



## **SEISMIC TORSION IN NON-LINEAR NOMINALLY SYMMETRIC STRUCTURES DUE TO RANDOM PROPERTIES**

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### **ABSTRACT**

In this paper the effect of uncertain structural parameters upon the torsional behavior of nominally symmetric structures is investigated. Considered parameters are the location of the center of mass and the stiffness and strength of the elements resisting lateral earthquake forces. A parametric study is carried out using models with a range of fundamental periods of interest for structures in Mexico City. Obtained results are used to evaluate the recommendation for accidental eccentricity included in the current Mexico City Design Code.

### **KEYWORDS**

Design codes; torsional behavior; accidental eccentricity; random structural properties.

### **INTRODUCTION**

Inherent uncertainties in the values of the structural parameters which define the lateral response of building structures, are some of the causes of accidental eccentricities, even in those structures designed and built as symmetric (Rosenblueth, 1979). These eccentricities produced by variations of the structural properties, of mass distribution on the building storeys, and by possible rotations induced by the ground motion are frequently cited as a cause of unexpected earthquake damage.

Up till now it has not been possible to assess, precisely, the size of the accidental eccentricities and the way they should be correctly included in the dynamic analyses of a building. They are frequently considered in the seismic design of structures by adding an additional static torsional moment obtained from the assumption that the storey shear force moves from its nominal position. This movement is a fraction of the in plan maximum dimension of the building and is measured perpendicular to the direction of seismic excitation.

The objectives of this paper are to determine which of the structural parameters considered as random, produce the maximum accidental eccentricity in nominally symmetric structures; and to investigate the effect of the random location of the center of mass, and the random stiffness and strength of the structural elements, upon the accidental eccentricity coefficient proposed in the current Mexico City Design Code (RDF93), (RDF, 1993).

## SEISMIC DESIGN OF ASYMMETRIC STRUCTURES

Current seismic design regulations allow, for buildings with eccentricities, the use of only translation static analyses, including the torsional effects through additional seismic storey shear forces distributed over all resistant frames. These forces are obtained from torsional moments, computed by multiplying the initial storey shear forces times some design eccentricities chosen to produce the most unfavorable effect on each resistant element. The design eccentricities prescribed by the codes for a building are expressed by the following equations (RDF93):

$$e_{d_1} = \alpha_1 e_s + \delta b$$

$$e_{d_2} = \alpha_2 e_s - \delta b$$

where  $e_s$  is the static eccentricity, defined as the distance between the center of mass and the center of torsion;  $\alpha_1$ ,  $\alpha_2$  are the dynamic amplification coefficients which consider the differences in responses between the static and dynamic methods of analysis;  $\delta$  is the accidental eccentricity coefficient, and  $b$  is the maximum in plan dimension of the structure perpendicular to the direction of excitation.

Accidental eccentricities are present in most building structures, even in those designed and built as nominally symmetric.

### STRUCTURAL MODEL AND EARTHQUAKE EXCITATION

The structural model used in this investigation is a one storey shear building. The behavior of structural elements is assumed elasto-plastic with no damping. The resistant elements are fixed to the base, and rigidly connected to the slab system.

The structural models are assumed to be located on the Lake Zone of Mexico City and subjected E-W component of the motion recorded at the SCT site during the 1985 Michoacan earthquake.

The non-linear dynamic analyses of the models are carried out using the computer program DYNDIR (Gillies, 1979). As a structural performance index the maximum ductility demand of the storey structural elements is considered.

The uncertainty of the structural properties of the models is introduced using the Monte Carlo method with a simulation space defined with the Multipoint Estimation Method (Ordaz, 1988), using three probability concentrations for each of the random variables.

Due to the large number of simulations required to define the statistical properties of the response to the long duration of the earthquake record, it is necessary to reduce its total duration without the losing precision in the final results. For this the effective duration of the record is defined as the part within which the maximum structural response occurs. Previous experimentations with this type of structures and this particular record have shown that an appropriate value of the effective duration is the part of the record between the 5 and the 95% Arias intensity, (Trifunac and Brady, 1975, and Escobar, 1994).

### INDUCED ECCENTRICITY DUE TO RANDOM STRUCTURAL PARAMETERS

The part of the accidental eccentricity due to the inherent uncertainties of the structural parameters and that caused by the rotational ground motion can be evaluated by establishing a functional relationship among the

induced eccentricity, the earthquake excitation, and the random structural parameters. This consideration turns the structural response to be a random variable. In the present investigation the earthquake excitation is considered as deterministic, and only the structural properties are random.

## ANALYSIS PROCEDURE

The induced eccentricity due to torsion in the structural model subjected to earthquake excitation in only one direction is after De la Llera and Chopra, 1994:

$$e_a(t) = \frac{m_r \ddot{\theta}(t)}{m \ddot{u}_g(t)}$$

where  $e_a(t)$  is the induced eccentricity by torsion;  $m$  and  $m_r$  are the translational and rotational mass, respectively;  $\ddot{\theta}(t)$  is the rotational acceleration of the storey, and  $\ddot{u}_g(t)$  is the ground acceleration.

The accidental eccentricity is defined as the maximum value of  $e_a(t)$  obtained from the earthquake response. This value is normalized respect to the dimension  $b$  of the building to give the accidental eccentricity coefficient.

$$\delta = \frac{\max |e_a(t)|}{b}$$

To determine which of the random variables (center of mass, stiffness and structural strength), has the greatest influence on the accidental eccentricity, an intuitive measure of the relative importance of each of the considered random variables is employed. This measure is defined as the absolute value of the relative error of the standard deviation subtracted from unity, it is

$$r_{x_i} = 1 - \left| \frac{\sigma_x - \sigma_{x_i}}{\sigma_x} \right|$$

where  $r_{x_i}$  relative importance factor of the random variable  $x_i$  (center of mass, stiffness and structural strength),  $\sigma_x$  is the standard deviation of the coefficient  $\delta$ , considering all the random variables, and  $\sigma_{x_i}$  is the standard deviation of the coefficient  $\delta$  considering only the random variable  $x_i$ .

## NUMERICAL SIMULATIONS

The procedure to carry out this study is to perform the step by step non-linear dynamic analyses of the structural models with simulated location of the center of mass and stiffness and strength properties. With the results from these analyses the statistical properties of the accidental eccentricity coefficient and of the parameter used to characterize structural behavior are obtained. It is applied to one storey shear building models with uncertainties in the location of the center of mass, and with random stiffness and strength.

## PARAMETERS STUDIED

The effect of the uncertainties of the structural parameters upon the coefficient  $\delta$  is evaluated using nominally symmetric structural models with two resistant elements. The fundamental periods  $T$ , of the models are 0.5, 1.0 and 1.5 sec. The structural overstrength factor is 1.5. The seismic behavior factors (ductility reduction factors),  $Q$ , are 2 and 4.

## STATISTICAL PROPERTIES OF THE RANDOM VARIABLES

For the location of the center of mass,  $CM$ , a Gaussian probability distribution function (pdf) is considered. Its statistical parameters are obtained considering the  $\pm 0.1b$  range values specified by the RDF93 values as discussed by Escobar, 1994.

For the stiffness,  $K$ , and strength,  $S$ , of the structural elements a lognormal pdf is considered, with a coefficient of variation of 20% (Escobar, 1994). The mean value of the strength, is obtained from its nominal value affected by the overstrength factor.

## RESULTS

Fig 1 shows the variation of the standard deviation of the coefficient  $\delta$  with respect to the translational fundamental period of the structural models. In general, structures designed to have a high nonlinear behavior, i.e.  $Q=4$ , present a standard deviation higher than those designed with  $Q=2$ , particularly when the vibration period is less than or equal to 1.0 sec. For relatively more flexible structures, i.e. natural period  $T=1.5$  sec, designed with  $Q=2$ , the values obtained for the standard deviation of the coefficient  $\delta$  are similar to those obtained for models designed with  $Q=4$  and the same translational fundamental period.

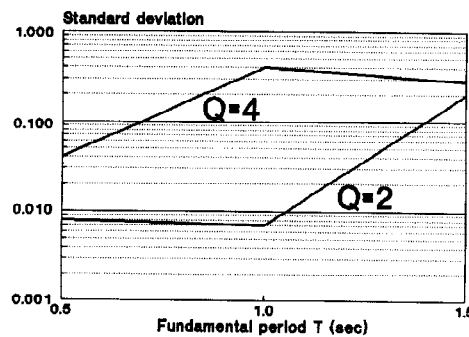


Fig. 1. Standard deviation of  $\delta$  vs. fundamental period  $T$ .

Figs. 2.a and 2.b show the variation of the relative importance factor of the structural parameters with respect to the natural period of the models studied. It can be seen that the influence of the structural strength over the accidental eccentricity, is less for structures in which the behavior is close to elastic (structures designed with  $Q=2$ , Fig. 2.a), than that observed in structures with high nonlinear behavior (structures designed with  $Q=4$ , Fig. 2.b), where the structural strength has the maximum influence over the accidental eccentricity coefficient in all cases. On the other hand, the effect of the structural stiffness is always less than the effect of the variation of the location of the center of mass. Moreover, for the relative flexible models ( $T=1.5$  sec), the effect of the structural strength over the coefficient  $\delta$ , is more important than that of other parameters.

For the models designed with  $Q=2$  (Fig. 2.a), the position of the center of mass is the parameter with the highest influence over  $\delta$ , for natural periods less than or equal to 1.3 sec. For models with a natural period higher than 1.3 sec, again the structural strength produces the higher effect over  $\delta$ .

Figs. 3.a and 3.b show in the vertical axis the probability that  $\delta$  has a value less than or equal to the corresponding value shown in the horizontal axis. In them, it may be observed that when the interval grows, the probability that  $\delta$  be less than or equal to this increases.

In Fig. 3.a, the models designed with  $Q=2$  and with  $T$  less than or equal to 1.0 sec are those which have

the largest probability for small values of  $\delta$ , having particularly high values for  $\delta = 0.1b$ . The models with  $T=1.5$  sec, are those that present the smallest values of probability.

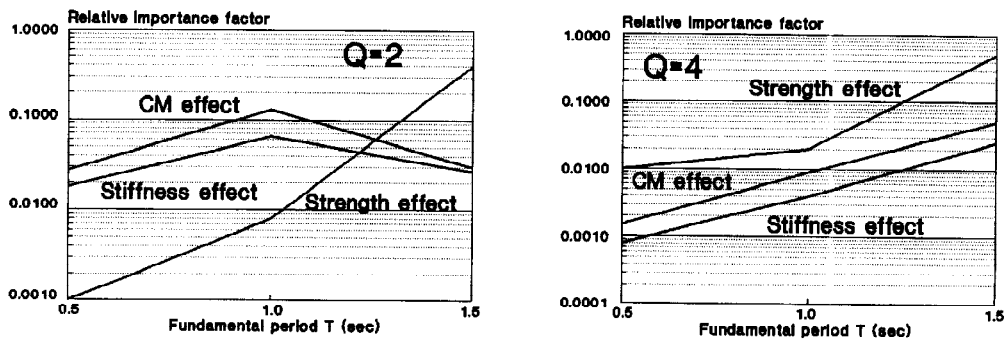


Fig. 2. Relative importance factor of the structural parameters vs. fundamental period  $T$ .

The corresponding results for the models designed with  $Q=4$  are shown in Fig. 3.b. It may be observed that only the models with  $T=0.5$  sec have a large probability value.

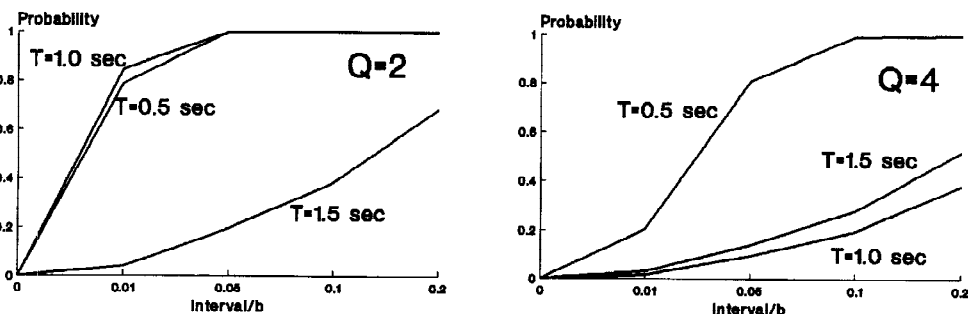


Fig. 3. Accumulative probability for  $\delta$  for models of different fundamental period  $T$ .

Table 1 presents for all the design ductilities and periods investigated the ratio of the maximum ductility demand of the structure, obtained at the time the maximum eccentricity is reached  $D_{tor}$ , and the mean value of the maximum ductility demanded  $D_{max}$ . It may be observed that this ratio is not always equal to one, implying that the maximum ductility demand, and the maximum value for the accidental eccentricity, do not occur at the same time, i.e. for the structural models, torsion is not always the cause of the largest ductility demand.

Table 1. Ratio of maximum ductility demand due to torsion and mean maximum ductility demand.

Q	T	$D_{tor} / D_{max}$
2	0.5	0.95
	1.0	1.00
	1.5	0.97
4	0.5	0.93
	1.0	0.81
	1.5	1.00

## CONCLUSIONS

The results in this investigation are similar to those obtained by De la Llera and Chopra, 1994, in which the limit value of the coefficient for the accidental eccentricity is exceeded. The later suggest that the values of  $\delta = \pm 0.1b$  underestimate this effect. Nevertheless, considering that the pdf of the location of the CM is Gaussian, and its variation occurs between  $\pm 0.1b$ , the computed confidence level, for the case of the maximum value of the standard deviation, is 99.73% for the models designed with  $Q=2$ , and 91.77% for those designed with  $Q=4$  which is acceptable, Figs. 4.a and 4.b.

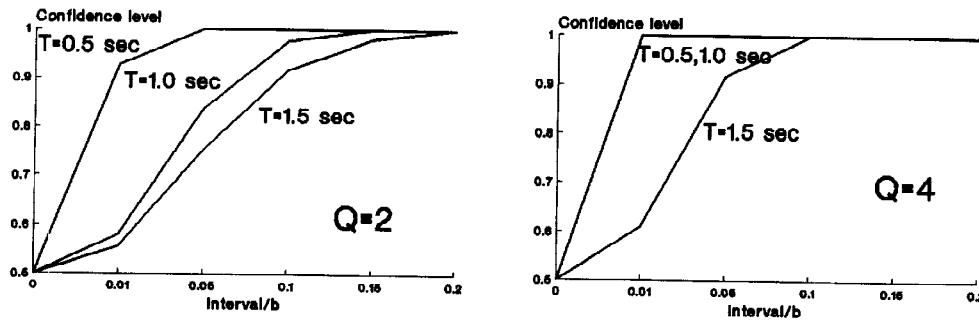


Fig. 4. Confidence levels for  $\delta$  for models of different fundamental period and designed with a)  $Q=2$ , and b)  $Q=4$ .

The accidental eccentricity coefficient is found to be function of the fundamental period of the structures. So, in order to establish the exact limits between which the code value is adequate, it is necessary to have more results, this time considering more complex models and different earthquake motions to simulate the seismic environment of the Valley of Mexico where the RDF93 is enforced.

The computed probability that the accidental eccentricity coefficient has the value proposed by the Mexican Code is relatively small for some structural models. This observation could suggest that the recommended value is not adequate. Nevertheless, if the structural performance of the models is examined, it can be observed that for almost all the cases in which the computed accidental eccentricity value exceeds that recommended by the code (RDF, 1993), the highest ductility demanded by the structure do not reaches its maximum value. So, it is not possible to judge how suitable the suggested value of the accidental eccentricity coefficient is, if the structural performance is not taken into account. Finally, the random stiffness, strength and location of the center of mass, greatly influence the accidental eccentricity coefficient, but fortunately do not affect at the same instant the behavior of the structural elements.

## ACKNOWLEDGEMENTS

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