Paper No. 139. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

STATISTICAL ESTIMATES OF SITE-SPECIFIC RESPONSE SPECTRA FOR DESIGN PURPOSES

B. MYSLIMAJ and Y. MATSUSHIMA

Institute of Engineering Mechanics, University of Tsukuba,
Tsukuba-shi, Ibaraki-ken, 305, JAPAN

ABSTRACT

This paper deals with quantitative evaluation of nonlinear soil deposit response estimates. Its main objective is the formulation of an approximate model for the evaluation of site-specific response estimates in an approximate but acceptable form for practical design purposes. The emphasis is on investigating the effect of the intensity of excitation on the earthquake motion characteristics at the free field. A suite of 30 soil profiles is analyzed using one-dimensional wave propagation model and random vibration theory concept. Results obtained from this analysis are statistically analyzed and numerical expressions for the estimation of equivalent fundamental period and damping coefficient are derived. Using these expressions it is possible to evaluate the free field response estimates as a function of local soil deposit parameters and input motion level. Results presented in this study indicate that this new formulation can be used for estimating input motion intensity dependent site-specific response spectra for design purposes and improving site-dependent building-code provisions.

KEYWORDS

nonlinear soil response; acceleration power spectrum; input motion intensity; site-specific response spectra.

EARTHQUAKE RESPONSE ANALYSIS OF SOIL DEPOSITS SUBJECTED TO RANDOM EXCITATION

In the first part of this study one-dimensional wave propagation model (1DM) and random vibration theory concept (Myslimaj, 1995) are used for the evaluation of nonlinear response estimates for all soil deposits considered, which are conveniently assumed to be horizontally layered. Strain dependence of shear modulus G and hysteretic damping coefficient β for each layer are taken into account by using the hysteretic model developed by Ohsaki et al. (Ohsaki *et al.*, 1978). The modified Markov spectrum (Aoyama and Matsushima, 1993), representing the double-sided shear wave acceleration power spectral-density function $S_A(\omega)$ and given by

$$S_A(\omega) = S_0 \cdot \{\frac{(\omega/\omega_c)^2}{1+(\omega/\omega_c)^2}\}^2 \cdot \frac{1}{1+(\omega/\omega_m)^2} \qquad (1)$$

is assumed as input motion at the top surface of the base layer. S_0 represents the level of power spectral-density, while ω_c and ω_m represent two corner frequencies. The later is related to the so-called ω_{max}, which appears in the observed strong ground motions.

The values of two corner frequencies, $\omega_c = 2\pi f_c$ and $\omega_m = 2\pi f_m$, were estimated by making use of nonlinear least squares technique in fitting the average acceleration power spectrum of widely used strong ground motions recorded on the free field to a model given by (Aoyama, 1994)

$$S_{FFA}(\omega) = S_0 \cdot \{\frac{(\omega/\omega_c)^2}{1+(\omega/\omega_c)^2}\}^2 \cdot \frac{1}{1+(\omega/\omega_m)^2} \cdot \frac{1+4\bar{h}^2(\omega/\bar{\omega})^2}{\{1-(\omega/\bar{\omega})^2\}^2+4\bar{h}^2(\omega/\bar{\omega})^2} \qquad (2)$$

where $\bar{\omega} = 2\pi\bar{f}$ and $\bar{h}$ are the soil deposit transfer function parameters and can be considered as some characteristic ground frequency and characteristic damping ratio, respectively. The values for the best-fit model parameters ω_c, ω_m, $\bar{\omega}$ and $\bar{h}$ were found to be (Aoyama, 1994) $0.886s^{-1}$, $47.04s^{-1}$, $13.125s^{-1}$ and 0.640, respectively.

The squared mean value of acceleration A can be obtained by the following integral of its double-sided power spectral-density function:

$$\sigma_A{}^2 = \int_{-\infty}^{\infty} S_A(\omega)d\omega = 2\pi S_0\omega_m \cdot \{\frac{2\xi^4 - 3\xi^3 + \xi}{4(1-\xi^2)^2}\} \qquad (3)$$

where $\xi = \omega_m / \omega_c$.

From the equation (3) S_0 can be derived as follows :

$$S_0 = \frac{1}{2\pi\omega_m} \cdot \frac{4(1-\xi^2)^2}{2\xi^4 - 3\xi^3 + \xi} \cdot \sigma_A{}^2 \qquad (4)$$

having σ_A as a variable parameter. In this study, 10 levels of σ_A are considered : $\sigma_A = 10,\ 20,\ ...,\ 100cm/s^2$.

SOIL DEPOSIT DATA CONSIDERED

30 soil profiles, selected from the structural design reports of high rise buildings submitted to The Building Center of Japan during 1990 ~ 1993, were chosen as object of this study (Myslimaj, 1995). Soil deposit data selection was carefully done in order to have wide variation in soil categories, i.e., from hard to soft ones, and in their location. Base layer was settled at the layer whose elastic shear wave velocity V_s satisfies the condition $V_S \geq 400m/s$. Each layer was classified to be either clay or sand, based on the material type content. Viscous damping of 2% was assumed for the material damping in soil at initial condition. The elastic first period of vibration T_1 was estimated for all soil deposits, by making use of transfer function analysis. Its dispersion range resulted to be within the interval 0.11s ~ 1.21s . On the basis of T_1 value, soil deposits were classified in three categories, as it is shown in Table 1.

Table 1. Classification of soil deposit data considered.

SOIL CATEGORY	T_1 VALUE	NUMBER OF SOIL DEPOSITS
HARD	$T_1 < 0.3s$	8
MEDIUM	$0.3s \leq T_1 \leq 0.7s$	13
SOFT	$T_1 > 0.7s$	9

THE APPROXIMATE MODEL FOR THE EVALUATION OF RESPONSE OF SOIL DEPOSITS

The formulation of the approximate model (APM) for the evaluation of site-specific response estimates is based on the lumped mass model. On the basis of this model, taking into account only the first two modes of vibration, an approximate estimation for the free field complex transfer function of a given soil deposit can be obtained by making use of expression (5), in which the variation of transfer function parameters due to the change of input motion level σ_A is also considered (Myslimaj, 1995).

$$\overline{H}_{N1}(\omega,\sigma_A) \approx \sum_{j=1}^{2} \Psi_j \cdot \frac{1+2\overline{h}_j(\sigma_A)\cdot[\omega/\overline{\omega}_j(\sigma_A)]\cdot i}{1-[\omega/\overline{\omega}_j(\sigma_A)]^2+2\overline{h}_j(\sigma_A)\cdot[\omega/\overline{\omega}_j(\sigma_A)]\cdot i} \quad ; \quad i=\sqrt{-1} \qquad (5)$$

From the expression (5) it can be seen that an approximate estimation of $\overline{H}_{N1}(\omega,\sigma_A)$ requires first the determination of free field participation function of the first and second mode of vibration Ψ_1, Ψ_2, angular frequencies $\overline{\omega}_1(\sigma_A)$, $\overline{\omega}_2(\sigma_A)$ and total damping coefficients of the first two modes of vibration $\overline{h}_1(\sigma_A)$, $\overline{h}_2(\sigma_A)$, which is shown in the next paragraph.

Frequency, Participation Function and Damping Estimation.

Having known in advance the transfer function for each layer one can easily evaluate the frequency, participation function and the damping ratio of a given soil deposit. In this paragraph, results of analysis obtained by one-dimensional model are used for this purpose.

The fundamental frequency $\overline{\omega}_1$ can be found out from the transfer function analysis, that is, finding the smallest frequency among all frequencies corresponding to the peak values appeared in the transfer function of free field $|H_{N1}(\omega,\sigma_A)|$. On doing this for linear analysis and nonlinear one ($\sigma_A = 10cm/s^2$ ~ $\sigma_A = 100cm/s^2$), the variation of fundamental frequency due to the change of the level of input motion σ_A can be obtained. Results of this analysis for all soil deposit models, normalized by the linear fundamental frequency $\overline{\omega}_{1,0} = 2\pi \overline{f}_{1,0}$, are shown in Fig. 1.

The total damping of first mode of vibration can be approximately evaluated by making use of the following expression:

$$\overline{h}_1 \approx \frac{1}{2}\cdot\frac{1}{\frac{1}{2}|H_{N1}(\overline{\omega}_1)|}\cdot\Psi_1 \qquad (6)$$

where $|H_{N1}(\overline{\omega}_1)|$ is the peak value of the free field transfer function corresponding to the fundamental frequency $\overline{\omega}_1$ and Ψ_1 is the free field participation function of fundamental mode of vibration, given by

$$\Psi_1 = |H_{N1}(\overline{\omega}_1)|\cdot\frac{\sum_{i=1}^{n} m_i\cdot|H_{Ni}(\overline{\omega}_1)|}{\sum_{i=1}^{n} m_i\cdot|H_{Ni}(\overline{\omega}_1)|^2} \qquad (7)$$

where

m_i - mass of the i-th layer;

$|H_{Ni}(\overline{\omega}_1)|$ - ordinate of the i-th layer transfer function at $\omega = \overline{\omega}_1$.

By making use of transfer function analysis results it is also possible to evaluate hysteretic damping of a soil deposit from its fundamental mode of vibration using the following relation:

$$\bar{h}_h \approx \frac{\sum_{i=1}^{n} \beta_i \cdot G_i \cdot \{|H_{Ni}(\bar{\omega}_1)| - |H_{Ni+1}(\bar{\omega}_1)|\}^2}{\sum_{i=1}^{n} G_i \cdot \{|H_{Ni}(\bar{\omega}_1)| - |H_{Ni+1}(\bar{\omega}_1)|\}^2} \tag{8}$$

where

β_i - hysteretic damping coefficient of i-th layer;
G_i - shear modulus of i-th layer;
$|H_{Ni}(\bar{\omega}_1)|$ - ordinate of the i-th layer transfer function at $\omega = \bar{\omega}_1$;
$|H_{Ni+1}(\bar{\omega}_1)|$ - ordinate of the (i+1)-th layer transfer function at $\omega = \bar{\omega}_1$.

Using the above presented expressions for the linear analysis and the nonlinear one ($\sigma_A = 10cm/s^2 \sim \sigma_A = 100cm/s^2$), the variation of $\bar{h}_1$, Ψ_1 and $\bar{h}_h$ due to the change of σ_A can be obtained. Results of this analysis for all soil deposit models are shown in Figs. 2 ~ 4. In case of $\bar{h}_1$ (Fig. 2), results are normalized by linear total damping $\bar{h}_{1,0}$ while in case of Ψ_1 (Fig. 3) and $\bar{h}_h$ (Fig. 4) no normalization has been done.

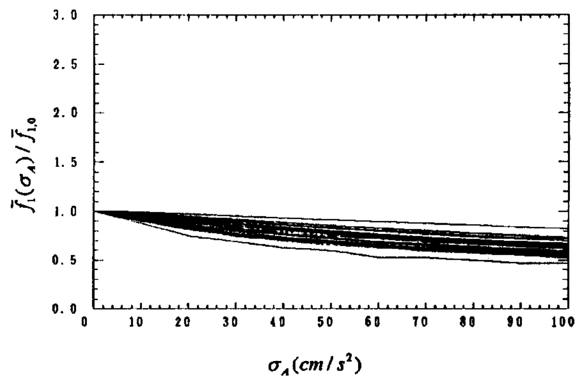


Fig. 1. Fundamental frequency variation.

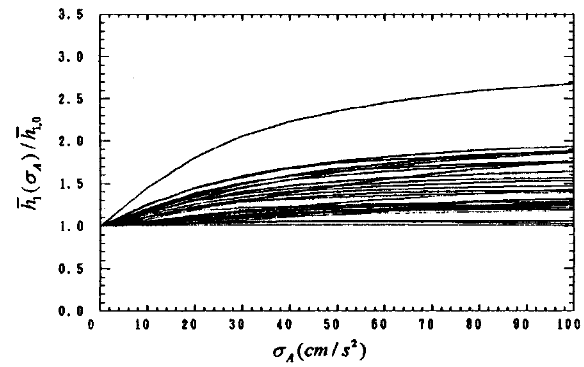


Fig. 2. Fundamental mode total damping variation.

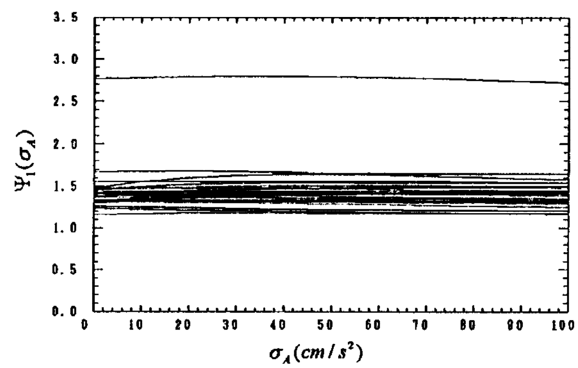


Fig. 3. Participation function variation.

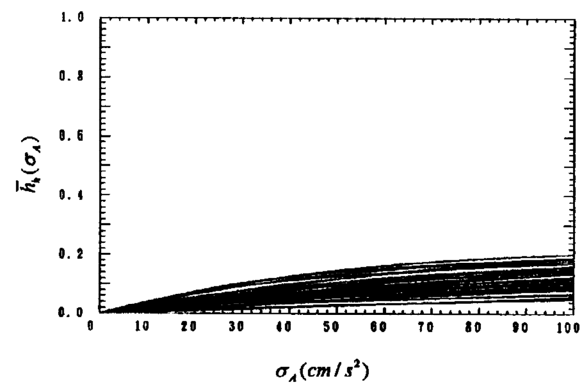


Fig. 4. Hysteretic damping variation.

Above presented results obtained for damping variation indicate a larger dispersion in case of total damping estimation (Fig. 2) compared to that one of hysteretic damping (Fig. 4). The least dispersion is seen in case of fundamental frequency estimation (Fig. 1). On the other hand, results of Ψ_1 estimation for all soil deposits considered (Fig. 3) do indicate that increase of σ_A doesn't affect almost at all the value of Ψ_1. In this case, for a single soil deposit Ψ_1 fluctuates around a value of 2.74, which is large compared to the Ψ_1 values of all other soil deposits considered. Most likely, this is related to the fact that in case of this soil deposit model there is an abrupt change between the rigidities of the upper part layers and the lower part ones. The mean value, standard deviation and coefficient of variation for Ψ_1 resulted to be 1.44, 0.271 and 0.189 respectively, when all soil deposit models were considered in their estimation. On the other hand, in case when from the total number of samples the above mentioned sample was excluded (being thought of as a sample of different type from all 29 others) the following respective values for mean, standard deviation and coefficient of variation were obtained: 1.39, 0.111, and 0.080.

It is important to notice here that the latter mean value of Ψ_1 is in satisfactory agreement with the estimation of Ψ_1 obtained on the basis of a cosine shape assumption for the fundamental mode of vibration of a soil deposit. This estimation gives for Ψ_1 the constant $4/\pi$ or 1.273. The assumption for a cosine shape of the modes of vibration can be further used for the estimation of the frequency of the second mode of vibration, providing that in this case $\overline{\omega}_2/\overline{\omega}_1 = 3.0$. This value for the ratio $\overline{\omega}_2/\overline{\omega}_1$ satisfactorily agrees with the mean value of $\overline{\omega}_2/\overline{\omega}_1$ obtained from the linear transfer function analysis of all soil deposits, which resulted to be 2.92.

From the above given comments it can be concluded as follows:

- Fundamental mode participation function at the free field Ψ_1 can be assumed $4/\pi$;
- σ_A dependency of fundamental frequency and hysteretic damping seem to be as more appropriate for the determination of σ_A dependency of total damping.

On the basis of the above presented numerical results, the following models were formulated for the σ_A dependency of fundamental frequency and hysteretic damping:

$$\frac{\bar{f}_1(\sigma_A)}{\bar{f}_{1,0}} = \frac{1}{1 + a\cdot(\sigma_A/g)^b} \quad (9)$$

$$\bar{h}_h(\sigma_A) = c\cdot(\sigma_A/g)^d \quad (10)$$

where

σ_A - standard deviation of incident acceleration;

g - gravity acceleration;

$\bar{f}_{1,0}$ - linear fundamental frequency, given by $V_e/4H$, where V_e is the average shear wave velocity of the soil deposit and H is its depth;

a,b,c,d - non-dimensional constants determined so that numerical data mostly agree with the proposed models .

The values for the best-fit models' parameters a,b and c,d were estimated by making use of nonlinear least squares technique in fitting respective numerical data to the models given by (9) and (10), respectively. They were found to be: $a = 5.85$, $b = 0.943$, $c = 0.661$ and $d = 0.722$. In the use of nonlinear least squares technique the standard deviation of $\bar{f}_1(\sigma_A)/\bar{f}_{1,0}$ and $\bar{h}_h(\sigma_A)$ were both assumed variable (Ang and Tang, 1975) and two nonlinear functions were respectively supplied in fitting their respective standard deviations of numerical data (Myslimaj, 1995).

In Figs. 5 and 6 are shown the numerical data (circles), best-fit model (solid line) and best-fit model $\pm$standard deviation (dotted line) for the frequency case and hysteretic damping case, respectively.

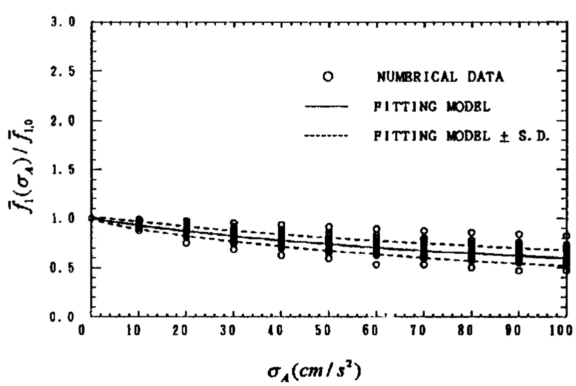


Fig. 5. σ_A dependency of fundamental frequency.

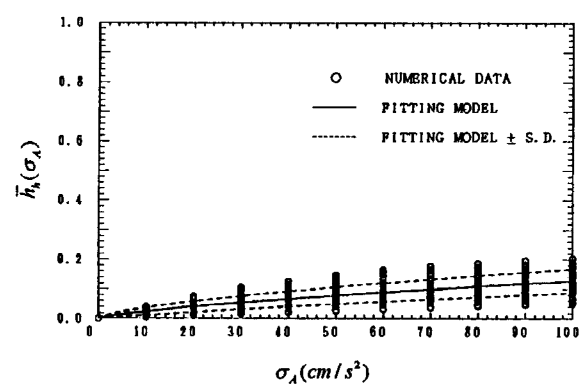


Fig. 6. σ_A dependency of hysteretic damping.

In case of soils, total damping is given by

$$\bar{h} = \bar{h}_h + \bar{h}_r + \bar{h}_v \quad (11)$$

where $\bar{h}_h$, $\bar{h}_r$ and $\bar{h}_v$ are respectively hysteretic damping, radiation damping and viscous damping. Therefore for the first mode of vibration the equation (11) becomes

$$\bar{h}_1(\sigma_A) = c\cdot\left(\frac{\sigma_A}{g}\right)^d + \frac{2}{\pi}\cdot\frac{\rho_e V_e}{\rho_B V_B}\cdot\frac{1}{1+a(\sigma_A/g)^b} + \bar{h}_{v,0}\cdot\left[1+a(\sigma_A/g)^b\right] \quad (12)$$

where

ρ_e - the average mass density;
ρ_B - mass density of the base layer;
V_B - shear wave velocity of the base layer;
$\bar{h}_{v,0}$ - initial viscous damping coefficient, assumed to be 0.02.

For the evaluation of the total damping of the second mode of vibration the following expression can be used:

$$\bar{h}_2(\sigma_A) = c\cdot\left(\frac{\sigma_A}{g}\right)^d + \frac{\bar{\omega}_1}{\bar{\omega}_2}\cdot\frac{2}{\pi}\cdot\frac{\rho_e V_e}{\rho_B V_B}\cdot\frac{1}{1+a(\sigma_A/g)^b} + \frac{\bar{\omega}_2}{\bar{\omega}_1}\cdot\bar{h}_{v,0}\cdot\left[1+a(\sigma_A/g)^b\right] \quad (13)$$

assuming that σ_A dependency of $\bar{\omega}_2$ holds the same as in case of $\bar{\omega}_1(\sigma_A)$ and $\bar{\omega}_2/\bar{\omega}_1 = 3.0$. Moreover, since hysteretic damping $\bar{h}_h$, radiation damping $\bar{h}_r$ and viscous damping $\bar{h}_v$ are respectively proportional to ω^0, ω^{-1} and ω^1, proportionality coefficients 1, $\bar{\omega}_1/\bar{\omega}_2$ and $\bar{\omega}_2/\bar{\omega}_1$ are used as multiplication factor for $\bar{h}_h$, $\bar{h}_r$ and $\bar{h}_v$, respectively first, second and third term on the right hand side of (13).

Evaluation of Free Field Response Estimates.

Since the lumped mass model is used for modeling the soil deposit, the acceleration power spectral-density function at the free field $\bar{S}_1(\omega,\sigma_A)$ will be given by

$$\bar{S}_1(\omega,\sigma_A) = \left|\bar{H}_{N1}(\omega,\sigma_A)\right|^2\cdot 4\cdot S_A(\omega,\sigma_A) \quad (14)$$

where $\bar{H}_{N1}(\omega,\sigma_A)$ is the approximate transfer function, evaluated in accordance with expression (5).

Free field velocity response spectrum $\bar{S}_V(T,h,\sigma_A)$, corresponding to $\bar{S}_1(\omega,\sigma_A)$, is obtained by the following approximate formula (Rosenblueth and Bustamante, 1962) :

$$\bar{S}_V(T,h,\sigma_A) = \sqrt{2\pi t_d \bar{S}_1(\omega,\sigma_A)}\cdot\sqrt{\frac{1-e^{-4\pi h\psi}}{4\pi h\psi}\cdot[0.424+\ln(4\pi h\psi+1.78)])} \quad (15)$$

where :

T - period in the response spectrum $(=2\pi/\omega)$;
t_d - input output time duration, assumed to be 10 seconds;
$\bar{S}_1(\omega,\sigma_A)$ - acceleration power spectral-density function at the free field;
ψ - non dimensional parameter $(=t_d/T)$;
h - damping coefficient in the response spectrum.

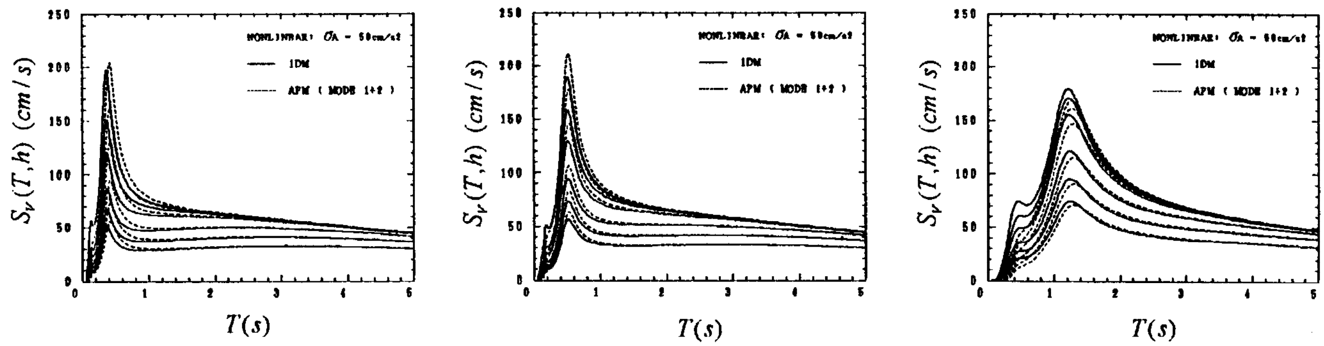
The standard deviation of velocity and acceleration at the free field are evaluated by the following integrals:

$$\bar{\sigma}_{\dot{u}_1}(\sigma_A) = \sqrt{\int_{-\infty}^{\infty}\left|\bar{H}_{N1}(\omega,\sigma_A)\right|^2\cdot\frac{4\cdot S_A(\omega,\sigma_A)}{\omega^2}d\omega} \quad (16)$$

$$\overline{\sigma}_{\ddot{u}_1}(\sigma_A)=\sqrt{\int_{-\infty}^{\infty}\left|\overline{H}_{N1}(\omega,\sigma_A)\right|^2\cdot 4\cdot S_A(\omega,\sigma_A)d\omega} \qquad (17)$$

RESULTS OF ANALYSIS

The level of agreement between the results obtained by APM and 1DM is first investigated through a comparative analysis. In Fig. 7.a,b,c are presented results of velocity response spectrum at the free field $S_V(T,h=0;0.01;0.02;0.05;0.1;0.2)$, for three typical soil deposits. In general, a good agreement between APM and 1DM results can be seen. Especially, in the long period range the agreement is very good.

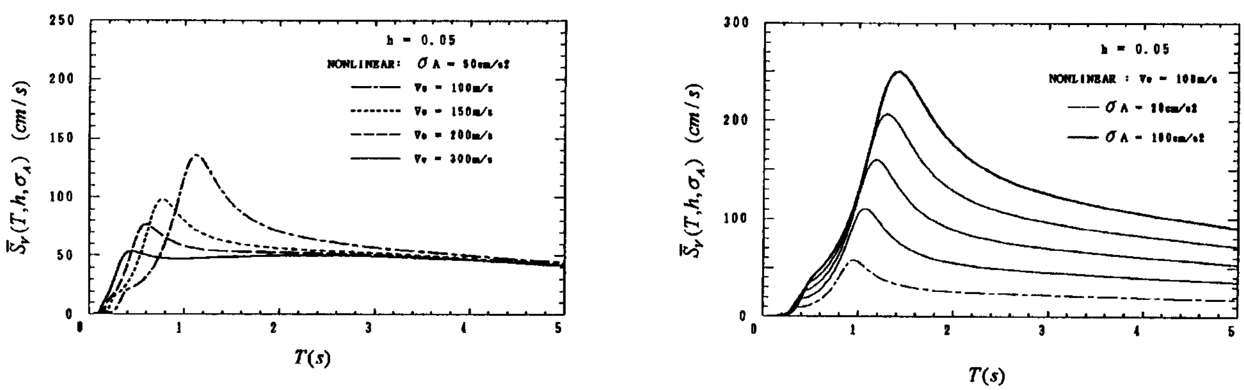


a. Hard soil deposit **b.** Medium soil deposit **c.** Soft soil deposit

Fig. 7. Velocity response spectrum at the free field for three typical soil deposits.

For above mentioned typical soil deposits, results obtained for velocity and acceleration standard deviation at the free field indicate also that APM is in satisfactory agreement with 1DM, especially when at least the first two modes of vibration are taken into consideration.

Results of a sensitivity study focusing on the effect of input motion intensity and "softness" or "hardness" of soil deposits on the velocity response spectrum, velocity standard deviation and acceleration standard deviation at the free field are shown in Figs. 8, 9 and 10 respectively. A soil deposit with $H=20\ m$, $\rho_e=1.75t/m^3$, $\rho_B=2.0t/m^3$, $V_B=400m/s$ and $\overline{h}_{v,0}=0.02$ is considered. Its average shear wave velocity V_e and input motion intensity level σ_A are held as variable parameters.



a. V_e variation **b.** σ_A variation

Fig. 8. Velocity response spectrum at the free field.

In order to investigate the effect of "softness" or "hardness" of soil deposits on the velocity response spectrum, the value of σ_A is kept fixed, while V_e is held as variable parameter. Four levels are assumed for V_e : $100m/s$; $150m/s$; $200m/s$ and $300m/s$. Results obtained from this analysis (Fig. 8.a) indicate that decrease of V_e is associated with the shift of spectral peak values toward long period range. From these results it is also noticed that V_e increase affects remarkably the peak values of velocity response spectrum. In case when the effect of input motion intensity on velocity response spectrum at the free field is investigated, V_e is kept fixed while σ_A is held as variable parameter. Five level are assumed for σ_A : 20, 40, 60, 80 and $100 cm/s^2$. Results of this analysis are shown in Fig. 8.b. Here again, it is noticed that peak location is shifted toward long period range and its amplitude is almost proportionally increased with the increase of σ_A . The later fact agrees also with the results obtained for velocity standard deviation at the free field, which are shown in one of the following figures.

In case of velocity standard deviation at the free field (Fig. 9), it is noticed that increasing softness of soil deposit is associated with higher values of $\overline{\sigma}_{\dot{u}_1}(\sigma_A)$, which is an increasing linear function. Results obtained for acceleration standard deviation (Fig. 10) indicate that increasing softness of soil deposit is associated with the decrease of $\overline{\sigma}_{\ddot{u}_1}(\sigma_A)$, which is also an increasing function but its increasing rate is sluggish.

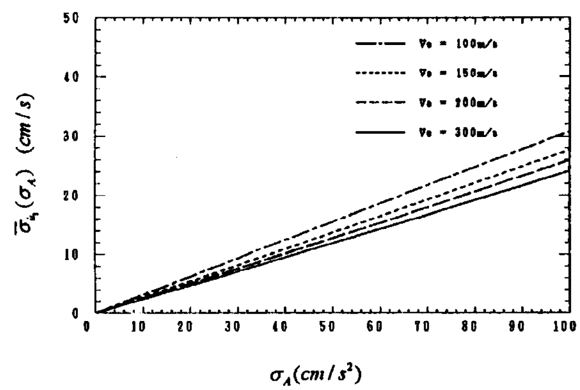


Fig. 9. Velocity standard deviation.

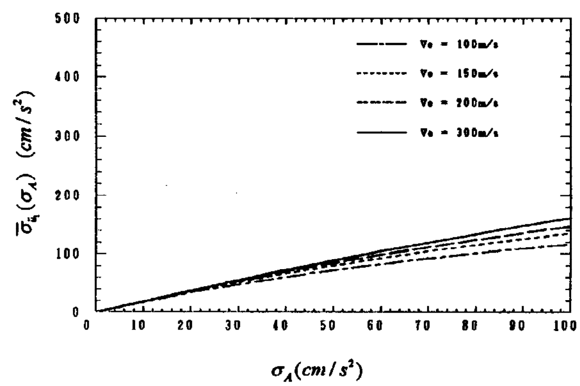


Fig. 10. Acceleration standard deviation.

CONCLUSIONS

The following conclusions can be drawn from the results of analysis presented in this study:

- The proposed new formulation can be used for estimating input motion intensity dependent site-specific response spectra for design purposes and improving site-dependent building-code provisions.
- The values of velocity response spectrum in the long period range increase with the increasing softness of soil deposit.
- Softer the soil deposit higher the values of velocity standard deviation at the free field.
- Harder the soil deposit higher the values of acceleration standard deviation at the free field.

REFERENCES

Ang, A.H-S. and W.H. Tang (1975). *Probability concepts in engineering planning and design*, John Wiley & Sons, New York.

Aoyama, T. (1994). Probability based analysis of seismic safety of structures, Master Thesis, University of Tsukuba, (in Japanese).

Aoyama, T. and Y. Matsushima (1993). A study on the power spectrum of strong ground motions, *Summaries of Technical Papers of Annual Meeting, Architectural Institute of Japan,* **Vol. B,** 75-76, (in Japanese).

Myslimaj, B. (1995). Earthquake motion in various kinds of soil deposits and its approximate estimation for design purposes, Master Thesis, University of Tsukuba.

Ohsaki, Y., A. Hara and Y. Kiyota (1978). Stress-strain model of soils for seismic analysis, *Proceedings of Fifth Japan Earthquake Engineering Symposium*, 697-704.

Rosenblueth, E. and J.I. Bustamante (1962). Distribution of structural response to earthquakes, *Proceedings of the American Society of Civil Engineers, Journal of the Engineering Mechanics Division,* **EM 3,** 75-106.