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FLOOR RESPONSE SPECTRUM AMPLIFICATION DUE TO YIELDING OF SUPPORTING STRUCTURE

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ABSTRACT

In some cases, seismic floor response spectra of inelastic structures have been observed to be higher than the elastic floor response spectra in the high frequency range. It is hypothesized that this happens due to the phenomenon of internal resonance. First, the conditions under which internal resonance can happen are examined through the numerical results obtained for a two degree of freedom nonlinear structure subjected to harmonic base inputs. It is shown that internal resonance can occur in base excited structures if some higher mode frequencies are odd integer multiples of the fundamental frequency, and if there is a strong input at this frequency. To demonstrate that international resonance can occur even in earthquake excited hysteretically inelastic structures, numerical results for a ten degree of freedom structure are also presented.

KEYWORDS

Secondary systems, inelastic floor spectra, hysteretic systems, internal resonance, earthquake response, yielding structures, nonlinear structures.

INTRODUCTION

It is a common knowledge that yielding in structural system due to seismic motions affects a reduction in the elastically calculated forces. This beneficial effect of yielding is currently utilized in code provisions (Building Seismic Safety Council, 1994) for seismic design of building structures. The elastically calculated forces and accelerations are reduced by a factor commonly referred to as the response reduction factor R. The choice of R-factor depends on the type of structure to be designed. A larger value of R is associated with a more ductile structures which can accommodate yielding deformations without apparent damage. For example, for specially designed moment resistant steel frames, the elastically calculated forces can be reduced by a factor of 8. An extension of this concept for reducing elastically calculated structural accelerations also seems natural. This has direct implication in the design of secondary systems, which are also called as nonstructural and architectural components in building design community.

For the design of light secondary systems in critical facilities such nuclear power plants, the floor response spectra are commonly used to define seismic design input. One simple approach to include the effect of yielding on floor response spectra will be to use the same response reduction factor with the elastically calculated floor response spectra as the one used to calculate inelastic force response from the elastic response. This may seem justified, as due to yielding, the maximum acceleration of a floor in a structure is reduced in about the same proportion as the forces in the structure.

To provide a quantitative description of the aforementioned beneficial effect of structural yielding of floor response spectra, a study on single degree of freedom yielding structures was conducted by Lin and Mahin (1985). This study provided a very valuable quantitative information about the level of response reduction one can expect for different primary structure and secondary system frequencies. Broad conclusions of this study were that (1) one can expect a reduction in the acceleration spectrum response for frequencies larger than the supporting structure frequencies, (2) the level of reduction in floor response spectrum was neither the same as in the base shear values and nor uniform for the entire frequency range of interest and, (3) one could also expect a minor increase in the response over the elastic response for secondary system frequencies lower than the supporting structure frequency. This study, although raised the possibility that inelastic response could be higher in a certain frequency range (which could be of practical interest for very flexible equipment), it did not reveal the possibility of obtaining increased inelastic response at frequencies higher than the fundamental frequency of the supporting structure, primarily because the study only considered single degree of freedom supporting structures. A later study by Sewell *et al.* (1986), however, raised some very puzzling questions. Through a comprehensive parametric study of a multi degree of freedom shear buildings, the study showed that in some yielding structures one can get a higher floor response spectrum than the corresponding elastic floor response spectrum. It was pointed out that this effect was more pronounced when the yielding was localized and the base input was narrow band centered around the fundamental frequency.

INTERNAL RESONANCE

Although not explicitly suspected in the Sewell study, the primary cause for a higher floor response spectrum amplification in inelastic structures is the phenomenon of internal resonance (Nayfeh and Mook, 1979), well-known in the field of nonlinear oscillations. It occurs when the higher frequencies of a first order linear structural system are equal to, or near, the integer multiple values of the fundamental frequency, and if there is an external energy input at the fundamental frequency of the structure. In such cases, because of modal interaction, there can occur a transfer of energy from the fundamental to higher modes. In stable structural systems, restoring force characteristics are odd functions of deformation. For structures with symmetric hysteresis loops for positive and negative deformations, one can approximately represent the average or equivalent (in some sense) restoring force as an odd power polynomial of deformation. In such cases, if one or more higher mode frequencies of the first order linear system are odd multiples of the fundamental frequency, then a transfer of energy from the first mode to the higher mode can occur. The five story structure and input motions considered in the Sewell study did have the conditions conducive for internal resonance.

To show the internal resonance analytically, here a two story shear beam model is considered. It is the simplest structural model through which one can show internal resonance without insurmountable analytical complexities. The schematics of the model are shown in Figure 1. The first story stiffness with small cubic nonlinear term is considered. The linear stiffness and mass characteristics are chosen such that the second mode frequency is about 2.958 times the first mode frequency. The two modal frequencies are given in Figure 1. To include some energy dissipation through viscous damping, a proportional damping matrix with modal damping ratios of 2% has been included in the model. The base input to the system is harmonic of frequency near the first mode frequency. The equations of motion for such a system with nonlinear restoring force can be written as:

$$
\begin{aligned}
m_1\ddot{x}_1 + c_{11}\dot{x}_1 + c_{12}\dot{x}_2 + f_{s1} - f_{s2} &= -m_1\ddot{x}_g \\
m_2\ddot{x}_2 + c_{21}\dot{x}_1 + c_{22}\dot{x}_2 + f_{s2} &= -m_2\ddot{x}_g
\end{aligned} \tag{1}
$$

where x_1 and x_2 are the relative displacements of the first and second floors, respectively; c_{ij} are the elements of the system damping matrix; m_1 and m_2 are the floor masses, and f_{s1} and f_{s2} are the nonlinear restoring forces in the stories, defined with cubic nonlinearities as follows:

$$f_{s1} = k_1 x_1 - \acute{k}_1 x_1^3 \quad ; \quad f_{s2} = k_2 (x_2 - x_1) - \acute{k}_2 (x_2 - x_1)^3 \tag{2}$$

We will assume that the cubic nonlinearity is small enough such that perturbation methods can be applied. To examine internal resonance, we will consider a nearly tuned harmonic input of frequency Ω. To solve eq.(1) for the forced response, the method of multiple scales can be used. It can be shown (Nayfeh and Mook, 1979) that the forced response solution for a near resonance case (that is, $\Omega \simeq \omega_1 =$ first frequency of the system) can be expressed as:

$$\begin{aligned} x_1 &= \phi_{11} a_1 \cos(\Omega t - \gamma_2) + \phi_{12} a_2 \cos(3\Omega t + \gamma_1 - 3\gamma_2) \\ x_2 &= \phi_{21} a_1 \cos(\Omega t - \gamma_2) + \phi_{22} a_2 \cos(3\Omega t + \gamma_1 - 3\gamma_2) \end{aligned} \tag{3}$$

where $\{\phi_{11}, \phi_{21}\}^T$ and $\{\phi_{12}, \phi_{22}\}^T$ are the first and second modal displacements for the first order linear system. For the steady state response, the parameters γ_1 and γ_2 are phase values which are related to amplitudes a_1 and a_2 according to the following equations:

$$\begin{aligned} 8\beta_1\omega_1^2 a_1 + \alpha_2 a_1^2 a_2 \sin\gamma_1 - 4 f_1 a_g \sin\gamma_2 &= 0 \\ 8\beta_2\omega_2^2 a_2 - \alpha_5 a_1^3 \sin\gamma_1 &= 0 \\ 8\omega_1\sigma_2 a_1 - \left(3\alpha_1 a_1^2 + 2\alpha_3 a_2^2\right) a_1 - \alpha_2 a_1^2 a_2 \cos\gamma_1 + 4 f_1 a_g \cos\gamma_2 &= 0 \\ 8\omega_2 a_2 (3\sigma_2 - \sigma_1) - \left(3\alpha_8 a_2^2 + 2\alpha_6 a_1^2\right) a_2 - \alpha_5 a_1^3 \cos\gamma_1 &= 0 \end{aligned} \tag{4}$$

In these equations, $\alpha_1, ..\alpha_6$ are related to the nonlinear stiffness coefficient and modal displacement; β_1 and β_2 are modal damping ratios; and σ_1 and σ_2 are the detuning parameters defined as:

$$\omega_2 = 3\omega_1 + \varepsilon^2\sigma_1 \quad ; \quad \Omega = \omega_1 + \varepsilon^2\sigma_2 \tag{5}$$

where ε^2 is the perturbation parameter, expressing the relative strength of the nonlinear term compared to the linear term.

These equations are solved for a_1, a_2, γ_1 and γ_2 by numerical procedures. Knowing x_1 and x_2, one can define the absolute acceleration time histories of the two floors. These two time histories are then applied to a single degree of freedom elastic oscillator to obtain floor response spectrum values. To calculate the floor spectra for the linear case, the nonlinear terms are dropped and a linear modal analysis is used.

NUMERICAL RESULTS

The following results are obtained to demonstrate the internal resonance and its effect on floor response spectra. Only nonlinearity, confined to the first story, is considered as systems with distributed nonlinearity do not exhibit internal resonance. The floor response spectra results are presented to show the effects of the input frequency detuning, placement of secondary oscillator, intensity of base motion, strength of the nonlinear stiffness term relative to the linear term, and equipment and structure dampings. These results are followed by similar results for a 10 story shear structure subjected to random earthquake inputs.

Figures 2 and 3 show the linear and nonlinear floor response spectra for the first and second floors for the case of exact tuning, that is, the frequency of the input harmonic motion being exactly equal to the first mode frequency. The nonlinear spectrum has an additional peak near the second structural frequency which is completely absent in the linear case. This peak is due to transfer of energy from the first mode to the higher mode caused by internal resonance. The quantitative differences between the linear and nonlinear spectra are perhaps better presented in terms of their ratio, call the floor response spectrum ratio (FRSR), shown in Figures 4 and 5 for the above two cases. This ratio is slightly higher than 1 for the entire frequency range, except near the frequency of excitation where it is slightly less than 1 and, of course, near the second mode frequency where it is significantly higher than 1. Comparing the spectra for the first and second floors, we observe that the effect of internal resonance is diminished for the second floor. It is because for the higher modes, though still receive energy through internal resonance, do not contribute much to the higher floor response. This characteristics is also shown quite clearly by the results obtained for a 10 story building, presented later.

Next we show the results for a slightly detuned input. The input frequency is now 2 Hz with the ratio of the first structural frequency to the input frequency = 0.957. Figures 6, 7 and 8 show the floor response spectra for increasing input amplitudes of 1/5g, 1/3g and 1/2g. We notice that higher the intensity of the input motion, the stronger the internal resonance effect, as shown by higher FRSR values at the second mode frequency.

Figures 9 and 10 show the effect of the nonlinearity strength. In the previous figures, the cubic nonlinear term was 10^{-4} time the linear term. For the cases of strong nonlinearity, the cubic terms are chosen as 10^{-3} and 10^{-2} times the linear terms. It is mentioned that increasing nonlinearity affects the accuracy of the perturbation approach; that is, stronger the nonlinearity, the less accurate the perturbation solution approach. However, for the level of nonlinearity considered here, the accuracy does not seem to be a problem. The floor response spectra and FRSR values depicted in these figures clearly indicate that stronger nonlinearity implies stronger internal resonance effect.

Next we show the effect of equipment and structural damping on the nonlinear floor response spectra in Figures 11 and 12. In figure 11, the structural damping ratio is increased to 5% from 2%, used in earlier cases; all other parameters are the same as for the results of Figure 2. Comparing the FRSR values in Figures 2 and 11 we observe that an increase in the structural damping is seen to diminish the effect of internal resonance as less energy is now available for transfer between modes. In Figure 12, the equipment damping ration is increased to 2% from 0.5%, used earlier. Comparing the nonlinear floor response spectrum in Figure 12 with the one in Figure 2, we again observe that an increase in equipment damping also tends to diminish the effect of internal resonance.

The above results were for a simple two degree of freedom system with cubic linearity excited by a harmonic input. To show that internal resonance phenomena can also manifest in a more realistic hysteretic structure excited by an earthquake induced ground motion, the following results are presented for a 10 story shear building structure. The elastic modal frequencies of the structure are: 1.666, 4.962, 8.147, 11.15, 13.90, 16.35, 18.42, 20.09, 21.31 and, 22.05 cycles per seconds. It is noted that the ratios of the higher mode frequencies to the fundamental mode frequency are: 2.978, 4.890, 6.693, 8.343, 8.814, 11.06, 12.06, 12.79 and, 13.23. Since several frequencies are odd multiples of the first frequency, there is a good possibility of interaction and internal resonance. Since internal resonance is more pronounced in systems with concentrated nonlinearity, here the strength characteristics of the structure have been chosen such that yielding can occur only in the first story. The force deformation characteristic of the yielding element is modeled by smooth Bouc-Wen model (Wen, 1976). The numerical results are obtained for both a narrow band recorded accelerogram (Parkfield event of 1966) and also for a broad-band input defined by 50 synthetically generated accelerograms with frequency response characteristics similar to those in RG-1.60 spectra (U.S. Nuclear Regulatory Commission, 1973). In the broad-band case, the floor response results are the average of the results for the 50 time histories.

First we show the results for the narrow-band Parkfield motion, the response spectrum of which, shown in Figure 13, has a very strong peak near the structural frequency. Figures 14 and 15 show the linear and nonlinear floor response spectra for the first and tenth floors, respectively, of the structure. In Figure 14 for the first floor, where higher modes do contribute significantly to the response, we observe some quite high floor response spectrum values in the high frequency range. This clearly indicates a rather very strong internal resonance effect. From the 10th floor response spectrum in Figure 15, we note that the effect of internal resonance is reduced since at the higher floor levels it is the first mode which dominates the response. The effect of internal resonance also appears to be less in the case of a broad-band input. This is noted from the results in Figures 16 and 17, obtained for the first and the tenth floor for the broad band input. A broad band input has significant energy at the higher frequencies. The higher modes receive this energy directly from the input, in addition to the energy transferred due to internal resonance from the fundamental mode . In such situations, the relative contribution of the energy transferred to a higher mode through internal resonance will not be very high. Internal resonance is, however, still present but its contribution is now masked.

CONCLUDING REMARKS

In some cases the floor response spectra for yielding structures can be higher in the high frequency range than the corresponding elastic response spectra, even though there is significant dissipation of energy due to yielding. Here it is shown that the primary reason for higher inelastic spectra is the phenomenon of internal resonance encountered in nonlinear vibrations. In civil structures subjected to seismic ground motion, the internal resonance can occur (1) if the higher mode frequencies are odd multiple of the fundamental frequency and (2) if there is significant energy in the input at the fundamental frequency. More perfect the tuning between the structural frequencies and also between the fundamental frequency and the input, the stronger the internal resonance effect. Structural and equipment dampings tend to reduce the internal resonance effect. Also if the input is broad-band such that it provides significant energy to the higher modes, then the internal resonance effect, though still there, appears masked.

ACKNOWLEDGMENTS

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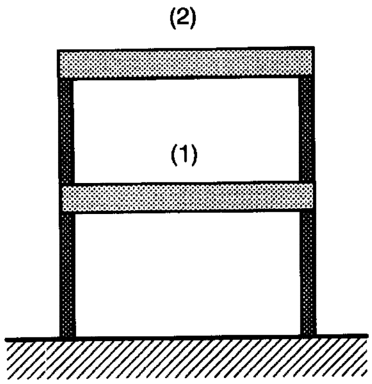

Mode	Frequency [Hz]
1	1.913
2	5.659

FIGURE 1: SCHEMATIC OF 2-DOF STRUCTURE

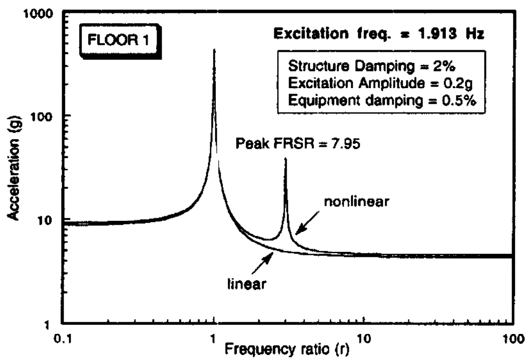

FIGURE 2: FIRST FLOOR, LINEAR AND NONLINEAR SPECTRA FOR RESONANCE CASE

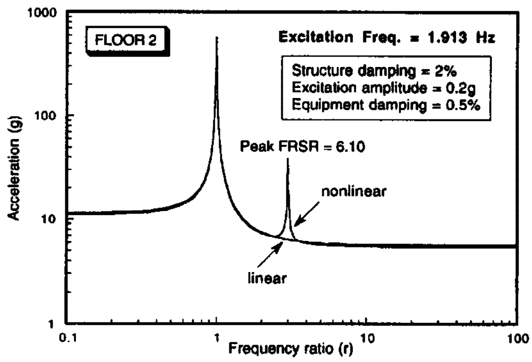

FIGURE 3: SECOND FLOOR, LINEAR AND NONLINEAR SPECTRA FOR RESONANCE CASE

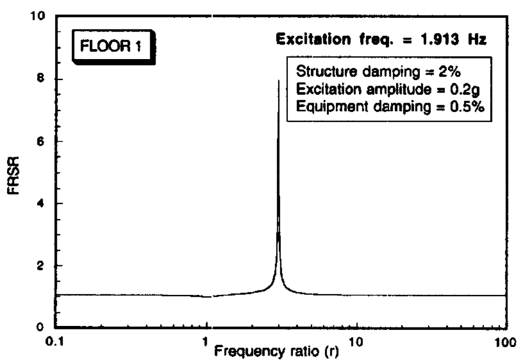

FIGURE 4: FIRST FLOOR RESPONSE SPECTRA RATIO FOR RESONANCE CASE

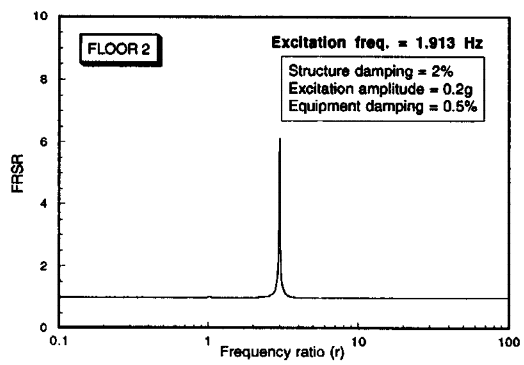

FIGURE 5: SECOND FLOOR RESPONSE SPECTRA RATIO FOR RESONANCE CASE

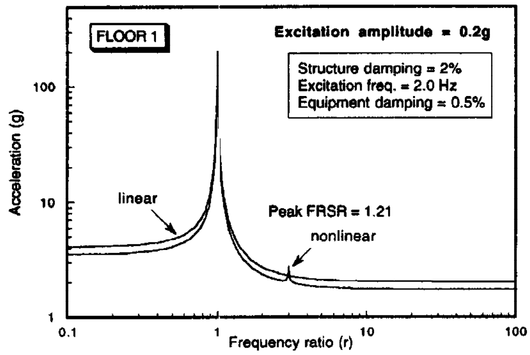

FIGURE 6: LINEAR AND NONLINEAR SPECTRA FOR EXCITATION LEVEL OF 0.2G, INPUT FRQ.= 2.0 HZ

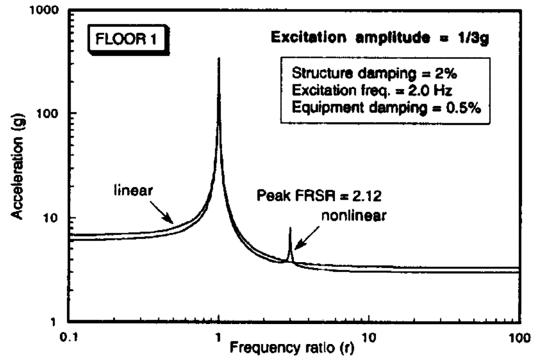

FIGURE 7: LINEAR AND NONLINEAR SPECTRA FOR EXCITATION LEVEL OF 1/3G, INPUT FRQ.= 2.0 HZ

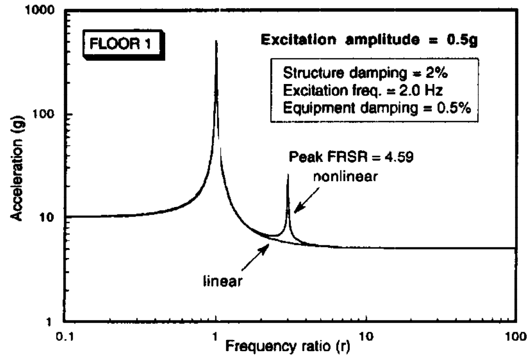

FIGURE 8: LINEAR AND NONLINEAR SPECTRA FOR EXCITATION LEVEL OF 0.5G, INPUT FRQ.= 2.0 HZ

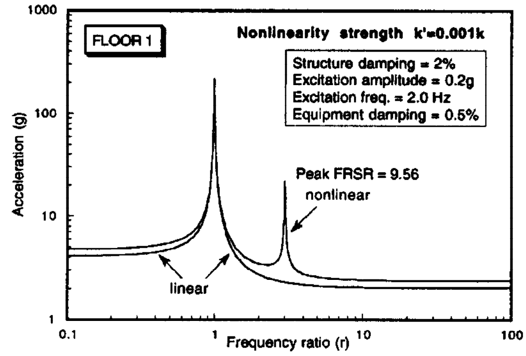

FIGURE 9: LINEAR AND NONLINEAR SPECTRA FOR NONLINEARITY LEVEL OF 0.001

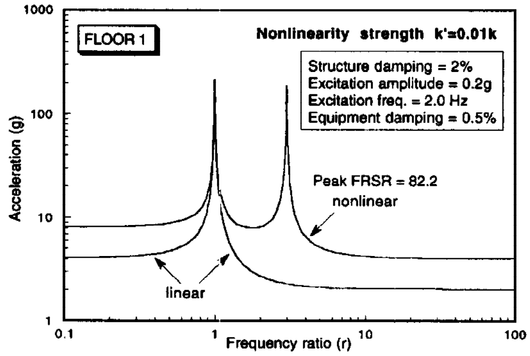

FIGURE 10: LINEAR AND NONLINEAR SPECTRA FOR NONLINEARITY LEVEL OF 0.01

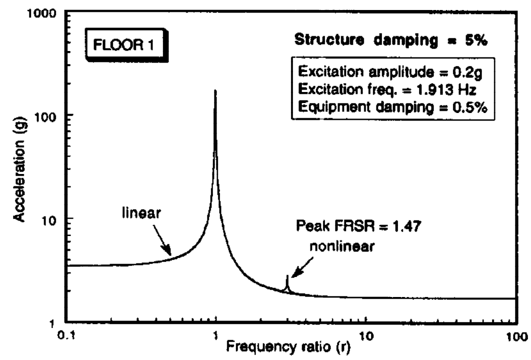

FIGURE 11: LINEAR AND NONLINEAR SPECTRA FOR STRUCTURE DAMPING RATIO OF 5%

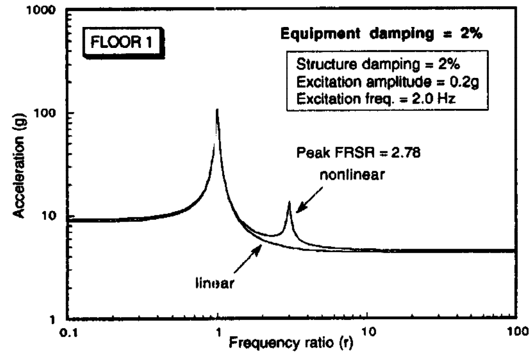

FIGURE 12: LINEAR AND NONLINEAR SPECTRA FOR EQUIPMENT DAMPING RATIO OF 2%

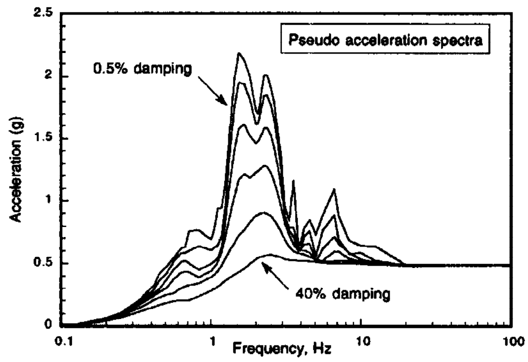

FIGURE 13: GROUND RESPONSE SPECTRA FOR PARKFIELD EARTHQUAKE

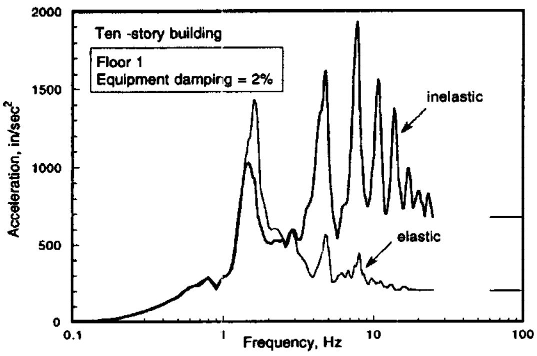

FIGURE 14: FLOOR 1 RESPONSE SPECTRA FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO PARKFIELD ACCELEROGRAM

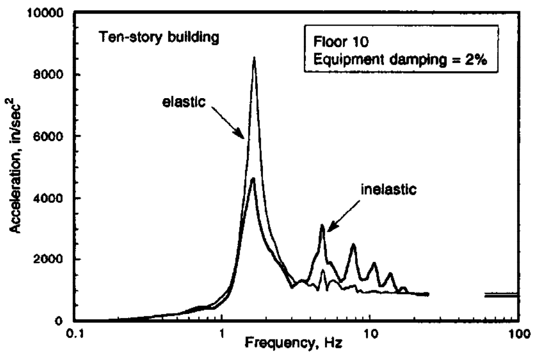

FIGURE 15: FLOOR 10 RESPONSE SPECTRA FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO PARKFIELD ACCELEROGRAM

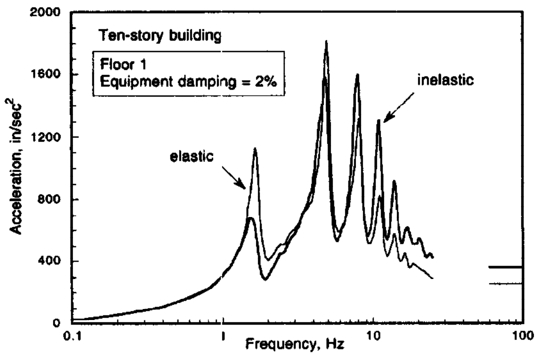

FIGURE 16: AVERAGE RESPONSE SPECTRA OF FLOOR 1 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO 50 ACCELEROGRAM

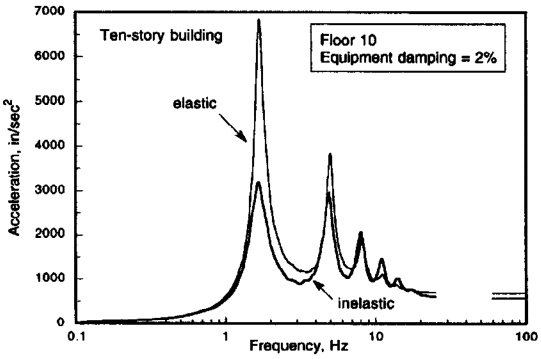

FIGURE 17: AVERAGE RESPONSE SPECTRA OF FLOOR 10 FOR ELASTIC AND YIELDING STRUCTURES SUBJECTED TO 50 ACCELEROGRAM