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A STUDY ON ENVELOPE CHARACTERISTICS OF STRONG MOTIONS IN A PERIOD RANGE OF 1 TO 15 SECONDS BY USING GROUP DELAY TIME

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ABSTRACT

We perform a fundamental study on an envelope model of strong motions which can represent variations of the frequency content with time. Principal idea of this study is that envelope is represented by Fourier phase spectra given by an integration of group delay time *tgr* which is modeled by physical parameters. Therefore in this study we examine the fundamental characterisitics of *tgr* and try to model *tgr* by phsysical parameters.We calculate μ_{tgr} and σ_{tgr} which represent an average and standard deviation of *tgr*, respectively, in moving windows with a narrow frequency band by using records at 26 sites in Japan during the 1990 Izu-Oshima Kinkai earthquake ($M_J = 6.5$, source depth = 6.0 km, epicentral distances = 13.5 to 340 km). We show that envelope of strong motions can be simulated by the frequency dependent *tgr* which is calculated from μ_{tgr} and σ_{tgr} assuming random number with normal distribution of *tgr*. Both μ_{tgr} and σ_{tgr} increase as epicentral distances increase, so we modeled μ_{tgr} and σ_{tgr} as a function of epicentral distance. The group velocity calculated from observed μ_{tgr} and epicentral distance is consistent with the theoretical group velocity of fundamental-mode Love waves in the period range from 5 to 15 sec.

KEYWORDS

Envelope of strong motions; group delay time; Fourier phase spectrum; group velocity of Love waves; dispersion of surface waves; the 1990 Izu-Oshima Kinkai earthquake

INTRODUCTION

It has been a common procedure that strong motions are synthesized from a model Fourier amplitude (or response) spectrum and a Fourier phase spectra composed of random number, assuming shapes of envelope function in time domain (e.g., Boore, 1983). This procedure cannot produce variations of the frequency content with time. This problem was not so important when short period strong motions or body-waves were primary concerns. However, recently new types of structures such as base-isolated buildings and long span buildings whose natural periods are larger than conventional ones have been constructed. In long period strong motions in a period range longer than 1 sec, surface waves are often predominant. Surface waves have dispersion characteristics, so variations of the frequency content with time become an important issue.

Therefore in this paper we perform a fundamental study on an envelope model of strong motions which can represent variations of the frequency content with time. Principal idea of this study is that the envelope is represented by Fourier phase spectra given by an integration of group delay time, *tgr* . It has been known that *tgr* which is a differential of Fourier phase spectra in frequency domain, means the centroid of signals in time domain (Papoulis, 1962). Katukura *et al.* (1978, 1983) pointed out that the average and the standard deviation of *tgr* of strong motions for a given frequency window in frequency domain correspond to the centroid and the duration of strong motions filtered with the same window in time domain, respectively.

Therefore envelope model which can represent variations of the frequency content with time could be represented by using the average and standard deviation of *tgr* calculated in moving windows with a narrow frequency band.

If strong motions observed at a site are aligned by origin time of an earthquake, group delay time depends on distance and velocity structure from the source to the site. Therefore we examine the relationship between distance and the average or standard deviation of *tgr* by regression analysis using observed records. Once the average and standard deviation of *tgr* is modeled as a function of distance, *tgr* at a site of interest is calculated from them assuming normal distribution of *tgr* within a narrow frequency window and then Fourier phase spectrum is calculated by an integration of *tgr*. Although conventional envelope function defined in time domain such as Jennings *et al.*(1968) must be changed for acceleration, velocity, and displacement waves, our envelope model based on *tgr* has an advantage that it can apply to all of them without any changes.

FUNDAMENTAL FORMULATION OF GROUP DELAY TIME

When we represent the Fourier transform of time history waves $f(t)$ as

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt = A(\omega)e^{-i\phi(\omega)}, \tag{1}$$

group delay time $tgr(\omega)$ is represented by

$$tgr(\omega) = \frac{d\phi(\omega)}{d\omega} \tag{2}$$

where ω is a circular frequency, $A(\omega)$ is the Fourier amplitude spectrum and $\phi(\omega)$ is the Fourier phase spectrum. The $tgr(\omega)$ means the centroid at a circular frequency ω.

We will show the fundamental relationships between $tgr(\omega)$ and envelope of $f(t)$ based on Katukura and Izumi (1983). The energy E of the complex envelope (Farnbach, 1972) $\bar{f}(t)$ of $f(t)$ is represented by

$$E = \int_{-\infty}^{\infty} |\bar{f}(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\bar{F}(\omega)|^2 d\omega\ , \tag{3}$$

based on Perseval's theorem. Here $\bar{F}(\omega)$ is the Fourier transform of $\bar{f}(t)$. The average of t, t_0 is represented by the centroid of $|\bar{f}(t)|^2/E$, in other words the 1st moment of $|\bar{f}(t)|^2/E$. The covariance of t, σ_t^2 is represented by the 2nd moment of $|\bar{f}(t)|^2/E$.

$$t_0 = \frac{1}{E}\int_{-\infty}^{\infty} t|\bar{f}(t)|^2 dt \tag{4}$$

$$\sigma_t^2 = \frac{1}{E}\int_{-\infty}^{\infty} (t-t_0)^2|\bar{f}(t)|^2 dt \tag{5}$$

The equations (4) and (5) become the equations(6) and (7) using the Perseval's theorem.

$$\frac{1}{E}\int_{-\infty}^{\infty} t|\bar{f}(t)|^2 dt = \frac{1}{2\pi E}\int_{-\infty}^{\infty} |\bar{F}(\omega)|^2 tgr(\omega)d\omega \tag{6}$$

$$\frac{1}{E}\int_{-\infty}^{\infty} (t-t_0)^2|\bar{f}(t)|^2 dt = \frac{1}{2\pi E}\int_{-\infty}^{\infty}\left[\left(\frac{d|\bar{F}(\omega)|}{d\omega}\right)^2 + |\bar{F}(\omega)|^2(tgr(\omega)-t_0)^2\right]d\omega \tag{7}$$

We express the term of the right-hand side of the equation (6) by μ_{tgr}, and the first and second terms of the right-hand side of the equation (7) by σ_{tgr}^2 and σ_{dA}^2, respectively. Since $\bar{f}(t)$ and $\bar{F}(\omega)$ satisfy causality, μ_{tgr}, σ_{tgr}^2, and σ_{dA}^2 are written as

$$\mu_{tgr} = \frac{1}{E}\int_0^{\omega_N} A^2(\omega)tgr(\omega)d\omega, \tag{8}$$

$$\sigma_{tgr}^2 = \frac{1}{E}\int_0^{\omega_N} A^2(\omega)\left(tgr(\omega)-\mu_{tgr}\right)^2 d\omega, \tag{9}$$

$$\sigma_{dA}^2 = \frac{1}{E}\int_0^{\omega_N}\left(\frac{dA(\omega)}{d\omega}\right)^2 d\omega, \tag{10}$$

where ω_N is the Nyqist circular frequency. The equations (8) and (9) show that μ_{tgr} and σ_{tgr} mean the average and standard deviation of *tgr*, respectively. By using the equations from (6) to (10) we can rewrite the equations (4) and (5) as follows,

$$t_0 = \mu_{tgr} \tag{11}$$

$$\sigma_t^2 = \sigma_{tgr}^2 + \sigma_{dA}^2 \tag{12}$$

The equations (11) and (12) mean that an envelope in time domain is modeled by μ_{tgr}, σ_{tgr}, and σ_{dA} in frequency domain. Since it is found that σ_{dA}^2 is much smaller than σ_{tgr}^2 (Satoh *et al.*, 1996), we do not examine σ_{dA} in this study. We focus on modeling of μ_{tgr} and σ_{tgr} from observed data and show an idea of a frequency-dependent envelope model based on μ_{tgr} and σ_{tgr} calculated in moving windows in a narrow frequency band.

DATA

We use JMA87 type accelerograms observed at 26 sites during the 1990 Izu-Oshima Kinkai earthquake of M_J6.5 because this shallow event (source depth of 6.0 km) generated well-developed surface waves at these sites. The location of the epicenter and sites are shown in Fig.1. The epicentral distances range 13.5 to 340 km. First the records are aligned by origin time of the earthquake. Next two horizontal component accelerograms are rotated into radial and transverse motions and are integrated into velocity data. Then we filter the records in the period range from 1 to 15 sec after confirming signal to noise ratio of the records in this period range. In this study we analyze the transverse components in which Love waves are predominant (e.g., Sato *et al.*, 1993). Before calculating *tgr*, we make uniform of data length as 655.36 sec by adding tailing zero and then add zero of 655.36 sec before records in order to avoid link effects in Fourier transform. We determine the width of moving windows for calculation of μ_{tgr} and σ_{tgr} as 0.02 Hz after preliminary parameter study.

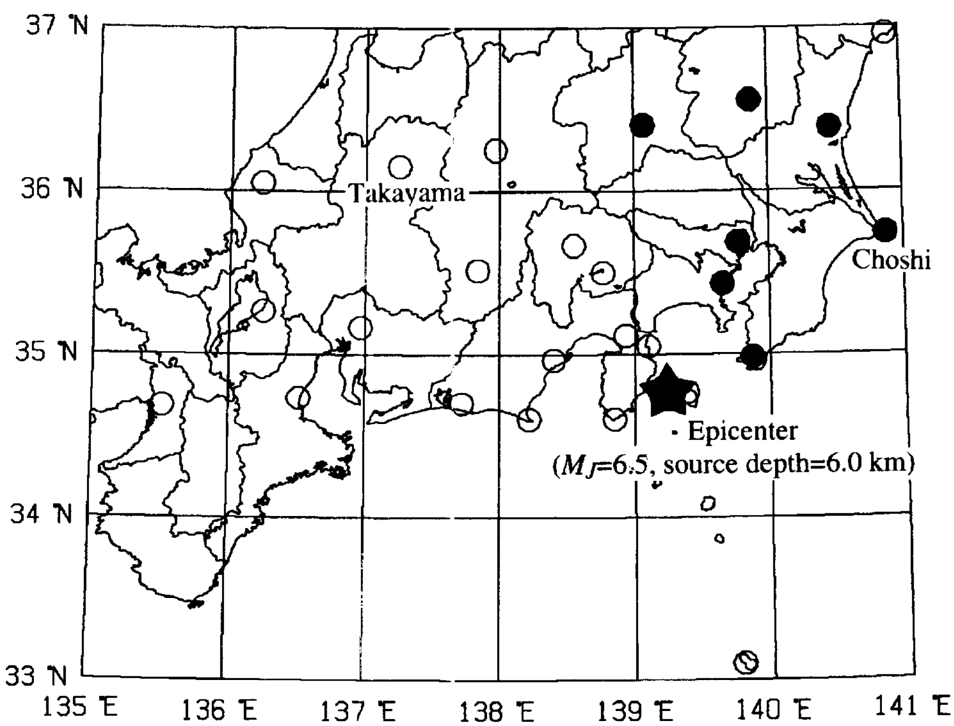
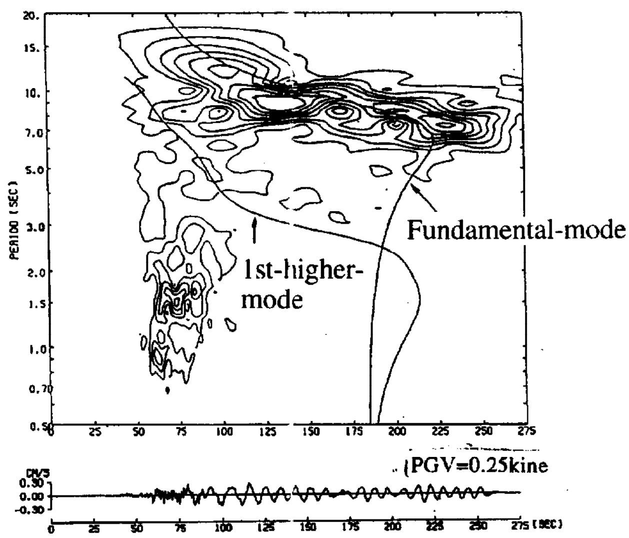


Fig.1 Epicenter of the 1990 Izu-Oshima Kinkai earthquake of M_J6.5 and the location of observation sites. (Circles indicate observation sites. Solid circles indicate observation sites in the Kanto district.)

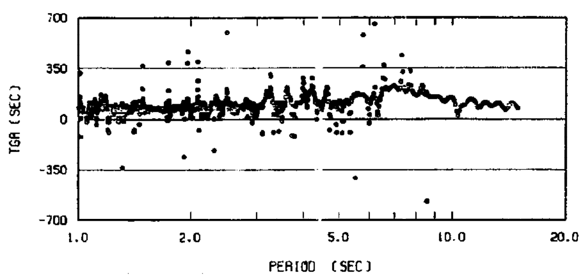
RESULTS

Observed Group Delay Time

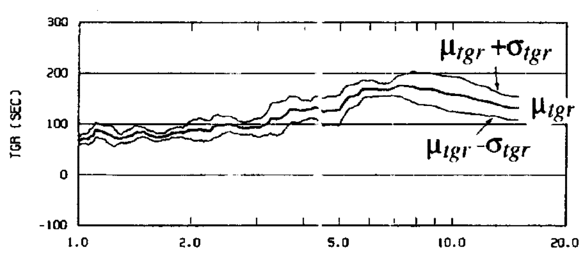
We show a velocity record and its nonstationary spectrum in Fig.2(a), *tgr* in Fig.2(b), and μ_{tgr} and $\mu_{tgr} \pm \sigma_{tgr}$ in Fig.2(c) for a record at Choshi (epicentral distance= 183 km) in the Kanto district as an example. In Fig.3(a) μ_{tgr} shown in Fig.2 (c) is shown together with the nonstationary spectrum. It is confirmed that μ_{tgr} agrees well with the dominant signal of the nonstationary spectra. This fact shows that μ_{tgr} represents the traveling time of waves which have variations of the frequency content with time. In Fig.3(a) we show theoretical *tgr* of fundamental- and the 1st- higher-mode Love waves assuming the flat-layered velocity model given in Table 1. This velocity model is assumed for the Kanto district based on previous studies (e.g., Yamanaka, 1990). By comparing Fig.2(a) and Fig.3(a), it is found that μ_{tgr} is consistent with the theoretical *tgr* of the fundamental-mode Love waves in the period range from 7 to 15 sec.



(a) Nonstationary spectrum and theoretical *tgr* of Love waves



(b) *tgr*



(c) μ_{tgr} and $\mu_{tgr} \pm \sigma_{tgr}$

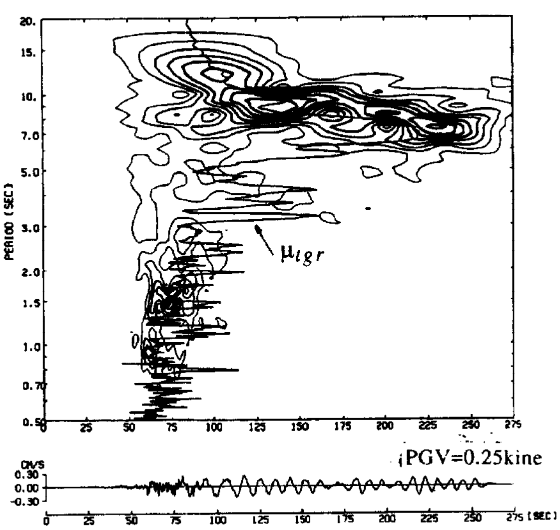
Fig.2 Nonstationary spectrum, *tgr*, μ_{tgr} and $\mu_{tgr} \pm \sigma_{tgr}$ of an observed record at Choshi.

Simulation of Envelope

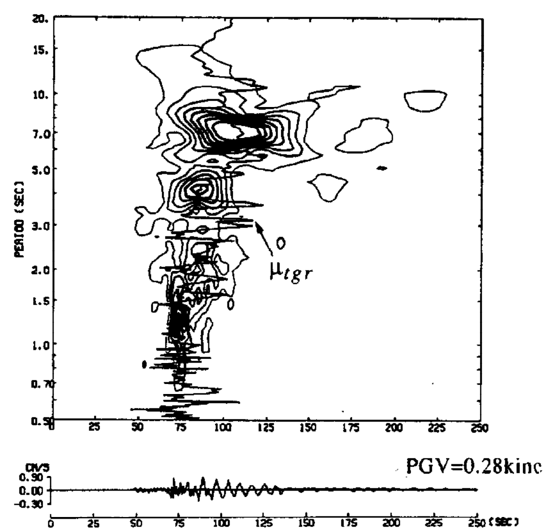
We simulate wave trains using an observed Fourier amplitude spectrum and a synthesized Fourier phase spectrum which is given by an integration of *tgr* calculated from μ_{tgr} and σ_{tgr} assuming random number with normal distribution of *tgr* within 0.02 Hz window. The simulated wave trains at Choshi for 3 different random numbers are shown in Fig.4(a). The observed and simulated waves at Takayama (epicentral distance= 236 km) are also shown in Fig.3 (b) and Fig.4 (b), respectively. The envelope of the simulated waves agree with that of observed both at Choshi and Takayama. This fact means the validity of the assumption of normal distribution of *tgr*. These results demonstrate that frequency-dependent envelope can be simulated by using μ_{tgr} and σ_{tgr}.

Table 1. Ground structure in the Kanto district

Depth (km)	S-wave Velocity (km/s)	Density (t/m^3)
0.0~1.5	1.0	1.9
1.5~2.5	1.5	2.0
2.5~5.5	2.5	2.4
5.5~8.5	3.0	2.5
8.5~15.0	3.5	2.7
15.0~30.0	3.9	3.0
30.0~	4.5	3.2

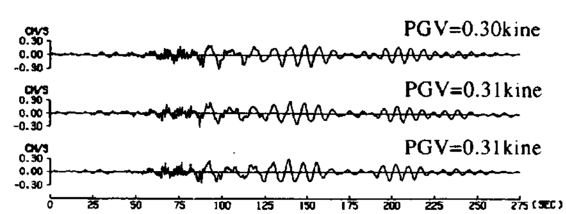


(a) Choshi

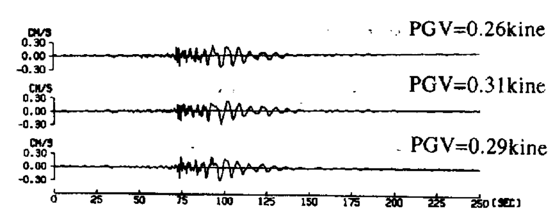


(b) Takayama

Fig.3 Observed wave and μ_{tgr}.



(a) Choshi



(b) Takayama

Fig.4 Simulated waves.

Relationship between Epicentral Distances and Group Delay Time

In Fig.5 we show the relationship between epicentral distances and μ_{tgr} calculated from records observed at 26 sites for the period of 1 and 12 sec. In Fig.6 the relationship between epicentral distances and σ_{tgr} is shown. We can see that both μ_{tgr} and σ_{tgr} increase as epicentral distances increase. Especially the μ_{tgr} has a good correlation with epicentral distances.

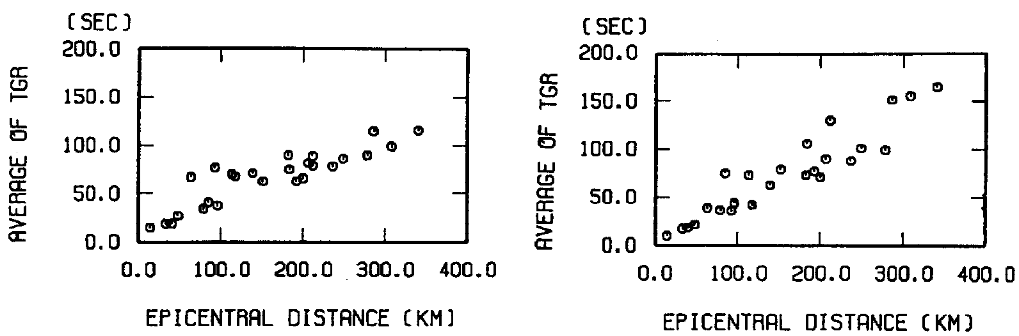
By using these data we perform a regression analysis of μ_{tgr} and σ_{tgr} by following equations,

$$\mu_{tgr}(f) = b_\mu(f)\Delta \tag{13}$$

$$\sigma_{tgr}(f) = b_\sigma(f)\Delta \tag{14}$$

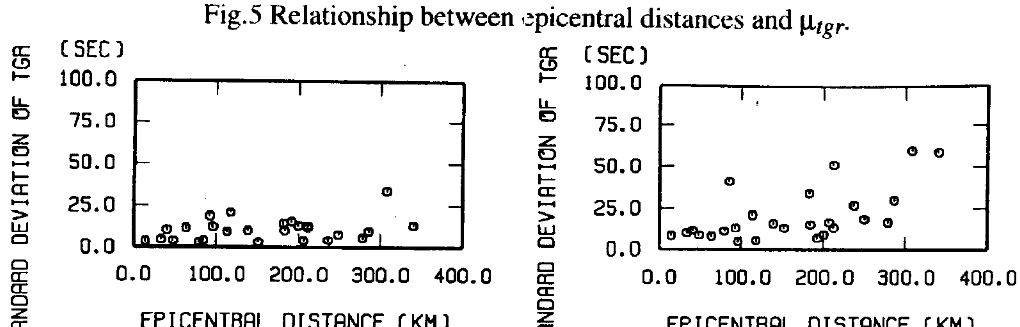
where f is the frequency, Δ is the epicentral distance, $b_\mu(f)$ and $b_\sigma(f)$ are regression coefficients of μ_{tgr} and σ_{tgr}, respectively. The inverse of $b_\mu(f)$ equals to group velocity (km/sec) from the source to the site in an averaged sense. Therefore it is found from Fig.5 that group velocity of waves of period of 1 sec is faster than that of period of 12 sec .

The inverse of $b_\mu(f)$ and the correlation coefficient between epicentral distances and μ_{tgr} are shown in Fig.7(a) and (b), respectively. The theoretical group velocity of fundamental-mode Love waves assuming flat-layered velocity model given in Table 1 is also shown in Fig.7(a). The dispersion characteristics of the inverse of $b_\mu(f)$ is similar to that of the theoretical group velocity in the period range longer than 5 sec. However, the inverse of $b_\mu(f)$ is a little larger than the theoretical group velocity. This result suggests that the group velocity reflecting velocity structure from the source to the 26 sites is larger than that reflecting velocity structure in the Kanto district where thick sedimentary layers form the Kanto planes. In the period range shorter than 5 sec the inverse of $b_\mu(f)$ is different from the theoretical group velocity. This can be interpreted that body-waves and
the higher-mode Love waves are predominant in this shorter period range as suggested in Fig.2(a). It is confirmed the correlation coefficient is high in all the period from 1 to 15 sec except in the period around 4 sec. Relatively low correlation in the period around 4 sec may be due to mixture of different type of waves, such as body-wave, fundamental-mode, and the higher-mode Love waves.



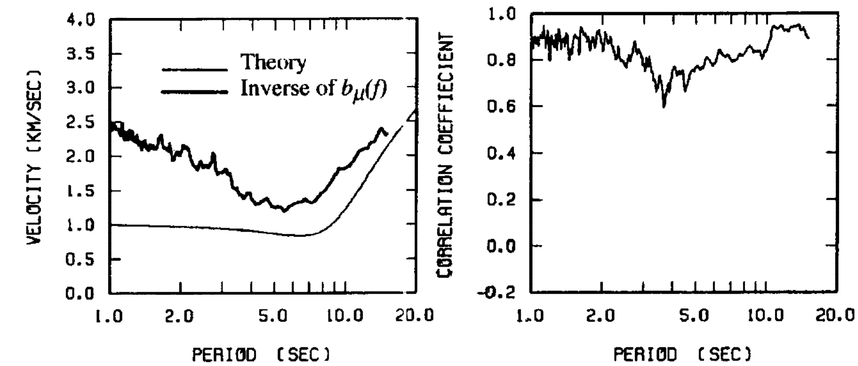
(a) Period of 1 sec (b) Period of 12 sec

Fig.5 Relationship between epicentral distances and μ_{tgr}.



(a) Period of 1 sec (b) Period of 12 sec

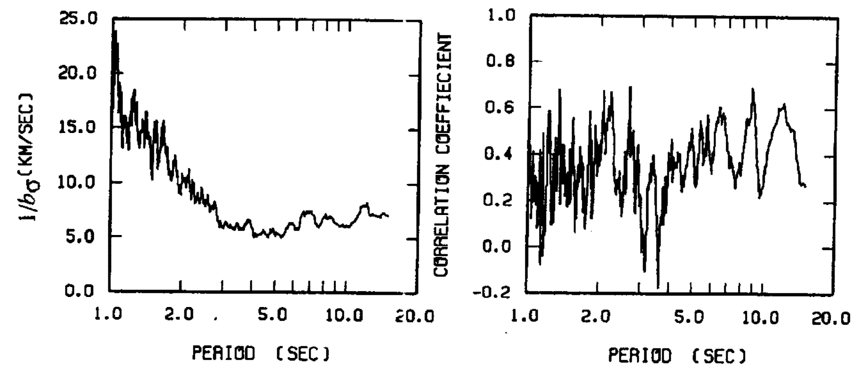
Fig.6 Relationship between epicentral distances and σ_{tgr}.



(a) Inverse of $b_\mu(f)$ and theoretical group velocity of fundamental-mode Love waves

(b) Correlation coefficient

Fig.7 Regression results of μ_{tgr}.



(a) Inverse of $b_\sigma(f)$

(b) Correlation coefficient

Fig.8 Regression results of σ_{tgr}.

The inverse of $b_\sigma(f)$ and the correlation coefficient between epicentral distances and σ_{tgr} are shown in Fig.8 (a) and (b), respectively. The inverse of $b_\sigma(f)$ in short period range is larger than that in long period range. This means that the difference of travel time of shorter period waves within 0.02 Hz window is smaller than that of longer period waves.

CONCLUSIONS

By using group delay time *tgr*, we perform a fundamental study on an envelope model which can represent variations of the frequency content with time. In this study we calculate μ_{tgr} and σ_{tgr} which represent an average and standard deviation of *tgr*, respectively, in moving windows of 0.02 Hz by using records at 26 sites in Japan during the 1990 Izu-Oshima Kinkai earthquake (M_J6.5, source depth=6.0 km, epicentral distances = 13.5 to 340 km).

(1) We show that envelope reprensented by Fourier phase spectrum which is given by an integration of *tgr* calculated from μ_{tgr} and σ_{tgr} assuming random number with normal distribution of *tgr* can represent variations of the frequency content with time.
(2) Both μ_{tgr} and σ_{tgr} increase as epicentral distance increase, so we modeled the average and standard deviation of group delay time as a function of distance. Therefore we could represent envelope by epicentral distance.
(3) The group velocity calculated from observed μ_{tgr} and epicentral distance is consistent with the fundamental mode of Love waves in the period range from 5 to 15 sec.

After we examine the path and magnitude dependency on μ_{tgr} and σ_{tgr} ,we will propose a final envelope model from many observed records.

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