



## EFFECT OF BEARING CONDITION ON THE DYNAMIC CHARACTERISTICS OF STRESS RIBBON BRIDGE

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### ABSTRACT

This paper describes the dynamic characteristics of stress ribbon bridge obtained from experiments and theoretical analyses. Generally, the stress ribbon bridge has a low natural frequency, because the stress ribbon is thin in comparison with its span length and has a low rigidity. For this reason, it is necessary to make clear the dynamic characteristics. In most stress ribbon bridges constructed recently in Japan, the stress ribbon is joined rigidly to the abutments. However, the supports are to be free from the end rotation of stress ribbon on old type bridges. The dynamic tests of the stress ribbon bridge named Usagibashi have been carried out by human forces. And the eigenvalue analysis by using FEM has been made to make clear the influences of different bearing conditions. It was obtained that the vibration mode of first natural frequency is point symmetric vertical vibration in case that the span to sag ratio is less than 60 and the bearing condition of the stress ribbon has effect on the dynamic characteristics.

### KEYWORDS

Stress ribbon; span to sag ratio; bearing condition; natural frequency; vibration mode; deflection theory; large deformation theory.

### INTRODUCTION

The stress ribbon foot bridges with more than 100m span length have been constructed recently because of the elegant scenery, high economy and ease of the construction (Strasky,1987,Noritake *et al.*,1992). About 20cm in thickness of the stress ribbon is very thinner in comparison with the span length (Arai and Nishiki,1992). So, these flexible bridges excel in elongation rigidity than in flexural rigidity. As the relation between actual load and displacement is not linear in this structural type, it is necessary to use the design theory which can deal with the geometric nonlinearity. The design method based on the deflection theory has been used until now. In case of the application of this design method, the stress ribbon is idealized to the cable structure with or without flexural rigidity. Rotational deformation of the stress ribbon due to the change of air temperature and action of live load varies remarkably at the neighborhood of the abutments. So, formerly, the curved surface bearing had been formed on the abutments in order to make it possible to move the support position of the stress ribbon according to the change of deflection. This bearing type will hereinafter be abbreviated to "pin connection".

The deflection theory shows an outstanding applicability to this structural type. But recently rigidly connected stress ribbon with the abutments has been constructed, because the adoption of rigid connection makes it possible to omit the above mentioned bearing system of the stress ribbon and expansion joints, and moreover, is desirable for the maintenance. In this structural type, the mechanical

characteristics near the connected region of the stress ribbon with the abutments can't be analyzed exactly by the deflection theory. So, we have examined the influence of the difference of bearing condition of the stress ribbon on the static characteristics (Nakazawa *et al.*, 1994). In this examination in which the main analytical factor is the span to sag ratio, we have used the large deformation theory. As a result, it becomes clear that there was no significant difference of the horizontal reaction and of the deflection between pin connection and rigid connection, and that the rigid connection had no mechanical superiority over the pin connection because of the appearance of large bending moment at the connective regions, and that it was necessary to examine the effective span length to apply the deflection theory to the stress ribbon bridge with the rigid connection. But, it has not made clear the influence of bearing condition on the dynamic characteristics of stress ribbon bridges. This paper describes the dynamic characteristics obtained by experiments and theoretical analyses. The eigenvalue analysis by using finite element method has been made to clarify the influences of different bearing conditions.

## ANALYTICAL MODEL

The prestressed concrete stress ribbon bridge named "Usagibashi" was used for analysis and experiment (Shibata *et al.*, 1993). This foot bridge shown in Fig.1 was constructed in August 1992. The location of this bridge is Kitakata Town, Miyazaki Prefecture, Japan. The span length is 115m, and the sag is 3.5m (span to sag ratio is about 33). The stress ribbon is rigidly connected with the abutments. The reverse wing section was adopted for the shape of the cross section of the stress ribbon to prevent the blowing up of the stress ribbon by side wind. Though the standard width of the stress ribbon is 2.0m, the width is widened to 5.0m gradually near both abutments over the 15m interval. Moreover, the stress ribbon in these intervals is thickened from 17cm to 120cm. These treatments were done to improve the stability to wind action.

Geometrical nonlinearity, in which the horizontal tensile force of cable due to the action of dead load was included, was introduced into the stiffness matrix. The horizontal tensile force of cable was

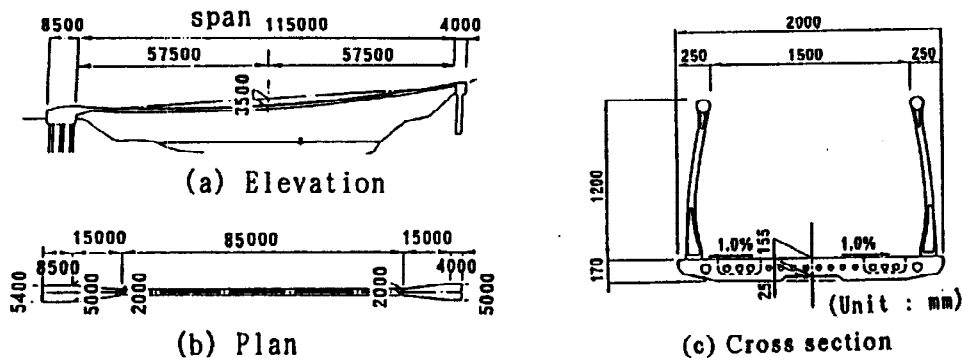


Fig. 1. General view of Usagibashi

Table.1 Coordinates of nodes

node	x(m)	y(m)	node	x(m)	y(m)	node	x(m)	y(m)	node	x(m)	y(m)
1	-28.000	-0.866	14	2.000	-0.040	27	34.500	-1.296	40	67.000	-4.789
2	-27.000	-0.808	15	4.500	-0.057	28	37.000	-1.485	41	69.500	-5.150
3	-25.500	-0.724	16	7.000	-0.088	29	39.500	-1.688	42	72.000	-5.525
4	-23.000	-0.596	17	9.500	-0.131	30	42.000	-1.904	43	74.500	-5.913
5	-20.500	-0.481	18	12.000	-0.188	31	44.500	-2.133	44	77.000	-6.314
6	-18.000	-0.379	19	14.500	-0.259	32	47.000	-2.375	45	79.500	-6.728
7	-15.500	-0.290	20	17.000	-0.342	33	49.500	-2.630	46	82.000	-7.155
8	-13.000	-0.215	21	19.500	-0.439	34	52.000	-2.899	47	84.500	-7.596
9	-10.500	-0.154	22	22.000	-0.548	35	54.500	-3.181	48	86.000	-7.865
10	-8.000	-0.104	23	24.500	-0.671	36	57.000	-3.476	49	87.000	-8.050
11	-5.500	-0.068	24	27.000	-0.808	37	59.500	-3.784			
12	-3.000	-0.045	25	29.500	-0.957	38	62.000	-4.106			
13	-0.500	-0.038	26	32.000	-1.120	39	64.500	-4.441			

obtained from the static analysis by the large deformation theory and the distributed mass was adopted for the mass matrix (Maeda *et al.*, 1974). In case of modeling, the stress ribbon was divided into 48 elements along the span length. Variable cross sectional region with 15m in length was divided into 7 elements, and standard cross sectional region with 85m in length was divided into 34 equal length partitions. The coordinates of these nodes are shown in Table 1. Table 2 shows the cross sectional area, moment of inertia of area and torsional constant of these elements. Modulus of elasticity, Poisson's ratio and unit weight of concrete are assumed as  $3.15 \times 10^5 \text{ kgf/cm}^2$ ,  $1/6$  and  $2.5 \text{ tf/m}^3$ , respectively.

Table 2. Analytical parameters of Usagibashi

Element No.	Cross-sectional area $A(\text{m}^2)$	Moment of inertia of area		Torsional constant $J(\text{m}^4)$
		$I_x(\text{m}^4)$	$I_y(\text{m}^4)$	
1 and 48	4.620	0.3654	9.186	0.3521
2 and 47	2.776	0.1131	5.024	0.1082
3 and 46	1.176	0.01123	1.790	0.01030
4 and 45	0.601	0.001402	0.6579	0.001140
5 and 44	0.520	0.001218	0.4288	0.0009883
6 and 43	0.438	0.001026	0.2596	0.0008363
7 and 42	0.354	0.0008240	0.1418	0.0006842
8 to 41	0.312	0.0007210	0.09465	0.0006082

### DYNAMIC CHARACTERISTICS OF USAGIBASHI

The bridges with two types of bearing condition of stress ribbon, namely, pin connection and rigid connection with the abutments, were analyzed theoretically to investigate the dynamic characteristics. The vibration modes and the natural frequencies obtained from the theoretical analysis are shown in Fig.2 and Table 3, respectively.

According to the considerations based on the analytical results, the following are pointed out as the dynamic characteristics of Usagibashi.

- 1) Many vibration modes have low natural frequency which may raise problems such as serviceability to passengers and stability for wind action.
- 2) In the two cases of pin connection and the rigid connection of stress ribbon with the abutments, the first point symmetric vertical mode has the lowest natural frequency. This result differs from the basic vibration mode of string and beam (axis symmetric). This may be caused by relatively smaller span to sag ratio.
- 3) Pin connection makes the natural frequency lower than the rigid connection. Second vibration mode coupled transverse to torsion appears in case of rigid connection, but third mode in case of pin connection. The difference may be caused by the difference of rigidity of stress ribbon at the end regions.

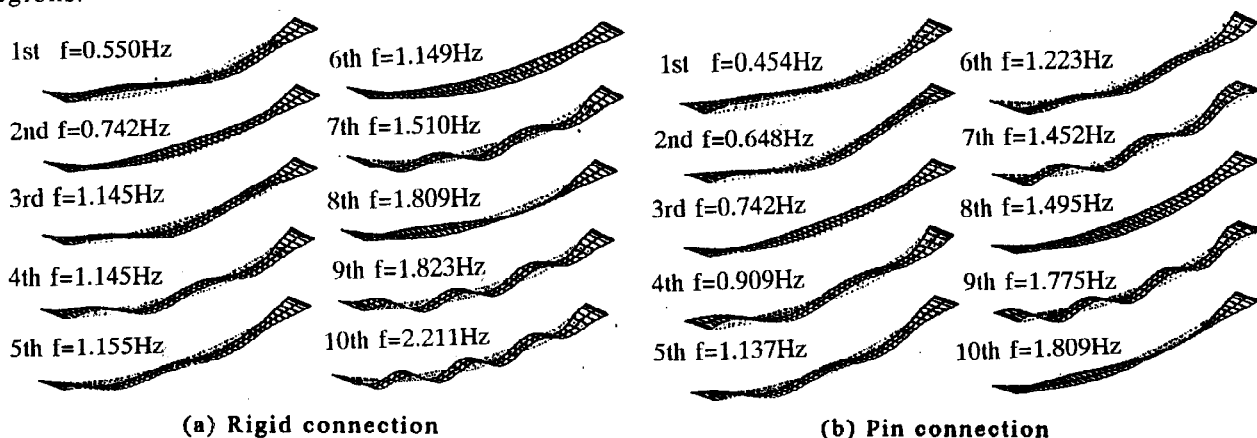


Fig. 2. Analytical vibration modes of Usagibashi

Table 3. Natural frequencies and vibration modes of Usagibashi

Mode in Fig.3	Rigid connection		Pin connection	
	Classification of Vibration mode	f(Hz)	Classification of Vibration mode	f(Hz)
1st	1st point symm. of vertical vib.	0.550	1st point symm. of vertical vib.	0.454
2nd	1st axis symm. of coupled vib.	0.742	1st axis symm. of vertical vib.	0.648
3rd	1st axis symm. of vertical vib.	0.766	1st axis symm. of coupled vib.	0.742
4th	2nd point symm. of vertical vib.	1.145	2nd point symm. of vertical vib.	0.909
5th	2nd axis symm. of vertical vib.	1.155	2nd axis symm. of vertical vib.	1.137
6th	2nd axis symm. of coupled vib.	1.495	3rd axis symm. of vertical vib.	1.223
7th	3rd axis symm. of vertical vib.	1.510	3rd point symm. of vertical vib.	1.452
8th	1st point symm. of coupled vib.	1.809	2nd axis symm. of coupled vib.	1.495
9th	3rd point symm. of vertical vib.	1.823	4th axis symm. of vertical vib.	1.775
10th	4th axis symm. of vertical vib.	2.211	1st point symm. of coupled vib.	1.809

Although it was confirmed that 3-dimensional beam model made it possible to analyze the dynamic characteristics of the stress ribbon bridge, it is pointed out that this model can't treat the variable parameters such as mass, rigidity, sag and the number of cable. The estimation method of the natural frequency which can deal with the above parameters, has been recently proposed by some researchers (Hejima et al., 1989). They developed the equation by solving the eigenproblem of the equation of motion obtained from Lagrange's equation which was composed of strain energy of cable calculated by the deflection theory, strain energy of concrete stress ribbon based on the beam theory, and the kinematic energy obtained by regarding the motion of stress ribbon as a rigid body. They further assumed the vibration mode by considering the bearing condition of the stress ribbon with the abutments as nearly rigid.

Table 4 shows the natural frequencies obtained from this method, experiments and the finite element method by using 3-dimensional beam element. The data used in the analysis is as follows:

- The number and the cross sectional area of cable for construction work : 6 and  $0.001387\text{m}^2$
- The number and the cross sectional area of cable for tension : 6 and  $0.003129\text{m}^2$
- Modulus of elasticity of cable :  $2.0 \times 10^6 \text{kgf/cm}^2$
- Cross sectional area of stress ribbon at standard region :  $0.3118\text{m}^2$
- Span length and sag : 115m and 3.5m,
- Horizontal tensile force of the cable due to the dead load : 451tf
- Weight of the stress ribbon per unit length : 0.78tf/m
- Moment of inertia of area :  $I_z=0.09465\text{m}^4$ ,  $I_y=0.000721\text{m}^4$
- Torsional constant :  $J=0.000608\text{m}^4$

It was confirmed that the natural frequency was obtained approximately from the estimation method. But,

Table 4. Natural frequencies (Hz) by experiment, by FEM and by estimation(Hejima et al., 1989)

Vibration mode	Experiment Analysis Estimation			
Vertical vib.	1st axis symm.	0.732	0.766	0.702
	2nd axis symm.	1.172	1.155	1.288
	1st point symm	0.586	0.550	0.416
	2nd point symm	1.123	1.145	1.145
Coupled vib.	1st axis symm.	0.781	0.742	0.757
	2nd axis symm.	2.344	1.495	1.484

the estimation method cannot consider the difference of bearing condition of stress ribbon with abutments and the influence of the variation of the cross section. Low estimation of rigidity at both ends of the stress ribbon connected with the abutments leads to slightly lower natural frequency in comparison with the results by the experiments and the finite element method.

### PARAMETRIC ANALYSIS

The following four structural models were considered to clarify the influence of the difference of connecting condition and of the variation of cross-sectional shape on the dynamic characteristics.

- Model A: Thickness and width of the stress ribbon vary at the neighbourhood of the abutments. Connecting condition of the stress ribbon with the abutments is rigid.
- Model B: Thickness and width of the stress ribbon vary like Model A. Connecting condition of the stress ribbon is pin.
- Model C: Thickness and width of the stress ribbon is uniform along the span length. Connecting condition of stress ribbon is rigid.
- Model D: Cross section of the stress ribbon is same as Model C. Connecting condition same as Model B.

The span to sag ratio is considered as the parameter in the analysis. The coordinates, the horizontal tensile force and sag are the values after the stress ribbon subjected to the dead load deformed.

In the case where the cross sectional shape of the stress ribbon is uniform along the span length and the connection of the stress ribbon with the abutments is pin, it is possible to apply the deflection theory to the analysis. The horizontal tensile force of cable  $H$  is denoted as  $H = qL^2/8f$  in which  $q$  is the intensity of load,  $L$  is the span length and  $f$  is sag. Substituting the values of the span length, and the intensity of dead load into this equation, the horizontal tensile force of cable varies from 112tf to 1346tf when the span to sag ratio changes from 10 to 120.

We analyzed the dynamic characteristics by using these data and 3-dimensional beam element model. Fig 3 shows the relation between the natural frequencies from first to third vibration modes and the span to sag ratio. It can be seen from this figure that the vibration mode with the lowest natural frequency changes from point symmetric mode to axis symmetric mode according to the increase of the horizontal tensile force of cable due to the increase of the span to sag ratio. In the case where the vibration mode is axis symmetric, the natural frequency takes a maximum value when the span to sag ratio is about 40. When the span to sag ratio is about 60, the natural frequencies of all the vibration modes approximately agree. But in the case where the span to sag ratio is over 60, the difference in the natural frequency between the point symmetric mode and axis symmetric mode becomes larger according to the increase of the span to sag ratio. This is caused by the increase of rigidity as the horizontal tensile force becomes larger.

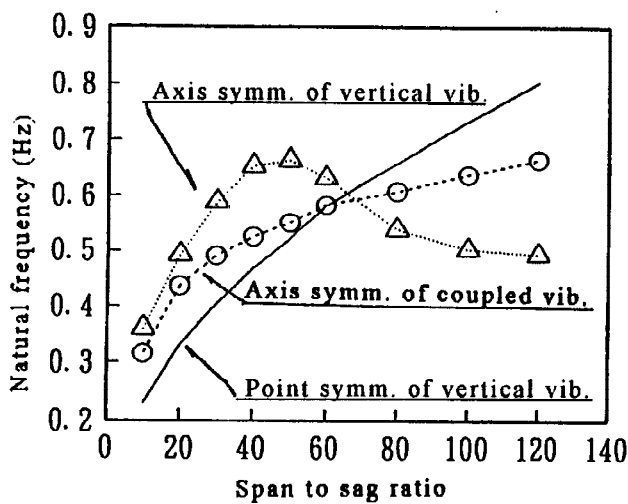


Fig. 3. Relation between natural frequency and span to sag ratio (Model D)

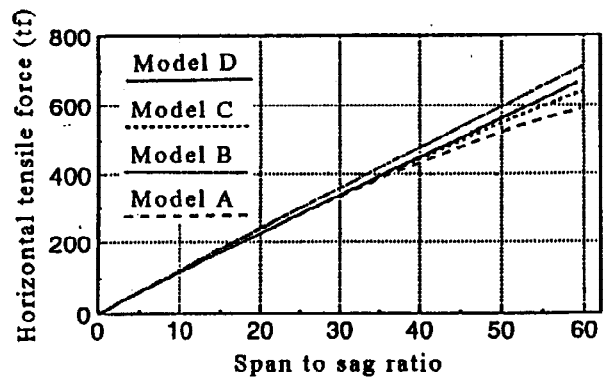


Fig. 4. Effect of bearing conditions and cross sectional shape on horizontal tensile force

The deflection theory gives linear relation between the horizontal tensile force of cable and the span to sag ratio. But, this relation becomes nonlinear when the cross sectional size of the stress ribbon varies along the span length or the stress ribbon is rigidly connected with the abutments. Influences of the bearing condition and the variation of cross sectional size along the span length on the relation between the horizontal tensile force due to the dead load and the span to sag ratio were analyzed by the large deformation theory. The results obtained are shown in Fig.4. In this analysis, the maximum span to sag ratio possible to analyze was limited to about 60 by the deflection due to the dead load. From this figure, it can be seen that nonlinear relation appears when the span to sag ratio becomes larger than 40.

Considering the influence of the bearing condition of the stress ribbon, it becomes clear that the pin connection makes the horizontal tensile force larger than the rigid connection when the span to sag ratio is the same. The horizontal tensile force becomes largest in the case that the cross sectional size varies along the span length and that the bearing condition of the stress ribbon is pin connection (Model B). This may be caused by following, namely, the rigidity in this case is smaller than that in the case where the bearing condition is rigid connection, and the intensity of dead load is larger than that in the case where the cross sectional size is uniform along the span length. Geometric rigidity obtained from the horizontal tensile force was introduced into the stiffness matrix. The relations between the natural frequency and the span to sag ratio obtained in this way are shown in Fig.5.

From these results, it becomes clear that (1) when the span to sag ratio is smaller than 60, the vibration mode having the lowest natural frequency is point symmetric vertical mode in all cases. (2) the natural frequencies of the point symmetric vertical mode and the vibration mode coupled transverse to torsion

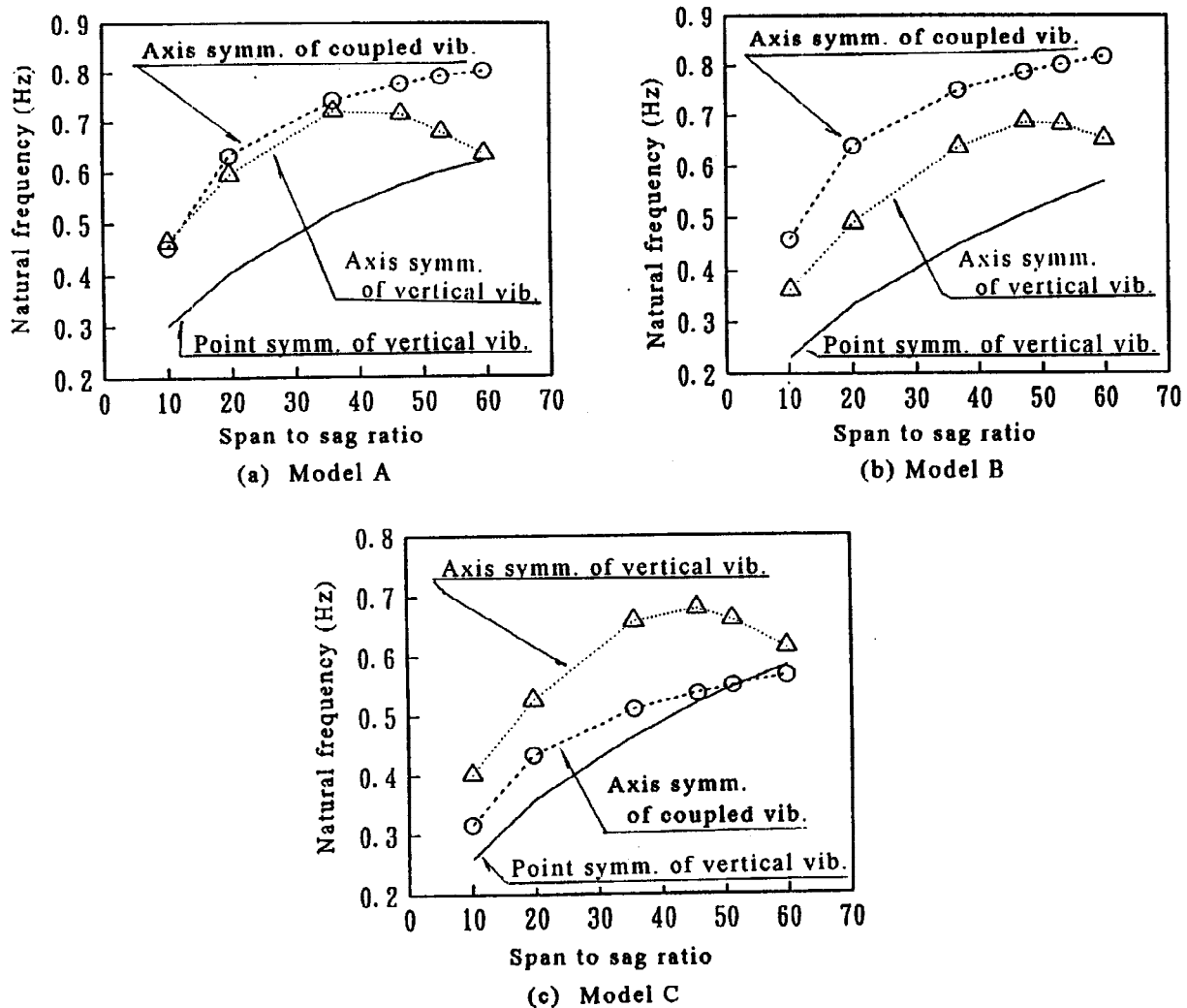


Fig. 5. Effect of bearing conditions and cross section shape on natural frequency

become high with the increase of the span to sag ratio, (3) the natural frequency of the vibration mode coupled transverse to torsion becomes high according to the increase of the rigidity, and the vibration shifts from second mode to third one, (4) there exists a peak value of natural frequency in the axis symmetric vertical vibration mode, and the span to sag ratio when the natural frequency reaches a peak value, decreases according to the increase of the rigidity. But, if the span to sag ratio exists in the range up to 20, second axis symmetric vertical vibration mode appears first. When the span to sag ratio is over 30, first axis symmetric vertical vibration mode appears. Considering that the difference of altitude of both abutments is 7.2m in Usagibashi and the steepest slope is about 20%, we further examined the effect of the difference of altitude of the abutments on the dynamic characteristics. But it became clear that there was no effect of the difference of altitude of the abutments.

## CONCLUSIONS

In this paper, we analyzed the effect of the bearing condition of the stress ribbon and the change of the span to sag ratio on the dynamic characteristics of the stress ribbon bridge, and compared the analytical results with the experimental ones obtained from the vibration test of Usagibashi. The principal findings are summarized as follows:

- (1) Bearing condition of the stress ribbon has no effect on the variation of the natural frequency of the vibration mode coupled transverse and torsion whether the cross sectional shape of the stress ribbon is uniform along the span length or not. But, in the case of axis and point symmetric vertical vibration mode, the rigid connection of the stress ribbon with the abutments make the natural frequency higher.
- (2) If the bearing condition of the stress ribbon is the same, the highest natural frequency in 1st vibration of each mode appears in the coupled vibration of transverse to torsion in case of variable cross sectional shape and in the axis symmetric vertical vibration in case of uniform cross sectional shape along the span length.
- (3) The natural frequencies of the point symmetric vertical vibration mode and of the vibration mode coupled transverse to torsion increase according to the increase of the span to sag ratio. This may be caused by the high rigidity due to the increase of the horizontal tensile force of cable.
- (4) The span to sag ratio when the natural frequency has a maximum value is about 40 in case of the axis symmetric vertical vibration mode. And, the natural frequency of the point symmetric vertical vibration mode agrees with that of the axis symmetric vertical vibration mode when the value of the span to sag ratio is about 60. In the range where the span to sag ratio is over 60, the axis symmetric vertical vibration mode has the lowest natural frequency. This may be due to the change of axis symmetric vibration mode from vertical vibration to longitudinal one.
- (5) Relatively small span to sag ratio like Usagibashi produces the point symmetric vertical vibration mode with the lowest natural frequency.

## REFERENCES

- Strasky, J. (1987). Precast stress ribbon pedestrian bridge in Czechoslovakia. PCI Journal, pp.52-73.
- Noritake, K., H. Arai, M. Takahashi, H. Hosono and H. Nishiki (1992). Design and construction of the Ishizuchi Bridge. Proceedings of the 5th Symposium on Developments in Prestressed Concrete, Japan Prestressed Concrete Engineering Association, pp.217-222. ( in Japanese )
- Arai, H. and H. Nishiki (1992). Stress ribbon bridge. Concrete Journal, Japan Concrete Institute, Vol.30, No.3, pp.42-52. ( in Japanese )
- Nakazawa, T., F. Imai, M. Akagi and F. Maeda (1994). Influence of support condition on the mechanical characteristics of stress ribbon bridge. Proceedings of the Japan Concrete Institute, Vol.16, No.2, pp.997-1002. ( in Japanese )
- Shibata, H., J. Yamaguchi, M. Makino and F. Maeda (1993). Design and construction of stress ribbon bridge "Usagibashi". Journal of Prestressed Concrete, Japan Prestressed Concrete Engineering Association, Vol.35, No.1, pp.14-21. ( in Japanese )
- Maeda, Y., M. Hayashi and M. Nakamura (1974). An acceleration approach for large deformation structural analysis by incremental method. Proc. of JSCE, No.223, pp.1-9. ( in Japanese )
- Hiejima, S., K. Haraguchi and Y. Fujino (1989). Approximate solution of inplane and out-of-plane natural frequency of stress ribbon bridge. Proc. of the 46th Annual Conf. of JSCE, I, pp.904-905. ( in Japanese )