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CHARACTERISTICS OF ROTATIONAL GROUND MOTIONS

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ABSTRACT

Rotational ground motions cause torsional and rocking responses of structures. Properly defining them is essential in calculating such structural responses. But rotational ground motions cannot be recorded directly due to some inherent difficulty. This paper discusses the characteristics of rotational ground motions with respect to those of the spatially propagating translational ground motions. A method to calculate the auto and cross power spectral density functions (PSD) of the rotational ground motions at a point on ground surface is also presented. Equations of rotational ground motion PSDs are derived as functions of the PSDs and coherency functions of spatial translational motions. Using the recorded spatial translational motions during Events 24 and 45 at the SMART-1 array, numerical values of the rotational motion PSDs are calculated. The rotational motion PSDs are also calculated by using the proposed method. Results from the two approaches are compared. Very good agreement is observed.

KEYWORDS

Rotational ground motion, power spectral density, coherency function

INTRODUCTION

At a point on ground surface, there are six components of ground motions, namely three translational and three rotational. Due to some inherent difficulty, only translational ground motions can be directly recorded in practice (Bolt et al. 1982). The three rotational components, which cause rocking and torsional responses of structures, are usually estimated from the translational motions. Some methods were proposed to estimate torsional ground motions (Newmark 1969, Nathan and Mackenzie 1975, Tso and Hsu 1978). In those methods, spatial translational motions are assumed to be horizontally propagating plane wave without coherency loss. The only variation in spatial translational motion is the phase shift due to the wave passage effect. Actual recorded spatial translational motions on the bases of building structures during recent earthquakes were also used to estimate Torsional motion time histories (De La Llera and Chopra 1994). All three components of rotational motions at a point on ground surface were estimated by using the recorded translational motions at the SMART-1 array (Oliveira and Bolt 1989). In the latter study, however, no explicit formula for estimating the rotational motion PSDs was presented. Relations between rotational and translational motions such as the PSDs and coherency functions were not defined. Recently, a method of estimating the torsional motion PSDs from the PSDs and coherency functions of the horizontal translational motions was derived and presented (Hao 1996). In that study, however, only the torsional component of the rotational motions was considered. The two rocking components were not included.

In the present paper, complete analyses of the three rotational ground motions, that is two rocking and one torsional components, are carried out. The equations of the rotational motion PSDs and the cross PSDs between any two components of the six ground motions at a point on ground surface are derived. They are expressed in terms of the PSDs and coherency functions of the translational motions. The empirical coherency functions of the horizontal and vertical spatial translational motions are obtained first by analyzing the recorded motions during Events 24 and 45 at the SMART-1 array. These coherency functions, together with the averaged PSDs of the recorded translational motions of the two events, are then used to estimate the rotational motion PSDs. Numerical values of the rotational motion PSDs are also approximately estimated by using the corresponding recorded spatial translational motions. Good agreement between the rotational motion PSDs obtained from the two approaches is observed. The validity of the derived formula on estimating the torsional motion PSDs is thus proved. The characteristics of the rotational ground motions are also discussed with respect to the corresponding characteristics of the translational ground motions.

TRANSLATIONAL MOTION PSDS AND COHERENCY FUNCTIONS

Standard approaches are used to estimate the PSDs and coherency functions of the recorded spatial translational motions (Kanasewich 1981). In this study, the PSDs and coherency functions of the recorded motions at the center and inner ring stations during Events 24 and 45 at the SMART-1 array (Bolt et al. 1982) are estimated.

Since the radius of the inner ring of the SMART-1 array is 200 m, which is much smaller than the epicentral distance, ground motions at the center and the inner ring stations are assumed having the same intensity. The PSDs of the two horizontal components are also assumed to be the same. Hence, the ensemble average of the PSDs of the horizontal ($S_g^H(\bar{\omega})$) and vertical ($Sg^V(\bar{\omega})$) accelerations for the two events are obtained by averaging the PSDs at the center and inner ring stations. Fig. 1 shows the calculated PSDs. As can be seen, the intensities of the horizontal motions are larger and concentrated in a narrower frequency band as compared to those of the vertical motions. The PSDs of ground motions corresponding to Event 45, which has magnitude $M_L = 7.0$ and epicentral distance 79 km, are larger than those of Event 24 of magnitude $M_L = 6.9$ and epicentral distance 84 km. The frequency contents of the horizontal motions from the two events are more or less the same, but those of the vertical motions are different. The vertical motions of Event 45 have a wider frequency band than those of Event.

Since the three translational motions at a point are statistically independent if the three directions coincide with the principal and minor directions (Penzien and Watabe 1975). In this study, the recorded horizontal motions (NS and EW components) are projected into the epicentral direction (X) and a horizontal direction perpendicular to the X direction (Y) as shown in Fig. 2. Because the principal direction usually coincides with the epicentral direction (Penzien and Watabe 1975), the translational motions in the X, Y and Z directions are statistically independent. Hence, only the translational motions in the same directions are cross correlated.

A few empirical coherency functions are available (Abrahamson 1985, Harichandran and Vanmarcke 1986, Loh and Yeh 1988, Hao el al. 1989, Hao 1996). As discussed in a previous study (Hao 1996), only the coherency functions which are function of the square of the separation distance can be used to model the rotational motion PSDs. In this study, the same coherency function as proposed earlier is used for both the horizontal and vertical motions. If ground motion propagates in the X direction, the coherency function between motions in the same direction on ground surface at points k and l is (Hao 1996)

$$\gamma_{kl}(x,y,i\bar{\omega}) = \mid \gamma_{kl} \mid exp(-\frac{i\bar{\omega}x}{v_a}) = exp\{-[\alpha_1(\bar{\omega})x^2 + \alpha_2(\bar{\omega})y^2]\bar{\omega}\}exp(-\frac{i\bar{\omega}x}{v_a}) \quad (1)$$

where $\mid \gamma_{kl} \mid$ is the coherency loss function and $exp(-\frac{i\bar{\omega}x}{v_a})$ is the phase shift between the motions at the points k and l, and $\bar{\omega}$ is the circular frequency, v_a is the apparent ground motion propagation velocity, x and y are the separation distances between the points k and l in the X and Y directions, respectively; $i = \sqrt{-1}$ is the imaginary unit; and $\alpha_j(\bar{\omega})$ are parameter functions that can be determined from the recorded motions.

By using the regression method, it is found that, for horizontal motions, it has

$$\alpha_j^H(\bar{\omega}) = \frac{a_j}{ln(\bar{\omega}) + b_j}, \qquad \bar{\omega} \geq 0.314 \ \ rad/s, \qquad j = 1,2 \quad (2)$$

whereas

$$\alpha_j^V(\bar{\omega}) = a_j\bar{\omega}^{b_j}, \qquad \bar{\omega} \geq 0.314 \ \ rad/s, \qquad j = 1,2 \quad (3)$$

for vertical motions; in which a_j and b_j are determined by best fitting the coherency of the recorded motions to Equation (1). For the two events considered, the constants a_j and b_j are given in Table 1.

Table 1. Constants in coherency functions

	Horizontal				Vertical			
Event	$a_1(\times10^{-6})$	b_1	$a_2(\times10^{-6})$	b_2	$a_1(\times10^{-6})$	b_1	$a_2(\times10^{-6})$	b_2
24	1.689	1.42983	1.741	1.57179	5.433	-0.7	2.099	-0.4765
45	1.117	1.66726	1.159	1.72263	0.678	-0.3498	0.610	-0.291

Figs. 3 and 4 show the coherencies of the two events estimated directly from the recorded motions and from Equation (1). As can be seen, Equation (1) and the constants given in Table 1 yield good estimates of the coherencies of the translational motions.

The apparent ground motion propagation velocities of the two events were estimated by calculating the cross correlations between motions recorded at different stations. It was found that (Hao 1996) $v_a = 770 \ m/s$ and $v_a = 875 \ m/s$ for Events 24 and 45, respectively.

ROTATIONAL MOTION POWER SPECTRAL DENSITY FUNCTIONS

Let $\ddot{u}(x,y,t)$, $\ddot{v}(x,y,t)$ and $\ddot{w}(x,y,t)$ be the three translational ground accelerations at point (x,y) on ground surface in the X, Y and Z directions. From the theory of mechanics of solids, the two rocking accelerations can be calculated by

$$\ddot{\phi}_x(x,y,t) = -\frac{\partial\ddot{w}(x,y,t)}{\partial y} \quad (4)$$

$$\ddot{\phi}_y(x,y,t) = \frac{\partial\ddot{w}(x,y,t)}{\partial x} \quad (5)$$

and the torsional acceleration by

$$\ddot{\phi}_z(x,y,t) = \frac{1}{2}[\frac{\partial \ddot{u}(x,y,t)}{\partial y} - \frac{\partial \ddot{v}(x,y,t)}{\partial x}] \tag{6}$$

Using Equations (4) to (6), the equations for PSDs of the rotational accelerations can be derived.

Since

$$\ddot{\phi}_x(x,y,t) = -\frac{\partial \ddot{w}(x,y,t)}{\partial y} = -\lim_{y\to 0} \frac{\ddot{w}(x,y,t) - \ddot{w}(x,0,t)}{y} \tag{7}$$

and its covariance function is

$$E[\ddot{\phi}_x(x,y,t)\ddot{\phi}_x(x,y,t+\tau)] = E[\lim_{y\to 0} \frac{\ddot{w}(x,y,t) - \ddot{w}(x,0,t)}{y} \lim_{y\to 0} \frac{\ddot{w}(x,y,t+\tau) - \ddot{w}(x,0,t+\tau)}{y}] \tag{8}$$

By assuming $\ddot{w}(x,y,t)$ to be stationary, continuous and differentiable, and transferring Equation (8) into the frequency domain, the PSD of $\ddot{\phi}_x(x,y,t)$ is derived as

$$S_{\ddot{\phi}_x}(\bar{\omega}) = 2\bar{\omega}\alpha_2^V(\bar{\omega})S_g^V(\bar{\omega}) \tag{9}$$

Similarly, the PSDs of $\ddot{\phi}_y(x,y,t)$ and $\ddot{\phi}_z(x,y,t)$ are derived and given as

$$S_{\ddot{\phi}_y}(\bar{\omega}) = 2\bar{\omega}[\alpha_1^V(\bar{\omega}) + \frac{\bar{\omega}}{2v_a^2}]S_g^V(\bar{\omega}) \tag{10}$$

and

$$S_{\ddot{\phi}_z}(\bar{\omega}) = \bar{\omega}[\alpha_1^H(\bar{\omega}) + \alpha_2^H(\bar{\omega}) + \frac{\bar{\omega}}{2v_a^2}]S_g^H(\bar{\omega}) \tag{11}$$

As can be noted, the PSD of $\ddot{\phi}_x(x,y,t)$ depends only on the coherency loss function of the vertical motions, which is governed by the two coefficient functions $\alpha_j^V(\bar{\omega})$, whereas those of $\ddot{\phi}_y(x,y,t)$ and $\ddot{\phi}_z(x,y,t)$ depend on both the coherency loss function and the ground motion phase shift.

The cross PSDs of the six ground motion components are also derived. Like the three translational motions, it is found that the three rotational components are statistically independent of each other, and $\ddot{\phi}_x(x,y,t)$ is also statistically independent of the three translational motions. $\ddot{\phi}_y(x,y,t)$ and $\ddot{w}(x,y,t)$, and $\ddot{\phi}_z(x,y,t)$ and $\ddot{v}(x,y,t)$ are, however, found to be cross correlated with the cross spectra

$$S_{\ddot{\phi}_y\ddot{w}}(i\bar{\omega}) = i\frac{\bar{\omega}}{v_a}S_g^V(\bar{\omega}) = \frac{\bar{\omega}}{v_a}S_g^V(\bar{\omega})e^{i\pi/2} \tag{12}$$

and

$$S_{\ddot{\phi}_z\ddot{v}}(i\bar{\omega}) = -i\frac{\bar{\omega}}{2v_a}S_g^H(\bar{\omega}) = \frac{\bar{\omega}}{2v_a}S_g^H(\bar{\omega})e^{-i\pi/2} \tag{13}$$

respectively.

To check the validity of the above derived equations for PSDs, the rotational accelerations are estimated by using finite difference to approximate the differentiations in Equations (4) to (6). Using the recorded spatial translational motions at the inner ring and the center stations during Events 24 and 45 at the SMART-1 array, the averaged PSDs of the three rotational accelerations are calculated. The results are shown in Figs. 5 and 6 for those of Events 24 and 45, respectively. The same rotational PSDs are also calculated by using Equations (9) to (11) with the constants given in Table 1. The corresponding results are also shown in Figs. 5 and 6 for comparison purpose. As can be seen, some discrepancies between the PSDs from the two approaches exist, which can be attributed to the approximation of the differentiation by finite difference in estimating the PSDs, and to the discrepancies in the model coherency loss values and the actual coherency loss between the spatial translational motions as shown in Fig. 1. Nevertheless, besides some minor discrepancies, the calculated PSDs from the two approaches generally agree well. It can be said that the derived equations can be used to estimate rotational ground motion PSDs.

From the results shown in Fig. 1 and Figs. 5 and 6, it can be noticed that the magnitudes of the rotational ground motions are much smaller than those of the translational motions, but the frequency contents of the rotational motions are wider than those of the translational motions, implying the energy of the rotational motions are more evenly distributed in a wider frequency band.

CONCLUSIONS

Equations of rotational ground motion power spectral density functions have been derived and presented. The rotational ground motion power spectral density functions are found to be functions of the translational ground motion power spectral density functions and their coherency functions. Constants in an empirical coherency function corresponding to the recorded motions during Events 24 and 45 at the SMART-1 array have been obtained by regression method. These constants, together with the averaged power spectral density functions of the recorded translational motions of the two events have been used to estimate the rotational motion power spectral density functions. The estimated rotational motion PSDs are compared with those obtained from direct calculation by using the recorded spatial translational motions. The derived equations have been proved to yield reliable estimate of the rotational motion power spectral density functions.

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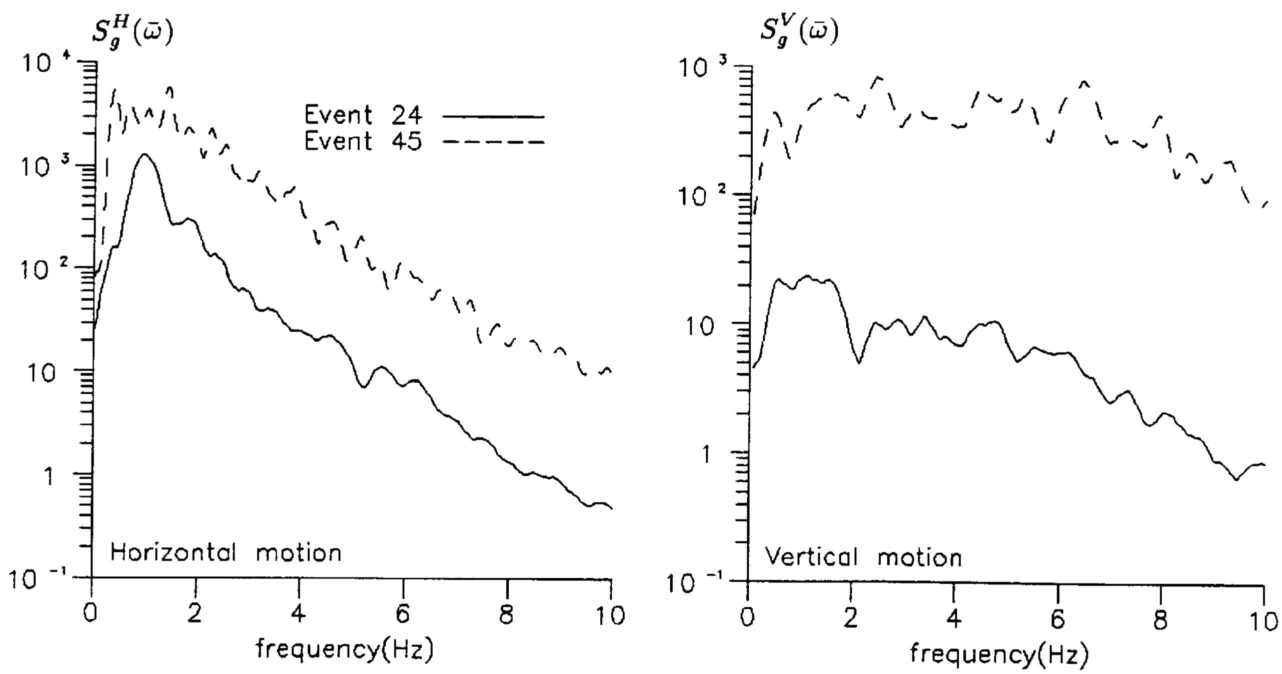

Fig. 1 Average power spectral density functions of the translational motions

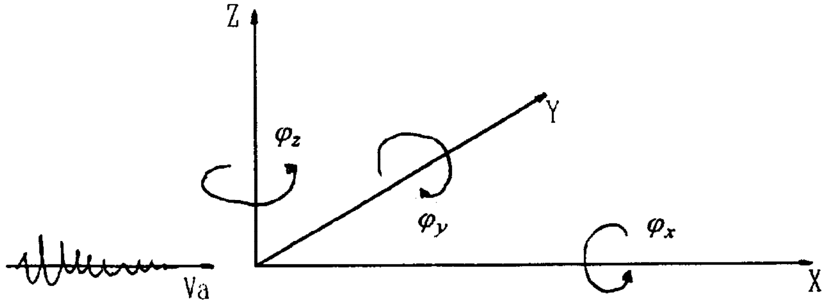

Fig. 2 Ground motion propagation direction and its six components at a point

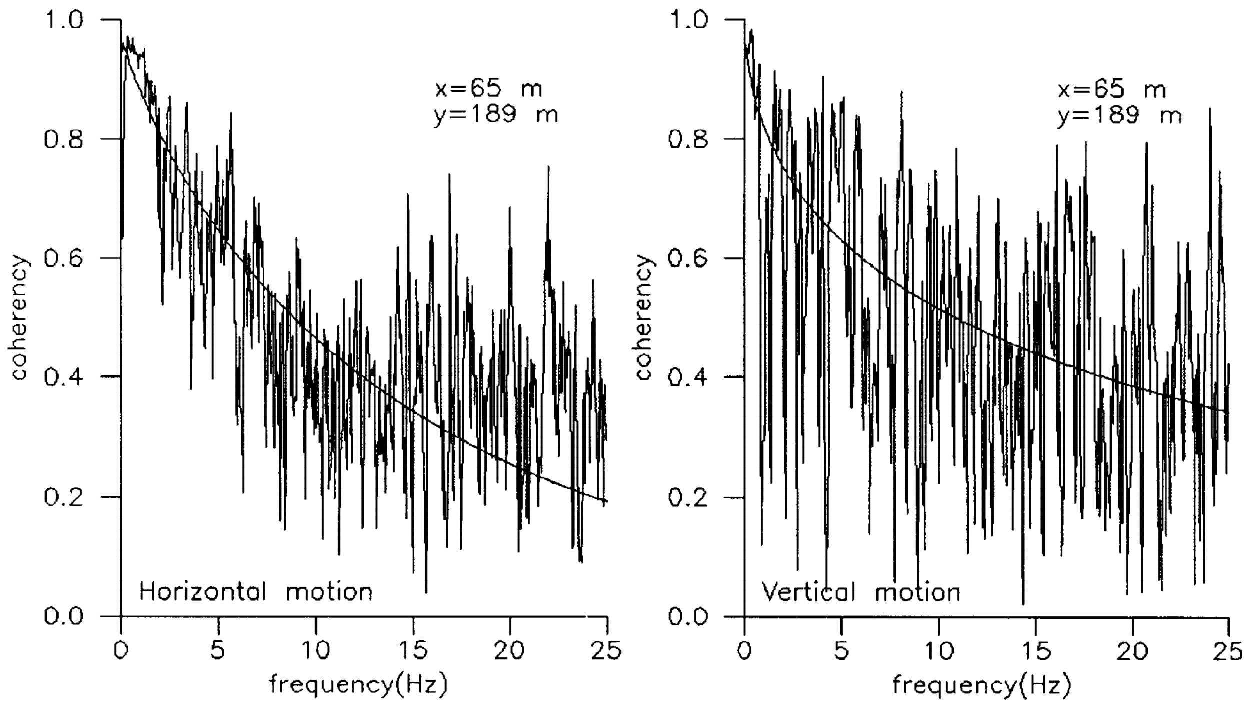

Fig. 3 Coherency values calculated from the two appraoches (Event 24)

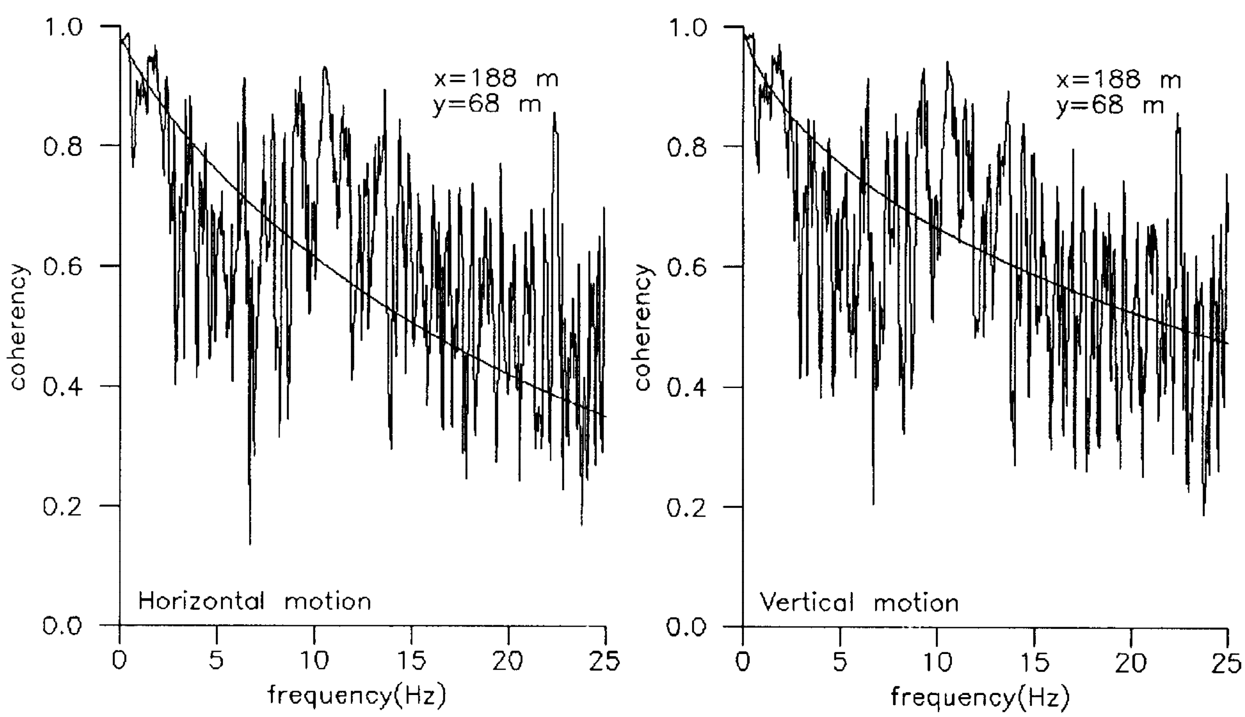

Fig. 4 Coherency values calculated from the two approaches (Event 45)

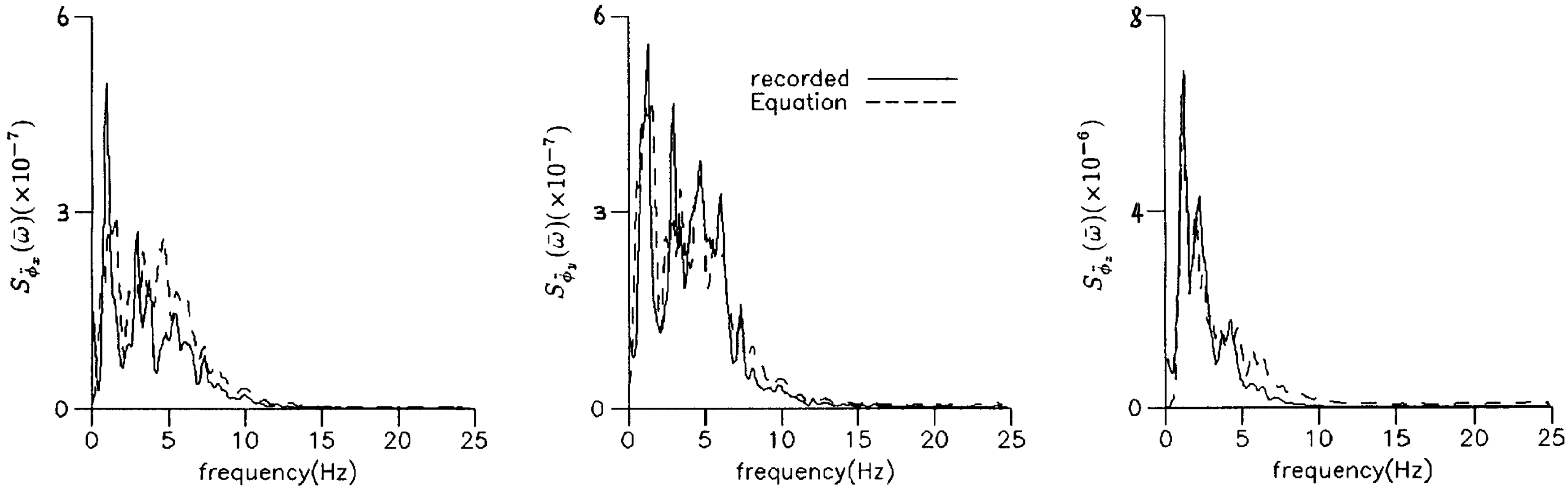

Fig. 5 Comparison between rotational power spectral density functions (Event 24)

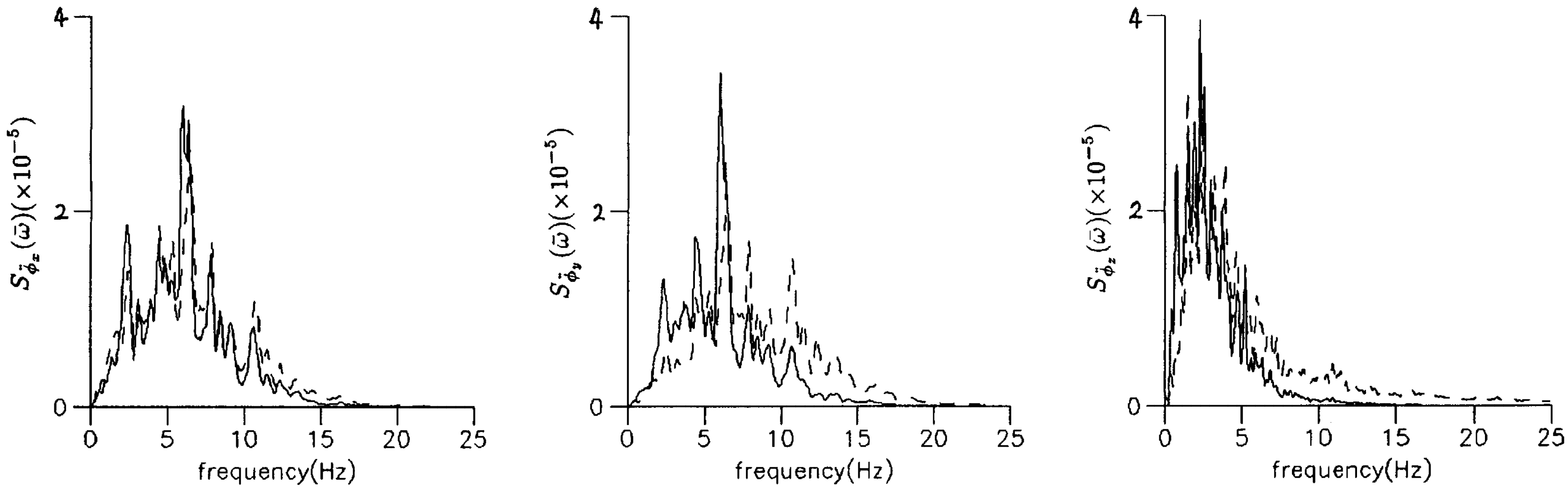

Fig. 6 Comparison between rotational power spectral density functions (Event 45)