STRAIN ENERGY CONCEPT FOR DEVELOPING LUMPED MASS STICK MODEL OF COMPLEX STRUCTURES

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ABSTRACT

The first step in the dynamic analysis of any engineering structure is the analytical modelling of the real structure with its stiffness and mass distribution simulated as closely as possible. The 3-D Finite Element model is the best possible model as of today. But often it is costly thus necessiating simpler models. It is also a common industrial practice to evolve a simple model at the beginning of design to have a feeling of the order of results and then do the subsequent analysis based on refined model. This paper presents the results of studies carried out for the Lumped Mass Beam Modelling (LMM) of complex 3-D structures based on strain energy equivalence. Two other conventional methods have been also discussed alongwith a few case studies. A comparison of the results obtained by these methods has been made.

KEYWORDS

Lumped Mass Beam Model; Bending Stiffness; Shear Stiffness; Calandria; Core; End-shield

INTRODUCTION

Lumped Mass Beam Models are popular on account of their simplicity and conservative response predictions which are highly appreciable at the design stage. The major task in this method of modelling is to find out the stiffness and mass distribution of this equivalent beam or stick so that it can reasonably simulate the behaviour of the original structure. Various methods exist to arrive at the estimation of the stiffness and mass of the equivalent beam, one of which has been developed and is presented herein.

Strain Energy Method (EM)

In this method , firstly the eigenvalue solution is performed to get the first fundamental mode using the 3-D FE model of the structure. Secondly, the flexural stiffness (EI) and shear stiffness (GA_s) of each element of the LMM are evaluated using the strain energy of the 3-D model. The simplified LMM for the dynamic response analysis is constructed from the obtained element stiffness. In order to calculate the strain energy, using the finite element model, a static analysis is carried out for the horizontal load weighted with the first

normal mode such that the static force applied to each layer has the distribution which is similar to the first mode.
Applied external force at i-th layer is given by the formula as follows ;

$$P = U_i . W_i . \Gamma_i \quad (1)$$

Where P_i : External force at i-th layer, U_i : Value of the 1st mode-shape corresponding to the i-th layer, W_i : Mass of the i-th layer, Γ_i : Mode-participation factor for the 1st mode
With the result of this static analysis, if the total axial energy is equal to the equivalent incremental rotation energy, the eq. corresponding to the i-th layer is

$$\sum_j (N_{ij} . \Delta V_{ij}) = \sum_j N_{ij} . (\Delta\theta_{ei} . l_{ij}) \quad (2)$$

Where, N_{ij} : Axial force of the j-th element in the i-th layer, ΔV_{ij} : Incremental axial displacement of the j-th element of the i-th layer ($\Delta V_{ij} = V_{ij} - V_{i-1j}$), V_{ij} : Axial displacement of the j-th element in the i-th layer, $\Delta\theta_{ei}$: Incremental rotation angle between the i-th and i-1th layer i.e ($\Delta\theta_{ei} = \theta_{ei} - \theta_{ei-1}$), θ_{ei} : Equivalent rotation angle of the i-th layer , l_{ij} : Distance between the j-th element and neutral axis in the i-th layer. The equivalent incremental rotation angle is expresed as follows

$$\Delta\theta ei = \int_{i-1}^{i} (M_i/EI_i)dx = M_i . h_i / EI_i \quad (3)$$

Where, M_i : Bending moment of the i-th layer, h_i : Height of the i-th layer, EI_i : Flexural rigidity of the i-th layer. As $M_i = \sum_j N_{ij} . li_j$ in eq.(3) , the equivalent stiffness is written as follows

$$EI_i = \sum_j \{(N_{ij} . li_j)/\Delta\theta_{ei}\} h_i \quad (4)$$

So, the shear deformation in the i-th layer, reducing the relative flexural displacement (δi_m) from the relative horizontal displacement (δ_{ti}) is expressed as follows

$$\delta_{si} = \delta_{ti} - \delta_{mi} = \delta_{ti} - (\theta_{ei} + \theta_{ei-1})h_i/2 \quad (5)$$

Then the equivalent shear-stiffness is expressed as follows

$$GA_{si} = Q_i . h_i / \delta_{si} \quad (6)$$

Where Q_i : Shear force at the i-th layer

Static Method (SM)

Arbitrary static load is applied to the 3-D FE model of the structure at different levels and from equilibrium condition, stiffness properties are calculated as follows. From the general system eq. $[K]\{\delta\} = \{F\}$, where, [K] is the stiffness matrix of the LMM which contains stiffness matrix of the LMM containing stiffness terms corresponding to both the bending and shear deformations. $\{\delta\}$ is the vector of displacements and rotations at each layer which is obtained from 3-D analysis and {F} is the applied force vector. Thus knowing the displacement vector $\{\delta\}$ and the {F}, the stiffness matrix for LMM is calculated using this equation (Soni *et al.*,1987).

Geometric Method (GM)

Cross-section of the structure at different elevations are considered and from the geometry of the cross-section, moment of inertia and shear-coefficient are calculated by closed-form formulae.

CASE STUDIES

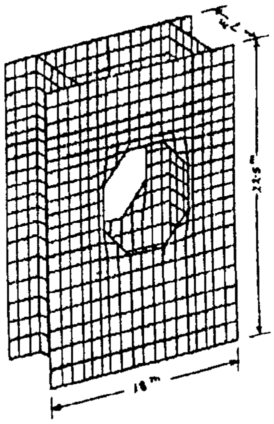

Fig.1. Calandria Vault

Case I (Calandria Vault) : It is basically a cantilever concrete box with octagonal opening on two opposite walls, which houses the nuclear core. The characteristics of the structure are varying wall thickness at different cross-sections and, large octagonal opening and low aspect ratio (Fig.1). The natural frequencies and mass participations are shown in Table. 1 and the forced responses i.e displacement, storey-shear and storey-moment for a given response spectrum are shown in Fig. 4 through 9.

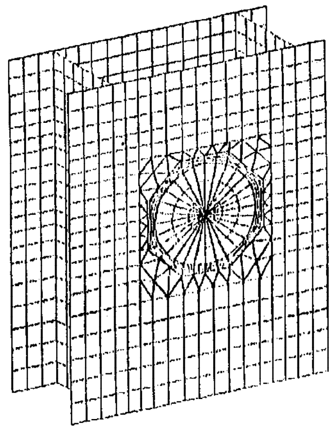

Fig. 2. Calandria Vault with End-shield

Case II (Calandria Vault with End-shield) : It is the same as previous structure but with octagonal opening filled up with the End-shield Assembly, which is an equipment made of steel thin diaphragm along the perimeter and central thick lattice section. The natural frequencies and mass participations are shown in Table. 2 and the forced responses are shown in Fig. 10 through 15.

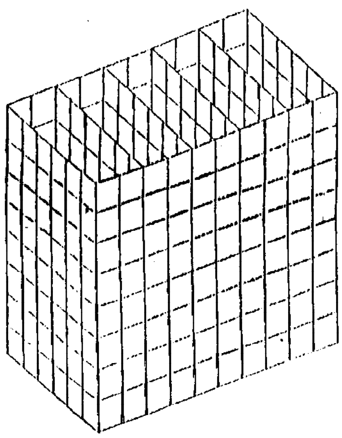

Fig. 3. A Processing Plant Building

Case III (A Processing Plant Building) : It is a concrete box-like structure with inside crosswalls, housing equipments. All the walls are of 0.5m thick. The characteristic of the structure is its low aspect-ratio. Here, two cases for 3-D has been considered and Guyan reduction has been applied in the analysis. In one case12 master nodes at each floor have been considered with all degrees of freedom constrained and other one with the same but only corresponding translational degree of freedom constrained. These cases have been mentioned as Guyan(a) and Guyan(b). The natural frequencies and mass participations are shown in Table. 3 and the forced responses for a given response spectrum are shown in Fig. 16 through 21. For comparison in figs., only the results of Guyan(b) have been used.

RESULTS

Natural Frequencies and Mass Participations

Table 1. Frequencies (f) in Hz and Participation Factors (τ) of the Calandria Vault (x-direction)

Mode	3-D	Beam (EM)	Beam (SM)	Beam (GM)
f1	11.79	11.87	11.81	15.26
f2	42.18	40.62	40.47	44.05
f3	59.10	62.25	56.80	63.36
τ1	15.24	15.37	15.43	15.81
τ2	8.88	9.40	9.31	9.26
τ3	4.53	4.59	3.29	4.78

Table 2. Frequencies (f) in Hz and Participation Factors (τ) of the Calandria Vault (z-direction)

Mode	3-D	Beam (EM)	Beam (SM)	Beam (GM)
f1	9.71	9.86	9.90	10.67
f2	34.83	35.18	34.86	36.13
f3	65.81	67.35	66.81	71.69
τ1	15.15	15.46	15.49	15.5
τ2	8.49	9.53	9.54	10.17
τ3	2.30	5.47	5.38	5.75

Table 3. Frequencies (f) in Hz and Participation Factors (τ) of the Calandria Vault with End-shield (x-direction)

Mode	3-D	Beam (EM)	Beam (SM)	Beam (GM)
f1	14.27	14.77	14.81	15.377
f2	40.12	42.94	42.19	43.98
τ1	18.03	18.40	18.38	18.44
τ2	8.50	8.33	8.42	8.89

Table 4. Frequencies (f) in Hz and Participation Factors (τ) of the Calandria Vault with End-shield (z-direction)

Mode	3-D	Beam (EM)	Beam (SM)	Beam (GM)
f1	9.35	9.90	10.01	10.19
f2	34.68	34.14	34.10	35.40
τ1	17.67	17.78	17.75	17.83
τ2	9.90	9.31	9.28	10.05

Table 5. Frequencies (f) in Hz and Participation Factors (τ) of Core (x-direction)

Mode	3-D Guyan (a)	3-D Guyan (b)	Beam (EM)	Beam (SM)	Beam (GM)
f1	11.17	11.77	11.77	11.73	12.12
f2	28.98	36.54	34.89	34.54	33.8
f3	--	--	68.41	69.1	35.2
τ1	224.8	207.3	230.9	230.61	231.8
τ2	24.36	66.45	45.00	44.74	48.0
τ3	--	--	10.65	11.23	6.16

Table 6. Frequencies (f) in Hz and Participation Factors (τ) of Core (z-direction)

Mode	3-D Guyan (a)	3-D Guyan (b)	Beam (EM)	Beam (SM)	Beam (GM)
f1	11.51	11.52	11.51	11.55	11.54
f2	38.21	40.07	38.15	38.13	37.94
f3	--	--	77.60	75.86	75.86
τ1	207.26	208.12	208.10	208.01	205.60
τ2	66.45	68.37	68.69	68.95	70.45
τ3	--	--	12.10	11.97	11.68

The Strain Energy Method has certain distinct merits. Firstly, in this method the incremental rotation $\Delta\theta$ of a layer is arrived at by the strain energy equation of the shape of the 3-D structure and so gives an average value of $\Delta\theta$. Secondly, the load applied in this method is not arbitrary but consistent with the first mode shape and thus with the dynamic behaviour of the structure.

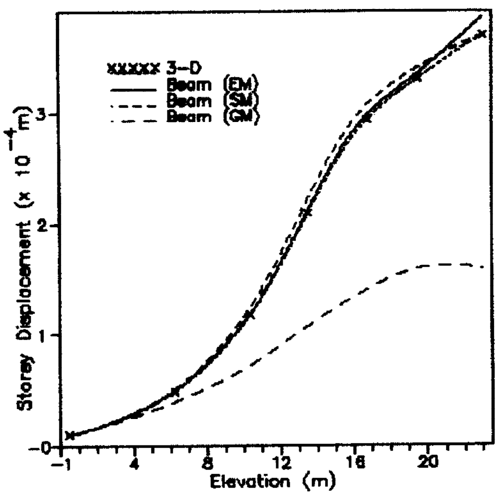

Fig.4 Storey Displacement vs Elev. of Calandria Vault (X-direction)

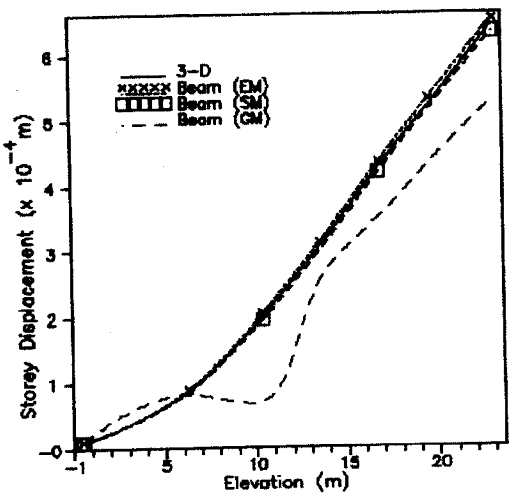

Fig.5 Storey Displacement vs Elev. of Calandria Vault (Z-direction)

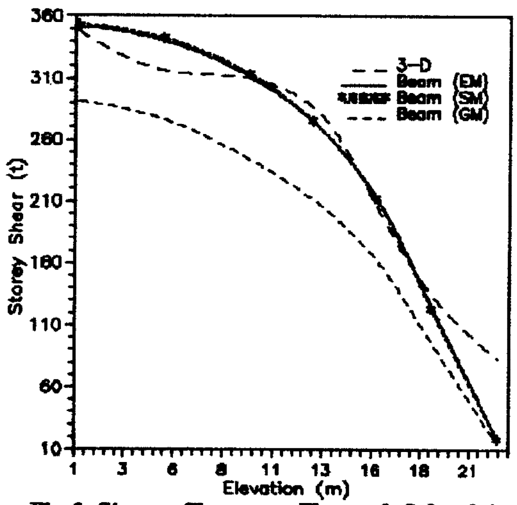

Fig.6 Storey Shear vs Elev. of Calandria Vault (X-direction)

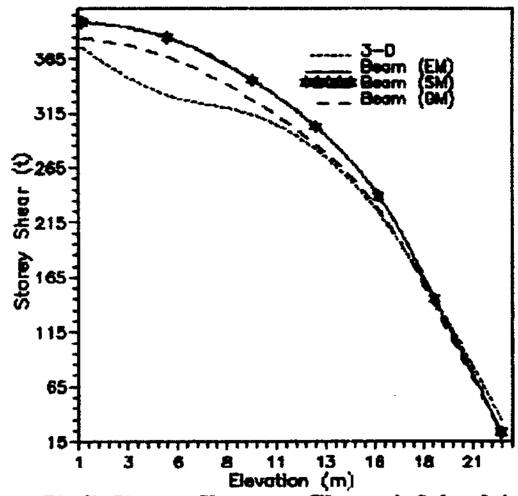

Fig.7 Storey Shear vs Elev. of Calandria Vault (Z-direction)

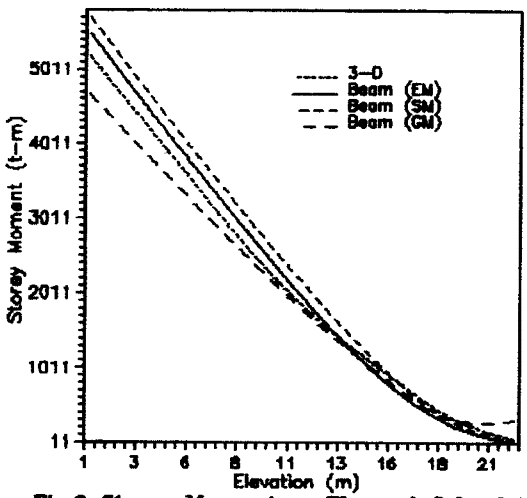

Fig.8 Storey Moment vs Elev. of Calandria Vault (X-direction)

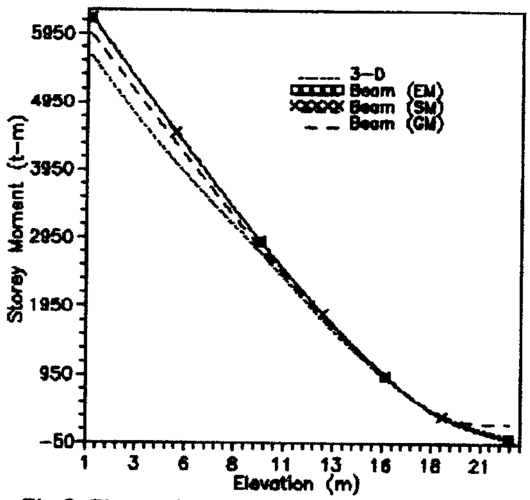

Fig.9 Storey Moment vs Elev. of Calandria Vault (Z-direction)

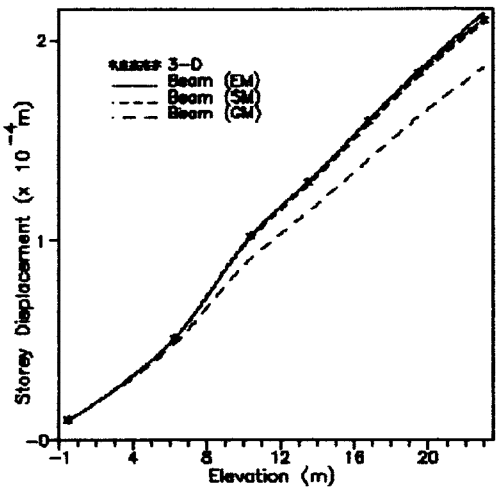

Fig.10 Storey Displacement vs Elev. of Calandria Vault with End-shield(X-direction)

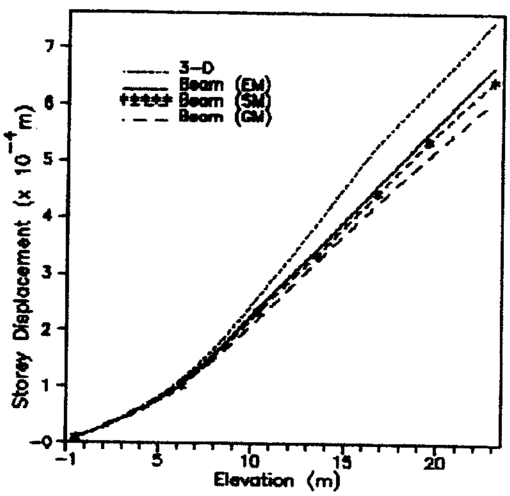

Fig.11 Storey Displacement vs Elev. of Calandria Vault with End-shield (Z-direction)

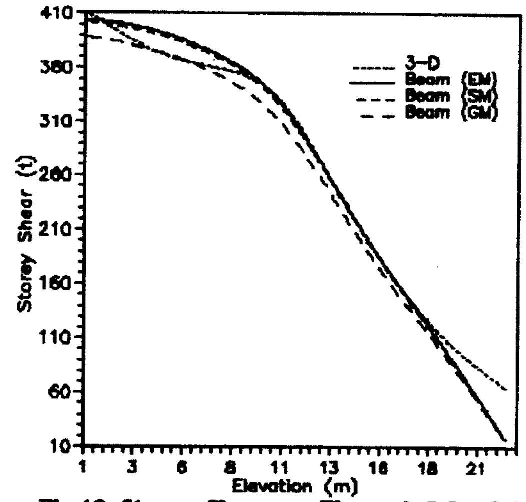

Fig.12 Storey Shear vs Elev. of Calandria Vault with End-shield(X-direction)

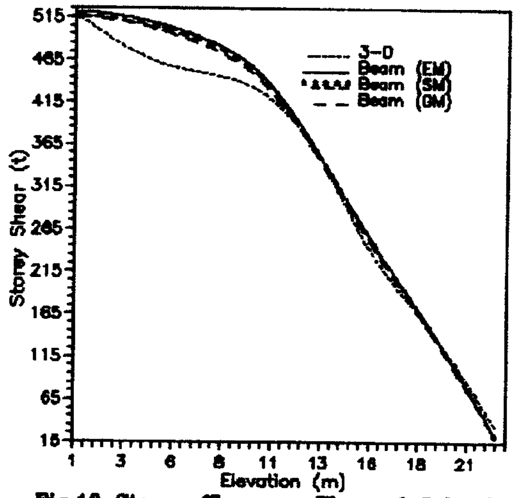

Fig.13 Storey Shear vs Elev. of Calandria Vault with End-shield(Z-direction)

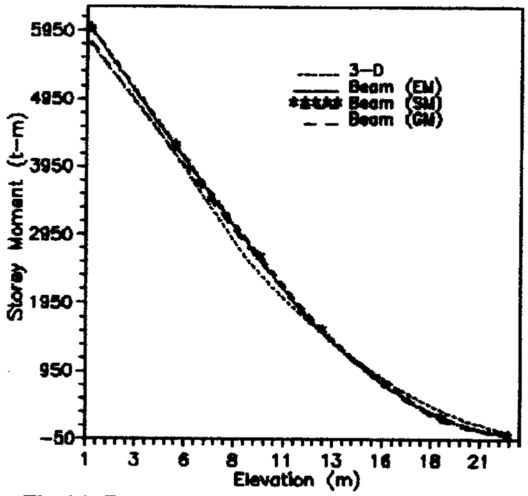

Fig.14 Storey Moment vs Elev. of Calandria Vault with End-shield (Z-direction)

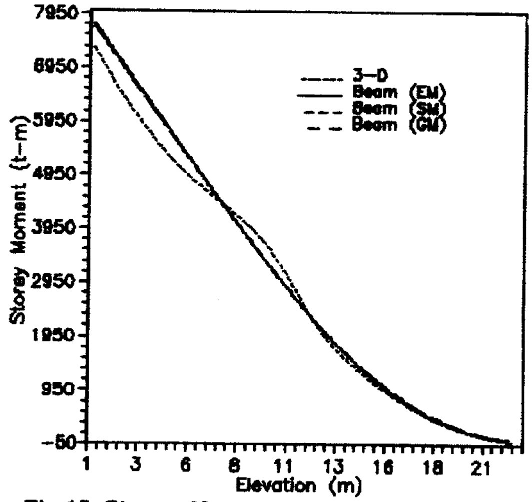

Fig.15 Storey Moment vs Elev. of Calandria Vault with End-shield(Z-direction)

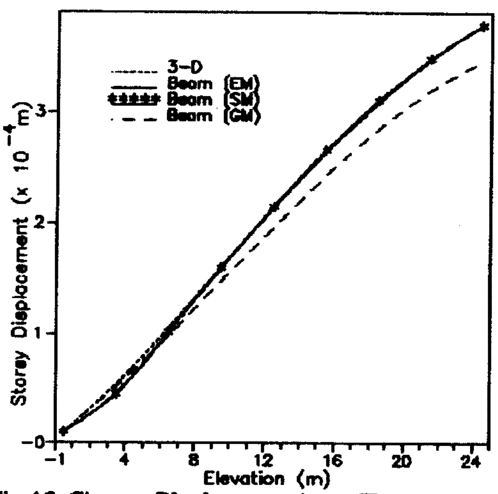

Fig.16 Storey Displacement vs Elev. of Core (X-direction)

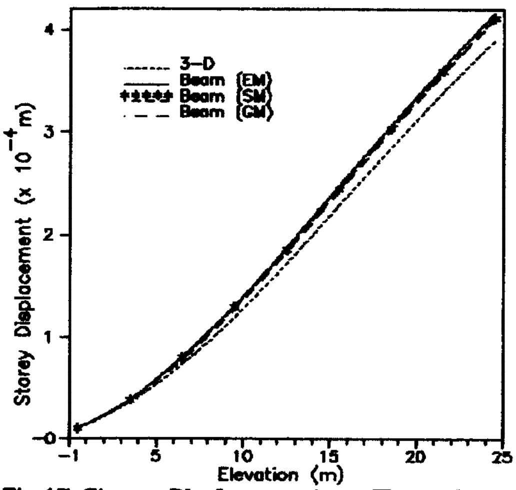

Fig.17 Storey Displacement vs Elev. of Core (Z-direction)

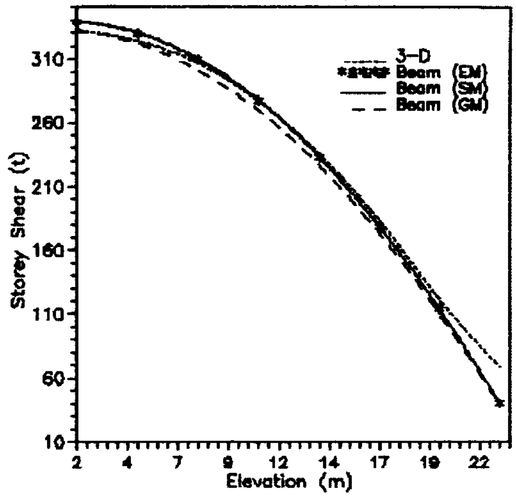

Fig.18 Storey Shear vs Elev. of Core (X-direction)

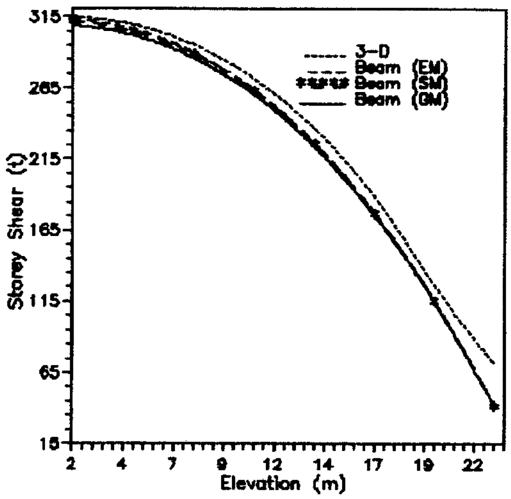

Fig.19 Storey Shear vs Elev. of Core (Z-direction)

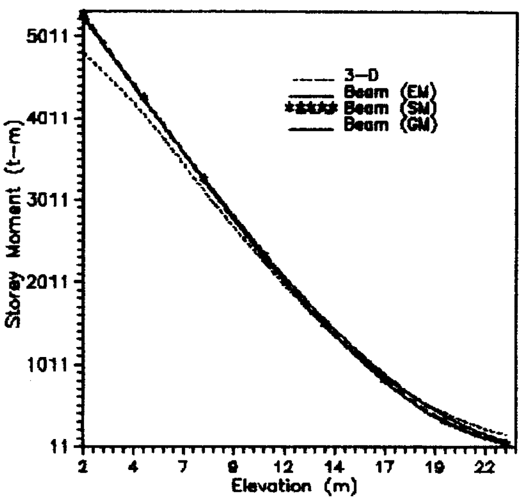

Fig.20 Storey Moment vs Elev. of Core (X-direction)

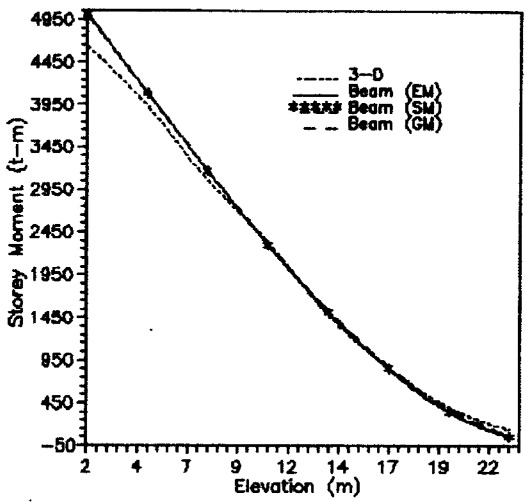

Fig.21 Storey Moment vs Elev. of Core (Z-direction)

CONCLUSIONS

From the results obtained for the three techniques of evoloving LMM of complex structures, following conclusions are arrived at : 1. Both the Energy and Static method can successfully model a complex structure as an equivalent beam. However the Energy method gives better results for complex structures.
2. The widely used Geometric method does not give correct results, for short and complex structure.

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