f)$ as $\bar{f}_v$. The central frequency $\bar{f}_v$ is expressed as the square root of the ratio of m_v^2 to m_v^0 and using Eqs. (7) and (8) we obtain

$$2\pi \bar{f}_v = \sqrt{m_v^2 / m_v^0}$$
$$= \{rms[a(t)]_{T_a} / rms[v(t)]_{T_v}\}\sqrt{T_a / T_v} \quad (9)$$

In addition to the central frequency of $V(f)$, we further introduce the central frequency $\bar{f}_a$ of $A(f)$. Then both central frequencies are connected with the durations T_v and T_a through the numbers of extrema N_v and N_a:

$$T_v = N_v / (2\bar{f}_v) \quad (10)$$

$$T_a = N_a / (2\bar{f}_a) \quad (11)$$

On the other hand, the method of random-vibration theory gives a relation between the peak value, rms value and number of extrema for a random motion(Joyner and Boore, 1988). That is, the expected value of peak velocity v_{max} is written as

$$E[v_{max}] = rms[v(t)]_{T_v}\{\sqrt{2\ln N_v} + \gamma/\sqrt{2\ln N_v}\} \tag{12}$$

where *E[ ]* means the expected value and γ is the Euler number.
Similarly for acceleration motion,

$$E[a_{max}] = rms[a(t)]_{T_a}\{\sqrt{2\ln N_a} + \gamma/\sqrt{2\ln N_a}\} \tag{13}$$

where a_{max} is the peak value of acceleration motion *a(t)*.
When we consider the duration satisfying the condition of $N_a = N_v = N$, Eqs. (12) and (13) give the relation

$$E[a_{max}]/E[v_{max}] = rms[a(t)]_{T_a}/rms[v(t)]_{T_v} \tag{14}$$

where $T_a = N/(2\bar{f}_a)$ and $T_v = N/(2\bar{f}_v)$.
Finally, the peak ratio $E[a_{max}]/E[v_{max}]$ is expressed by the central frequencies $\bar{f}_a$ and $\bar{f}_v$ from Eqs. (9), (10) and (11) under the condition of $N_a = N_v = N$ as follows:

$$E[a_{max}]/E[v_{max}] = 2\pi\sqrt{\bar{f}_a\bar{f}_v} \tag{15}$$

The above relation is also applied to velocity and displacement motions:

$$E[v_{max}]/E[d_{max}] = 2\pi\sqrt{\bar{f}_v\bar{f}_d} \tag{16}$$

where d_{max} is the peak value of displacement motion and $\bar{f}_d$ is the central frequency of displacement spectrum.

Meanwhile, in the case of a harmonic motion with a frequency f, the amplitudes of acceleration, velocity and displacement motions have the well-known relations

$$a/v = 2\pi f \tag{17}$$
$$v/d = 2\pi f \tag{18}$$

where a, v and d are the amplitudes of harmonic acceleration, velocity and displacement motions respectively.
A comparison of Eqs. (15) and (16) with Eqs. (17) and (18) shows that the expected peak values of random motions are interrelated in terms of the central frequencies of their spectra similarly to the relation between amplitudes and frequency of a harmonic motion, regarding the harmonic mean of the two relevant central frequencies as an effective frequency of the random motions.

<u>Bounded central periods of acceleration, velocity and displacement spectra</u>

The above relation does not give a point value of central frequencies, but only the interrelations of them. However, Eq. (4), a differential relation and/or integral relation, gives an inequality $\bar{T}_a \le \bar{T}_v \le \bar{T}_d$ where $\bar{T}_a$, $\bar{T}_v$ and $\bar{T}_d$ are the central periods, respectively, for acceleration, velocity and displacement spectra. This inequality can provide a way for specifying the bounds of these central periods in cooperation with the above relation. In the following, the expected value symbol | | used in Eqs. (15) and (16) is omitted for the sake of simplicity. Since $\bar{T}_v > 0$, the inequality becomes

$$\bar{T}_a\bar{T}_v \le (\bar{T}_v)^2 \le \bar{T}_v\bar{T}_d \tag{19}$$

With regard to $\bar{T}_a\bar{T}_v$ and $\bar{T}_v\bar{T}_d$ in Eq. (19), both are expressed in terms of the peak values as shown in Eqs. (15) and (16), and then the inequality of Eq. (19) is transformed to

$$2\pi v_{max}/a_{max} \le \bar{T}_v \le 2\pi d_{max}/v_{max} \tag{20}$$

Following the bounded $\bar{T}_v$ in Eq. (20), we can further bound $\bar{T}_a$ and $\bar{T}_d$ as a form of inequality by using Eqs. (15) and (16). The results are

$$2\pi v_{max}^3/(a_{max}^2 d_{max}) \le \bar{T}_v \le 2\pi v_{max}/a_{max} \tag{21}$$

$$2\pi d_{max}/v_{max} \le \bar{T}_d \le 2\pi a_{max} d_{max}^2/v_{max}^3 \tag{22}$$

Eqs. (20), (21) and (22) show that the bounds of central periods for acceleration, velocity and displacement motion spectra can be specified merely in terms of the peak values of their time histories.

VALIDITY OF THE THEORETICAL RELATION IN COMPARISON WITH OBSERVED DATA

In an attempt to establish validity of the above relation, we here apply it to strong ground motions observed in Japan. Fig.1 shows, as an example, a set of strong-motion records observed at Kushiro, Hokkaido, during the 1973 Nemuro-hanto-oki earthquake(Kurata et al., 1974). In Fig.1, peak values of acceleration, velocity and displacement are also given along with their time histories. The Fourier spectra of these records are shown in Figs.2 (1), (2) and (3) together with the central period bounds which were obtained from the peak values using Eqs. (20), (21) and (22). It is seen in Figs.2 (1), (2) and (3) that the theoretical bounds of central periods due to the peak values move toward longer periods in accordance with the spectral characteristics of acceleration, velocity and displacement. Judging from the spectral features, these theoretical bounds seem to correspond to the shift of the central periods of acceleration, velocity and displacement spectra.

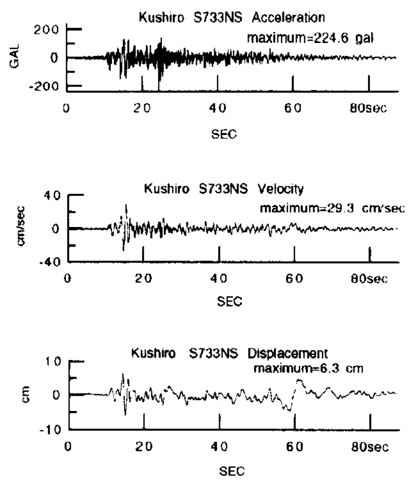

Fig.1. Strong ground motion records at Kushiro during the 1973 Nemuro-hanto-oki earthquake.

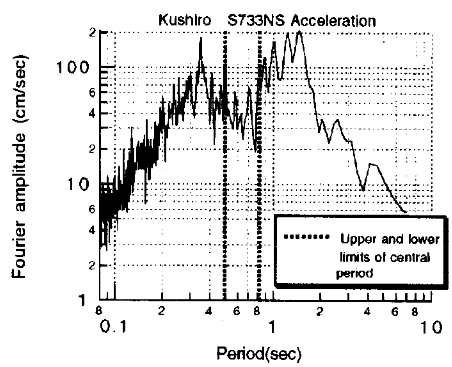

Fig.2(1). Fourier spectra of acceleration record at Kushiro.

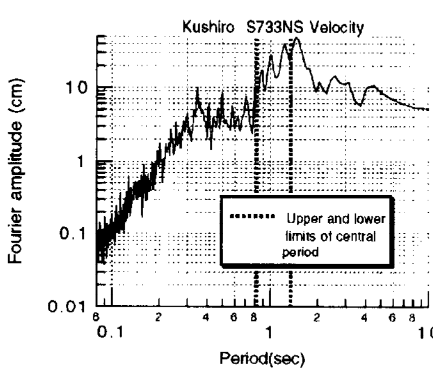

Fig.2(2). Fourier spectra of velocity record at Kushiro.

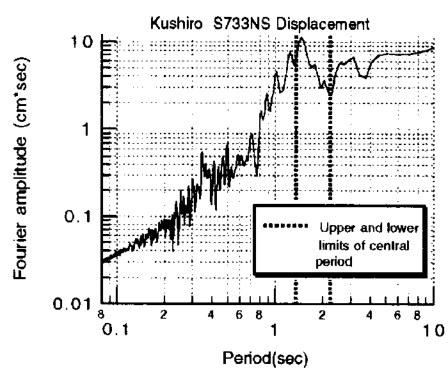

Fig.2(3). Fourier spectra of displacement record at Kushiro.

Fig.1 and Figs.2 (1), (2) and (3) are a representative example of comparison on the basis of spectral figure. We further obtained central periods both due to spectra and peak values from many strong ground motions observed in Japan. Both central periods are plotted, as a scatter diagram, in Figs.3, 4 and 5 separately for acceleration, velocity and displacement. The strong ground motions used in these figures are the same as the data set used by Kamiyama and Yanagisawa(1986). In Figs.3, 4 and 5, the central periods due to the observed spectra plotted in the horizontal axis were obtained by the central period of spectra as defined in Eq.(9) while the central periods due to the peak values plotted in the vertical axis were estimated as the upper and lower limits of the bounded central periods given in Eqs.(20), (21) and (22). Figs.3, 4 and 5 systematically demonstrate that the theoretical bounds due to peak values almost include the central periods by spectra. Namely, the 1-to-1 lines ,which denote that the two types of central periods perfectly coincide, almost fall within a range between the upper and lower limits in each comparison of acceleration, velocity and displacement. Thus, as a whole, the theoretical bounds provide a reasonable estimate of central periods.

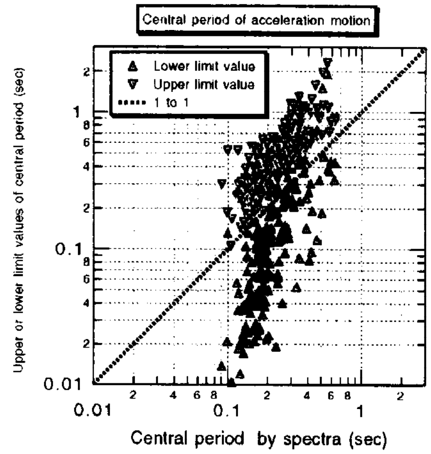

Fig.3. Comparison between central period due to observed spectra and central period due to peak values for acceleration motions.

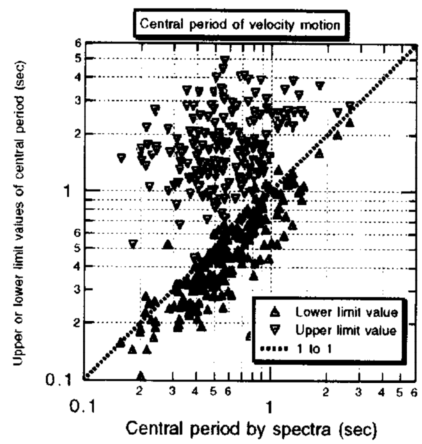

Fig.4. Comparison between central period due to observed spectra and central period due to peak values for velocity motions.

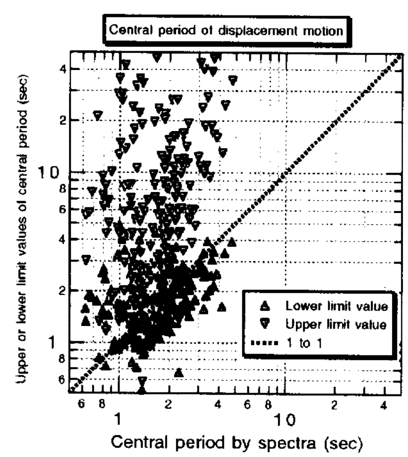

Fig.5. Comparison between central period due to observed spectra and central period due to peak values for displacement motions.

THEORETICAL SPECTRA OF STRONG MOTIONS ASSOCIATED WITH PEAK VALUES

We made an attempt to derive a scaling law of acceleration, velocity and displacement spectra, using and expanding the relations between time and frequency domains shown above. Random-vibration theory, for example, Eq.(12) provides that peak value of motions is connected with their spectral content through the Parseval theorem. This means that we can inversely determine spectral amplitudes of motions in terms of peak values. In order to perform such an inverse determination of spectra, we costructed a basic structure of spectral features for acceleration, velocity and displacement motions based on the ω-square model(Kamiyama, 1996). Fig.6 shows the derived spectral structures of acceleration, velocity and displacement. Using Eqs.(20) through (22), the characteristic periods for these spectral structures are determined as follows:

$$\hat{T}_a = 2\pi\, v_{max}^3 / (a_{max}^2 \times d_{max}) \qquad (23)$$

$$\hat{T}_{av} = 2\pi\, v_{max} / a_{max} \qquad (24)$$

$$\hat{T}_{vd} = 2\pi\, d_{max} / v_{max} \qquad (25)$$

In addition, the spectral amplitudes are also given with the aid of the random-vibration theory and Parseval theorem, following such a determination of the characteristic periods(Kamiyama, 1996). Thus we can obtain the parameters for scaling acceleration, velocity and displacement spectra provided that strong-motion peaks are given as an attenuation law in terms of earthquake magnitude, source-to-site distance, etc. Kamiyama et al.(1994) derived a semi-empirical attenuation law of acceleration, velocity and displacement peaks based on a faulting source model and a statistical method. In this study, their attenuation law of strong-motion peaks was used to scale spectra.

Figs.7(1), (2) and (3) show spectra scaled by such a semi-empirical technique. In Figs.7(1), (2) and (3), acceleration, velocity and displacement spectra in the source area are scaled by earthquake magnitude with no effects of path and local site, using the attenuation law of strong-motion peaks by Kamiyama et al.(1994).

In order to confirm the adequacy of these scaled spectra, the semi-empirical technique for predicting spectra was applied to an actual event and compared with observed spectra. Herein we adopted the 1989 Loma Prieta earthquake in the US. for the event of comparison because it provided a strong-motion record at a rock site in an epicentral area. Fig.8(1) and (2) show two horizontal components' records at the Santa Cruz site during the Loma Prieta event(Huang et al., 1990). The Santa-Cruz records were obtained at a limestone site with an epicentral distance of 16 km. Therefore they are considered to nearly satisfy the condition of the source area record that has no effects of path and local site. Note here that these records were directly cited from the reports of the California Strong Motion Instrumentation Program, CSMIP(Huang et al., 1990). The accelerometer and data processing used by CSMIP are different from those by Kamiyama et al. whose attenuation law of strong-motion peaks constitutes main parameters for the present technique for predicting spectra. In particular, the filter band of data processing by CSMIP is much broader in the long period range, so the displacement motions which are mainly determined by long period characteristics might have less reliability of comparison.

The Fourier spectra of the main parts of the Santa Cruz records are shown in Fig.9(1), (2) and (3). With the information of a magnitude of 7.0 for the Loma Prieta event, we can estimate spectra theoretically expected in the source area for the event. These theoretical estimates of spectra are plotted as thick solid lines in Fig.9(1), (2) and (3). Comparisons of the theoretical spectra with the observed ones in Figs.9(1), (2) and (3) indicate that the present technique of spectra gives a relatively good prediction within the period range between about 0.1 sec and about 3 sec. The degree of agreement in Figs.9(1), (2) and (3) may be acceptable in view of the fact that the present method uses only information of earthquake magnitude without any other detailed source parameters related to faulting process.

SUMMARY AND CONCLUSIONS

The central periods of spectra for seismic acceleration, velocity and displacement motions are related with the motions peaks similarly to the relations between the amplitudes and periods of a harmonic motion as expressed in Eqs.(15) and (16). The harmonic mean of central periods for the two neighboring peak parameters plays the same role as the frequency of a harmonic motion. The bounds of central periods of acceleration, velocity and displacement

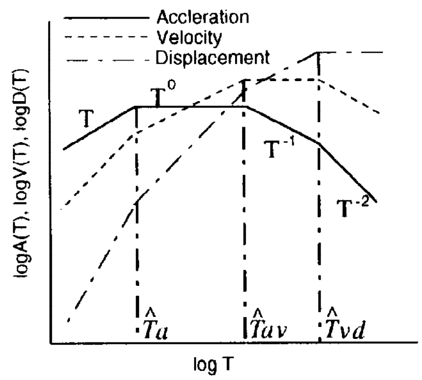

Fig.6. A schematic figure of scaling model for acceleration, velocity and displacement spectra

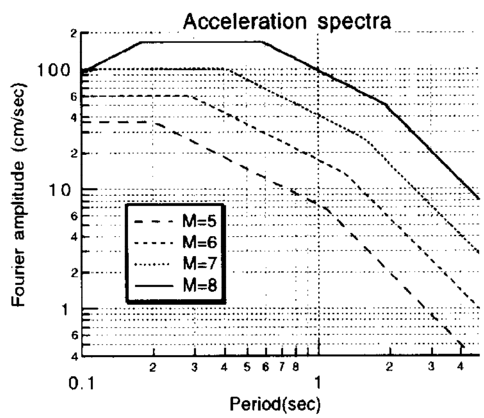

Fig.7(1). Acceleration spectra in the source area scaled by the the empirical law of strong-motion peaks by Kamiyama et al.

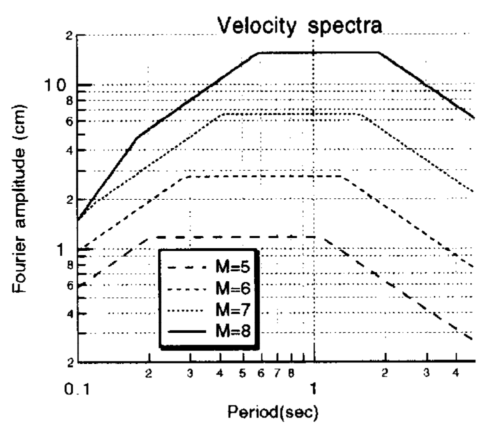

Fig.7(2). Velocity spectra in the source area scaled by the empirical law of strong-motion peaks by Kamiyama et al.

spectra were specified as a form of inequalities whose upper and lower limits are determined in terms of peak values of time history. The inequalities are expressed in Eqs.(20) through (22). As an expansion of the theoretical peak-period relation, a method for scaling acceleration, velocity and displacement spectra was presented using the empirical attenuation law of strong-motion peaks by Kamiyama et al. Along with the theoretical peak-period relation, the method for scaling spectra was also confirmed to be valid in comparison with observed strong-motion records.

Although the method for scaling spectra presented here gives a simplified prediction relatively compatible to the observed within the limited period range, it seems to have an insufficient estimate in longer periods. This is due to the over-simplification of the method as well as the peak values of velocity and displacement motions used in this paper which were obtained in a period-limited manner to avoid numerical errors. Accordingly, more sophisticated spectral shape reflecting complicated faulting process of seismic source is needed to enhance the applicability of the theoretical peak-period relation presented in this paper while velocity or displacement records reliable in much longer periods are indispensable for future studies.

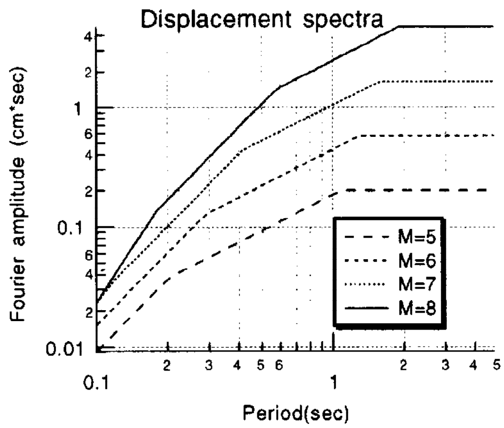

Fig.7(3). Displacement spectra in the source area scaled by the empirical law of strong-motion peaks by Kamiyama et al.

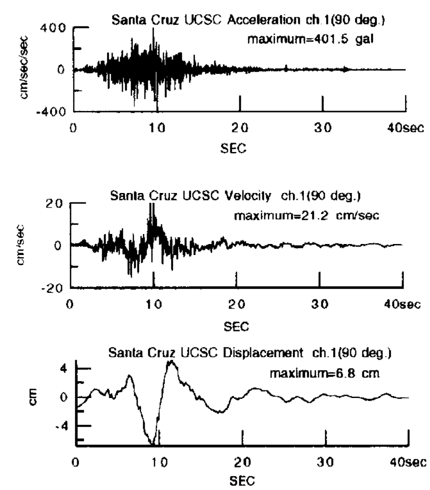

Fig.8(1). Strong-motion records in the 90 degree horizontal direction at Santa Cruz.

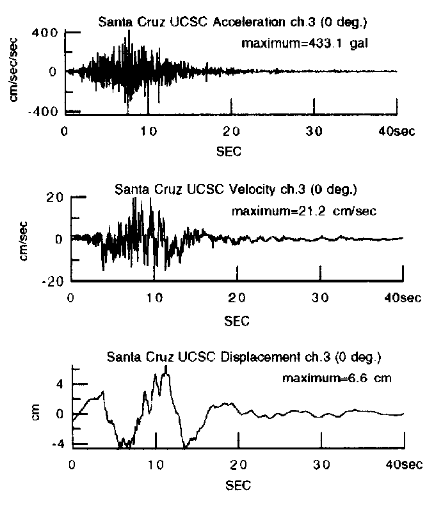

Fig.8(2). Strong-motion records in the 0 degree horizontal direction at Santa Cruz.

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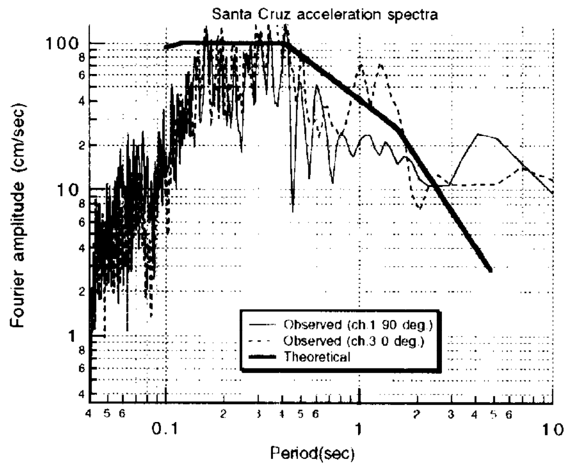

Fig.9(1). Fourier spectra of the Santa Cruz acceleration records and theoretical acceleration spectrum estimated by the scaling method of spectra.

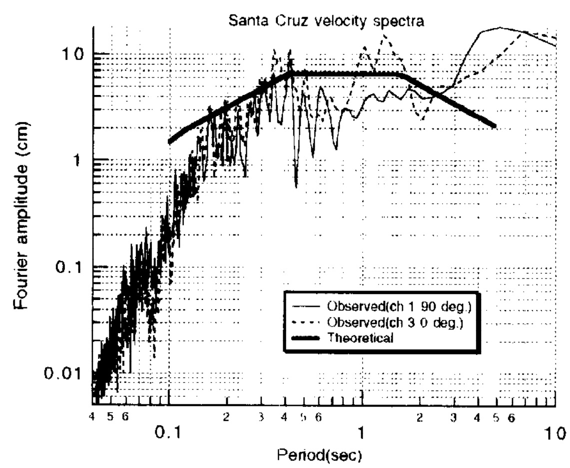

Fig.9(2). Fourier spectra of the Santa Cruz velocity records and theoretical velocity spectrum estimated by the scaling method of spectra.

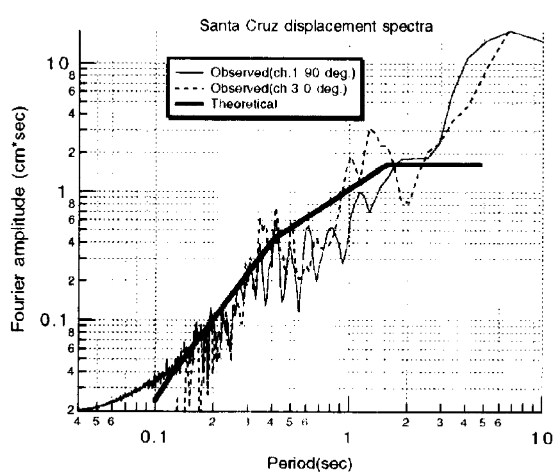

Fig.9(3). Fourier spectra of the Santa Cruz displacement records and theoretical displacement spectrum estimated by the scaling method of spectra.