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SEISMIC RESPONSE OF EXTENDED SYSTEMS TO MULTIPLE SUPPORT EXCITATIONS

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ABSTRACT

The response of general multiple-degrees-of-freedom systems subjected to several components of non-stationary random seismic excitation of multiple supports is studied. It is assumed that a measure of the seismic intensity is cumulative square accelerations, or Arias Intensity, which depends from the surface wave magnitude and the focal distance of earthquake. The parameters characterizing the stochastic ground motion model are defined considering this measure. The seismic response analyses of extended structures for spatial effects are based on the idea that the total displacement is separated into quasi-static and vibrational components (Clough and Penzien, 1975). In this paper the procedure is obtained for quickly estimating the root mean square and peak seismic responses of the extended systems that allows to take into account the spatial correlation of the ground motion components, the strong-motion duration, the cross-correlation of modal coordinates closely spaced in the frequency domain *etc.*

KEYWORDS

Extended systems, multiple support nonstationary random seismic excitation, spatial correlation.

INTRODUCTION

Seismic response of extended systems like long-span bridges, pipe-lines or other lifeline structures depends on many factors, such as spatial variations in the earthquake ground motion under the support points of the system, the duration of excitation, the arrangement of supports *etc.* In the earlier studies (Bogdonoff *et al,* 1965; Petrov, 1967, 1974, 1978; O' Rourke *et al,* 1980; Abdel-Ghaffar, 1980; Abdel-Ghaffar and Rubin, 1982, 1983 and other authors) it was found that the seismic response values associated with spatial variations of the ground motion may differ significantly from those obtained through uniform ground motion. For the solution of the problem, time history or spectral analysis are utilized. Offen the apparent propagation velocity is introduced and the travelling wave hypothesis is utilized. Another way to describe the spatial variation of the ground motion is using the sumultaneous records collected by closely spaced arrays of strong-motion instruments (Harada, 1984). However, at present the necessary seismological data are not sufficient. General expression for spatial correlation of ground motion (Petrov, 1978) permits to introduce different assumptions, for example, travelling wave (or "frozen wave") hypothesis or uncorrelated (or statistically independent) unput support motions *etc.*

The parameters of the stochastic ground motion model are defined considering cumulative square ground accelerations.

The proposed approximative procedure for quickly estimating the root mean square and peak seismic responses of the extended systems allows to take into account the spatial correlation of ground motion components, the strong-motion duration, the relatively long natural periods of the system and very small damping, a large number of modes and cross-correlation of modal coordinates closely spaced in the frequency domain.

EQUATIONS OF MOTION

The motion of a structure which is subjected to multiple support excitations can be governed by the differential equations (Clough and Penzien, 1975)

$$M\ddot{u} + C\dot{u} + Ku = -MR\ddot{u}_0, \qquad (1)$$

where M., C and K are mass, damping and stiffness matrices respectively; u - the vibrational displacement vector; u_0 - the ground displacement vector of support points; R - the quasi-static influence matrix which represents the displacements of structure due to unit displacement at each support, while the other support are held fixed.

The total displacement vector of the degrees of freedom is the sum of the quasi-static (u_s) and the relative or vibrational displocements

$$u_t = u + u_s = u + Ru_0 \qquad (2)$$

The vibrational displacement at point j of the structure due to displacement at k-th support may be decomposed into its modal solution

$$u_{kj}(t) = \sum_{i=1}^{N} \alpha_{ij} f_{ik}(t), \qquad (3)$$

where α_{ij} - the i-th mode shape of the structure; $f_{ik}(t)$ - the i-th generalized coordinate, which satisfies the decoupled equation

$$\ddot{f}_{ik}(t) + 2\xi_i p_i \dot{f}_{ik}(t) + p_i^2 f_{ik}(t) = -\ddot{u}_{ok}(t)\delta_{ik} / M_i, \qquad (4)$$

in which ξ_i, p_i - the damping ratio and the natural circular frequency, respectively, of the i-th mode;

$M_i = \sum_{s=1}^{n} m_s \alpha_{is}^2$ - the i-th generalized mass; m_s - the mass, concentrated at point s of the structure;

$\delta_{ik} = \sum_{s=1}^{n} m_s r_{ks} \alpha_{is}$; r_{ks} - the quasi-static influence function which gives the displacement at point s due to a unit displacement at the k-th support, while the other supports are held fixed.

In right part of the Eq.(4) the velocity terms are ignored because its contribution to the total response is usually small when compared to that of the acceleration.

MEAN SQUARE RESPONSE

The mean square response at point j of the structure is given by

$$\overline{u_j^2(t)} = \sum_{i,r=1}^{N} \alpha_{ij}\alpha_{rj}\overline{f_i(t)f_r(t)}; \quad (5)$$

$$\overline{f_i(t)f_r(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_{Q_iQ_r}(\omega)\Phi_i(i\omega,t)\Phi_r(-i\omega,t)d\omega \ ; \quad (6)$$

$\Phi_i(i\omega,t),\Phi_r(-i\omega,t)$ - i-th complex frequency response and r-th complex conjugate frequency response ; if $t \to \infty$, $\Phi_i(i\omega,t) \to \left[p_i^2 - \omega^2 + 2\xi_i p_i i\omega\right]^{-1}$; $i\omega = \sqrt{-1} \bullet \omega$; $G_{Q_iQ_r}(\omega)$ - the cross-spectral density of the generalized forces Q_i and Q_r, which is given by

$$G_{Q_iQ_r}(\omega) = \sum_{k,l=1}^{n_0} G_{kl}(\omega)\delta_{ik}\delta_{rl} / M_iM_r \quad (7)$$

The cross-spectral density function between the inputs at points k and l is

$$G_{kl}(\omega) = \sigma_{\ddot{u}_{ok}}\sigma_{\ddot{u}_{ol}}\sqrt{G_{ok}^N(\omega)G_{ol}^N(\omega)}R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega), \quad (8)$$

where $\sigma_{\ddot{u}_{ok}}, \sigma_{\ddot{u}_{ol}}$ - the root mean square (rms) of the ground accelerations at points k and l;

$G_{ok}^N(\omega)$, $G_{ol}^N(\omega)$ - the normalized power spectral density of the ground accelerations at points k and l;

$R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega)$ - the cross-correlation of the ground accelerations.

Now, substituting Eq.(7), (8) into Eq.(6), yields

$$\overline{f_i(t)f_r(t)} = \sum_{k,l=1}^{n_0} \sigma_{\ddot{u}_{ok}}\sigma_{\ddot{u}_{ol}} I_{kl}^{(i,r)}\delta_{ik}\delta_{rl} / M_iM_r; \quad (9)$$

$$I_{kl}^{(i,r)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \sqrt{G_{\ddot{u}_{ok}}^N(\omega)G_{\ddot{u}_{ol}}^N(\omega)}R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega)\Phi_i(i\omega,t)\Phi_r(-i\omega,t)d\omega \quad (10)$$

Then mean square of vibrational displacement can be represented as

$$\overline{u_j^2(t)} = \sum_{i,r=1}^{N}\sum_{k,l=1}^{n_0} \frac{\delta_{ik}\delta_{rl}\alpha_{ij}\alpha_{rj}}{M_iM_r p_i^2 p_r^2}\sigma_{\ddot{u}_{ok}}\sigma_{\ddot{u}_{ol}}\beta_{ik}(t)\beta_{rl}(t)A_{ir}C_{irkl}; \quad (11)$$

$$\overline{\beta_{1k}^2(t)} = p_1^4 \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{\ddot{u}_{ok}}^N(\omega) |\Phi(i\omega,t)|^2 d\omega ; \qquad (12)$$

$$A_{ir} = I_k^{(i,r)} / \sqrt{I_k^{(i)} I_k^{(r)}}; \quad C_{irkl} = I_{kl}^{(i,r)} / \sqrt{I_k^{(i,r)} I_l^{(i,r)}} . \qquad (13)$$

In the low frequency range of the power spectral density the mean square resonant magnification factor is

$$\overline{\beta_{1k}^2(t)} \approx (p_1 / 4\xi_1) G_{\ddot{u}_{ok}}^N(p_1)[1 - \exp(-2\xi_1 p_1 t)] , \qquad (14)$$

where $t = \bar{t}$ - the strong-motion duration.

If $\xi_1 \to 0$, $\overline{\beta_{1k}^2(t)} = \to 0.5 p_1^2 \bar{t} G_{\ddot{u}_{ok}}^N(p_1)$.

Then $\beta_1(\xi_1) / \beta(\xi = 0) = \sqrt{[1 - \exp(-2\xi_1 p_1 \bar{t})] / 2\xi_1 p_1 \bar{t}}$

For example, at ξ_1=0.005 (0.5% damping), $\bar{t}$=15s, $p_1 = 1.7 s^{-1}$ (the suspension bridge across the Amudarya-river with span length 660m) $\beta_1(0.005) / \beta_1(0) = 0.945$. At ξ_1=0.05 (5% damping) $\beta_1(0.05) / \beta_1(0) = 0.62$. Accordingly $\beta_1(0.5\%) / \beta_1(5\%) \approx 1.5$.

The factor A_{ir} describes the cross-correlation of i-th and r-th modal coordinates. The solution of integral $I_k^{(i,r)}$ may be computed using the concept "white noise" for the random loading (Petrov and Bazilevsky, 1978; Kiureghian A.D., 1980).

The factor C_{irkl} accounts the spatial correlation of accelerations at the k-th and l-th supports. The function $R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega)$ in Eq.(10) can be written as it was assumed earlier (Petrov, 1978)

$$R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega) = \exp\left[-\frac{\omega}{v} L_{kl}(c_1 + c_2 i)\right], \qquad (15)$$

where L_{kl} -the length of span between k-th and l-th supports; v - the shear wave velocity; c_1, c_2 - the constants; $i = \sqrt{-1}$.

For example, the travelling wave (or "frozen wave") hypothesis corresponds to $c_1 = 0$, $c_2 = 1$; the uncorrelated multiple-support excitations - $c_1 \to \infty$, $c_2 = 0$ and the full correlation case - $c_1 = 0$, $c_2 = 0$.

Then $C_{irkl} \approx \mathrm{Re} R_{\ddot{u}_{ok}\ddot{u}_{ol}}(p_{ir})$; $p_{ir} = 0.5(p_i + p_r)$.

The mean square of quasi-static displacement can be now found using the following expressions

$$\overline{u_{sj}^2} = \sum_{i,r=1}^{N} \sum_{k,l=1}^{n_0} \alpha_{ij} \alpha_{rj} I_{kl} \delta_{ik} \delta_{rl} / M_i M_r , \qquad (16)$$

$$I_{kl} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \omega^{-4}\sqrt{G_{\ddot{u}_{ok}}(\omega)G_{\ddot{u}_{ol}}(\omega)}R_{\ddot{u}_{ok}\ddot{u}_{ol}}(\omega)d\omega = \sigma_{u_{ok}}\sigma_{u_{ol}}R_{kl}. \quad (17)$$

The cross-correlation fimction of the ground displacements at points k and l can be approximated by

$$R_{kl} \approx e^{-c_1\theta L_{kl}/v}\cos(c_2\theta L_{kl}/v), \quad (18)$$

where θ - predominant frequency of ground displacements.

Using the data of SMART-1 arrays (Harada, 1984), we may obtain wave length $\lambda = 2\pi v/\theta \approx 1600 \div 4800\text{m}$ and $c_1 \approx 0.25$, $c_2 \approx 1$. If $v \approx 1000$ m/s, $\theta \approx 1.2 \div 4s^{-1}$.

Ignoring cross-correlation vibrational and quasy-static components, the mean square total response can be written as

$$\overline{u_t^2} = \overline{u^2} + \overline{u_s^2} \quad (19)$$

Assuming $\sigma_{\ddot{u}_{ok}} = \sigma_{\ddot{u}_{ol}} = \sigma_{\ddot{u}_0}$, $G_{\ddot{u}_{ok}} = G_{\ddot{u}_{ol}}$, $A_{11} = 1$, $A_{1r} = 0$ ($i \neq r$), $\delta_{11} = \pm\delta_{12}$, for symmetrical bridge with two supports $(n_0 = 2)$ and span length L the mean square of the total displacement at point j can be expressed as

$$\overline{u_j^2(t)} = \sigma_{\ddot{u}_0}^2 \sum_{i=1}^{N} \overline{\beta_1^2(t)\mu_1^2}\frac{\alpha_{1j}^2\delta_{11}^2}{M_i^2 p_i^4} + \sigma_{u_0}^2 \sum_{i=1}^{N}\delta_{11}^2\overline{\mu_s^2} \quad (20)$$

$$\overline{\mu_1^2} = 2(1 \pm C_{i1kl}); \ \overline{\mu_{(s)}^2} = 2(1 \pm R_{kl}) . \quad (21)$$

Then for travelling wave $\overline{\mu_1^2} = 2[1 \pm \cos(p_i L/v)]$; $\overline{\mu_s^2} = 2[1 \pm \cos(\theta L/v)]$, for full correlation case $\mu_1^{(+)} = \mu_s^{(+)} = 2$; $\mu_1^{(-)} = \mu_s^{(-)} = 0$; for uncorrelated case $\mu_1 = \mu_{(s)} = \sqrt{2}$.

The mean square of the bending moments, shearing forces responses *etc* can be obtained by replacing the α_{1j}, α_{rj} with the equivalen t values responses accordingly.

The proposed procedure is utilizied for analysis of some extended systems: the suspension bridge across the Amudarya-river (span length L=660 m); the row suspension bridges for mountain regions of Tajikistan; the Rogun multispan highway bridge; the arch bridge across the Arpa-river (L=120 m) in Armenia and other structures. The seismic response analyses implies that the suspension bridge's vertical response is not affected only by the vertical ground displacements, but also by the longitudinal ground displacements.

ASSESSMENT OF THE RMS GROUND ACCELERATION AND DURATION OF EXCITATION

Cumulative square accelerations, or Arias Intensity, may be a measure of the seismic intensity at time t

$$I_A = \int_0^\infty \ddot{u}_0^2(t)dt, \qquad (22)$$

where $\ddot{u}_0(t)$- nonstationary random earthquake acceleration, represented by

$$\ddot{u}_0(t) = A(t)\varphi(t), \qquad (23)$$

where φ(t)- the stationary random process with the power spectral density function Kanai-Tajimi or Barstein-Bolotin;
A(t) - the intensity parameter, which has been modelled as time-dependent function

$$A(t) = A_0 \frac{t}{t_m} e^{1-t/t_m}. \qquad (24)$$

The suitable value $\bar{t}$ of mean duration of excitation is selected using conditions that the limits of the integral (22) comprise 80 per of the total energy. Then

$$I_A = I_A(t_{0.9}) - I_A(t_{0.1}) = \sigma^2_{\ddot{u}_0}\bar{t}, \qquad (25)$$

where $\sigma_{\ddot{u}_0} = 0.5A_0\sqrt{e}$- the rms acceleration; $t_{0.1}, t_{0.9}$- the limits of the integral (22) comprising 10 per cent and 90 per cent of the total energy, respectively; $\bar{t} = 0.8et_m$; $t_{0.9} = t_{0.5A}$ is the time, when $A(t)=0.5A_0$.

In seems the next approximations can be suitable

$$\bar{t} \approx \exp(0.66M-2.34); \sqrt{I_A} = c_1 g e^{c_2 M} R^{-c_3}, \qquad (26)$$

where M, R - the magnitude of the earthquake and distance of the source from the site; c_1, c_2, c_3- the regional constants; g - acceleration of gravity.

PEAK RESPONSE FACTORS

The expected spectral peak value of responses (displacements, stresses *etc*) is equal to rms responses multiplied by peak factors:

$$u_{max} = r\sigma_u. \qquad (27)$$

The peak factor can be given by equation of Vanmarcke (1976)

$$r = \sqrt{2\ln\{2n[1 - \exp(-\delta_e\sqrt{\pi \ln 2n})]\}}, \qquad (28)$$

in which $n = (\omega_0\bar{t}/2\pi)(-\ln\bar{p})^{-1}$;

δ_e- a measure of the spread in the frequency content of the power spectral density function; ω_0- the central frequency; $\bar{p}$- the probability that the response does not exceed the peak value at time $\bar{t}$, determined as $\bar{p} = 1 + (T/\tau)\ln R_*$ (Bolotin, 1980); T - return period of the earthquake with the magnitude M; τ- lifetime of structure; R_*- rate of reliability, $R_*=1-Q_*$(Q_*- rate of the seismic risk). If $Q_* \ll 1$, $\ln R_* \approx -Q_*$. If $Q_*T/\tau \ll 1$, $-\ln\bar{p}=Q_*T/\tau$.
The values *r* are equal 2.5÷ 4.

CONCLUSION

It is suggested that uniform ground motion is not a good assumption for extended systems. The total displacement is the sum of the quasi-static and the vibrational displacements. In general, the spatial variability of longitudinal components of ground motion may excite significant quasi-static contribution. The longitudinal components of ground motion are likely to excite both vertical and longitudinal vibrations of the extended suspension structures.

The analyses of seismic response to multiple earthquake excitation show that the contribution of summetrical modes usually decreases as the span length increases in comparison with the effective wavelengdth and contribution of anti-symmetrical modes increases accordingly. The efficiency and simplicity of the proposed approach are obtained without inducing significant errors, if realistic data are provided.

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