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MULTI-STAGE TUNED MASS DAMPER

GENDA CHEN

Steinman Consulting Engineers, Parsons Transportation Group
110 William Street
New York, NY 10038, U.S.A.

ABSTRACT

A multi-stage tuned-mass-damper system is presented in this paper, which can solve several problems inherited in the conventional tuned-mass-dampers. It consists of many oscillators that are attached at different floors of a building structure. All oscillators are tuned to the fundamental frequency of the structure, which completely decouples the vibration modes of the structure. The proposed system can thus suppress the seismic responses of the higher modes as well as the fundamental mode of the structure. The system is also robust in seismic performance and so light that it will generate insignificant overstroke impact on the structure in an unexpected earthquake event.

KEYWORDS

Tuned-mass-damper; power flow; power input; effective damping; seismic performance.

INTRODUCTION

A tuned mass damper (TMD) is an auxiliary system, attached to a main structure such as building, that can help mitigate the structural dynamic responses by tuning its frequency to the predominant frequency (fundamental frequency) of the structural system. It has been applied on about ten high-rise buildings in the world to reduce acceleration level primarily for tenants comfort reason during strong windy seasons. Its performance has been verified with extensive analyses, wind tunnel experiments and full-scale tests. The TMD was also successfully added in some buildings such as theaters to suppress floor vibrations from human movements. However, the TMD has not yet found its application for earthquake loads since its seismic performance records are not consistent (Chowdhury et al., 1987). The primary reasons for the discrepancies in the results are attributable to: a) in-phase motion between the TMD and the structure; b) non-optimal design parameters of the TMD for various seismic inputs; c) energy transmission among different vibration modes of the structure.

The in-phase motion is inherited in any TMD since the inertia force on the TMD is always in phase with the effective load on main structure and certain time is needed to set the TMD in motion. This action becomes even more significant when the structure and the TMD are subjected to an impulsive type of earthquake loads with short ascending period before reaching its maximum shaking intensity. The heavier the TMD, the stronger the in-phase action.

The optimal design parameters of the TMD are dependent upon combination of the type of external loads and the interested response quantities (Warturbon, 1982). Due to the uncertain nature of earthquake, the "optimal" design parameters can only be selected approximately based on some postulated earthquakes. This approximation defeats the TMD performance to a certain degree.

A conventional TMD installed atop a building conveys energy flow between the damper and all natural modes of the building structure due to coupling. It absorbs vibration energy from the tuned fundamental mode of the structure, but it transfers energy into higher modes. This energy transmission amplifies the higher mode responses and counteracts the performance of the TMD for the mitigation of total structural responses.

The objective of this paper is to present a multi-stage tuned mass damper (MSTMD) to solve the above issues. The proposed MSTMD involves many identically-tuned mass dampers installed at different floor of a building.

MECHANISM OF TUNED MASS DAMPER

Consider a single degree-of-freedom structural system (SDOF) subjected to a base acceleration. The structural responses can be reduced by adding an oscillatory mass (TMD) connected in series with the structure.

The equations of motion for the TMD and the structural system can be written as:

$$m_s\ddot{x}_s + (c_s + c_d)\dot{x}_s + (k_s + k_d)x_s - c_d\dot{x}_d - k_d x_d = -m_s\ddot{x}_g(t) \tag{1}$$

$$m_d\ddot{x}_d + c_d\dot{x}_d + k_d x_d - c_d\dot{x}_s - k_d x_s = -m_d\ddot{x}_g(t) \tag{2}$$

where x_s and x_d are displacements of the SDOF structure and the damper with respect to the ground. Quantities m_i , c_i and k_i (i=s or d) are masses, damping coefficients and stiffness. Summation of Eqs. (1) and (2) leads to

$$(m_s + m_d)\ddot{x}_s + c_s\dot{x}_s + k_s x_s = -(m_s + m_d)\ddot{x}_g(t) - m_d\ddot{y} \tag{3}$$

in which $y = x_d - x_s$, denoting the relative displacement of the damper with respect to the SDOF structure. Equation (3) indicates the addition of a 'force term' $-m_d\ddot{y}$ for an SDOF system. When $\ddot{x}_g(t)$ is considered as a harmonic excitation or a stationary random input, Eq. (3) can be rewritten into the following power balance equation (Chen and Soong, 1991):

$$c_s < \dot{x}_s^2 > = -(m_s + m_d) < \ddot{x}_g\dot{x}_s > - m_d < \ddot{y}\dot{x}_s > \tag{4}$$

in which <•> is the time average in one cycle for the harmonic excitation and the mathematical expectation for the stochastic input. The quantity $c_s < \dot{x}_s^2 >$ is the dissipated power due to the structural damping; $-(m_s + m_d) < \ddot{x}_g\dot{x}_s >$ is the input power from the ground motion; $m_d < \ddot{y}\dot{x}_s >$ is the power flow transmitting from the structural system to the damper. The power flow is an appreciable scale to measure the damper performance. The larger the power flow, the smaller the mean-square velocity of the structure. The maximum power flow is obtained when the relative displacement of the damper with respect to the structure lags the structural displacement by 90°. In this case, the relative acceleration $\ddot{y}(t)$ is in phase with the velocity response of the structure and the power flow is equivalent to a dissipated power. The effective damping coefficient can therefore be written as:

$$c_{eff} = c_s + m_d \frac{< \ddot{y}\dot{x}_s >}{< \dot{x}_s^2 >} \tag{5}$$

The optimal design parameters for the damper can be determined by maximizing the effective damping coefficient in Eq. (5). They can be expressed as (Warturbon, 1982):

$$\alpha_{opt} = 1/(1+\mu) \ , \qquad \xi_{opt} = \sqrt{3\mu/[8(1+\mu)]} \tag{6}$$

where $\mu = m_d / m_s$; α_{opt} and ξ_{opt} are the optimal damper-to-structure frequency ratio and the optimal damping ratio.

MULTI-STAGE TUNED MASS DAMPER

Equations of Motion

Consider an n-story building with n-oscillator respectively installed at each floor of the building. The oscillators are of identical damping and frequency properties. They are all tuned to the fundamental frequency of the building structure and therefore are named as the multi-stage tuned mass damper (MSTMD).

The equations of motion of the structure-damper system in Fig. 1 can be expressed as

$$\begin{bmatrix} M_s & O \\ O & M_d \end{bmatrix}\begin{Bmatrix} \ddot{X}_s \\ \ddot{X}_d \end{Bmatrix} + \begin{bmatrix} C_{ss} & C_{sd} \\ C_{ds} & C_{dd} \end{bmatrix}\begin{Bmatrix} \dot{X}_s \\ \dot{X}_d \end{Bmatrix} + \begin{bmatrix} K_{ss} & K_{sd} \\ K_{ds} & K_{dd} \end{bmatrix}\begin{Bmatrix} X_s \\ X_d \end{Bmatrix} = -\begin{bmatrix} M_s & O \\ O & M_d \end{bmatrix}\begin{Bmatrix} E_s \\ E_d \end{Bmatrix}\ddot{x}_g(t) \tag{7}$$

in which s and d are associated with the degrees-of-freedom of the structure and dampers; $X_s = \{x_{1s}, x_{2s}, \ldots, x_{ns}\}^T$ and $X_d = \{x_{1d}, x_{2d}, \ldots, x_{nd}\}^T$, representing displacements with respect to the ground; M_s and M_d are mass matrices; K_{ik} and C_{ik} (i, k=s, d) respectively represent stiffness and damping matrices between system i and system k; and E_s and E_d are respectively the load vectors that take the unit elements at all entries for the structure-damper system under consideration. The damper masses are considered to be proportional to the building floor masses, i.e., $M_d = \mu M_s$. The stiffness matrix K_{ss} can be decomposed into $K_{ss}^{(0)} + K_{dd}$ in which $K_{ss}^{(0)}$ is the structural stiffness without presence of the dampers. The cross stiffness matrices, K_{sd} and K_{ds} are diagonal matrices and they can be expressed by $-K_{dd}$. The damping matrices C_{ss}, C_{dd} and C_{sd} have exactly the some structure as stiffness K_{ss}, K_{dd} and K_{sd}, respectively.

The dampers stiffness and damping matrices, K_{dd} and C_{dd}, are diagonal matrices and they can be respectively expressed by $\omega_d^2 M_d$ and $2\xi_d\omega_d M_d$ with ω_d and ξ_d representing the damper frequency and damping ratio. By denoting the mode shape matrix of the structural system as Φ_s that is normalized with structure mass matrix M_s, the displacements of the structure and dampers can be represented into:

$$\begin{Bmatrix} X_s \\ X_d \end{Bmatrix} = \begin{bmatrix} \Phi_s & O \\ \Phi_s & \frac{1}{m}\Phi_s \end{bmatrix}\begin{Bmatrix} q_s \\ q_d \end{Bmatrix} = \Phi q \tag{8}$$

in which $m = \sqrt{\mu/(1+\mu)}$. Substituting Eq. (8) into Eq. (7) and pre-multiplying both sides of the resulting equation by Φ^T leads to the following equations in the modal space after employing orthogonal conditions:

$$\ddot{q}_{ks} + \frac{2\xi_{ks}\omega_{ks}}{1+\mu}\dot{q}_{ks} + \frac{\omega_{ks}^2}{1+\mu}q_{ks} + m\ddot{q}_{kd} = -\Gamma_{ks}\ddot{x}_g(t) \tag{9}$$

$$\ddot{q}_{kd} + 2\xi_d\omega_d\dot{q}_{kd} + \omega_d^2 q_{kd} + m\ddot{q}_{ks} = -m\Gamma_{ks}\ddot{x}_g(t) \tag{10}$$

In the above, q_{ks} and q_{kd} are the k-th modal displacements of the structure and the dampers. The quantities ξ_{ks}, ω_{ks} and Γ_{ks} are damping ratio, frequency and mass participation factor of the k-th mode of the structure.

Transfer Functions for Structural Responses

When $\ddot{x}_g(t) = \ddot{X}_g(\omega)e^{j\omega t}$, the steady-state modal displacement of the structure in Eqs. (9) and (10) can be expressed as $q_{ks}(t) = -\Gamma_{ks}H^*_{ks}(\omega)\ddot{X}_g(\omega)e^{j\omega t}$ in which

$$H^*_{ks}(\omega) = \frac{1/H_d(\omega) + m^2\omega^2}{1/[H_{ks}(\omega)H_d(\omega)] - m^2\omega^4} \tag{11}$$

$$H_d(\omega) = 1/(\omega_d^2 - \omega^2 + j2\xi_d\omega_d\omega)\ ,\quad H_{ks}(\omega) = 1/[\omega_{ks}^2/(1+\mu) - \omega^2 + j2\xi_{ks}\omega_{ks}/(1+\mu)\omega] \tag{12}$$

The total displacements of structure can then be expressed by $x_{is}(t) = X^*_{is}(\omega)\ddot{X}_g(\omega)e^{j\omega t}$ where

$$X^*_{is}(\omega) = -\sum_{k=1}^{n}\Gamma_{ks}H^*_{ks}(\omega)\phi_{ks}(i) \qquad (i=1, 2, \ldots, n) \tag{13}$$

In order to show improvement of the proposed MSTMD, the conventional TMD is considered to be installed at the second floor from the building roof. Since the equations of motion in frequency domain are linear algebraic equations, the dynamic displacement at the second floor from the roof can be derived without any difficulty as $x_{(n-1)s}(t) = X^{**}_{(n-1)s}(\omega)\ddot{X}_g(\omega)e^{j\omega t}$ in which

$$X^{**}_{(n-1)s}(\omega) = -\sum_{k=1}^{n}\Gamma_{ks}H^{**}_{ks}(\omega)\phi_{ks}(i) \tag{14}$$

and

$$H^{**}_{ks}(\omega) = \frac{1 + \phi_{ks}(n-1)m_d[1+\omega^2H_d(\omega)]/\Gamma_{ks}}{1 - m_d\omega^2[1+\omega^2H_d(\omega)]\sum_{r=1}^{n}\phi^2_{rs}(n-1)H^{(0)}_{rs}(\omega)}H^{(0)}_{ks}(\omega) \tag{15}$$

$$H^{(0)}_{ks}(\omega) = 1/(\omega_{ks}^2 - \omega^2 + j2\xi_{ks}\omega_{ks}\omega) \tag{16}$$

The dynamic displacements of structure without presence of damper can be simply represented by $x_{is}(t) = X^{(0)}_{is}(\omega)\ddot{X}_g(\omega)e^{j\omega t}$ where

$$X^{(0)}_{is}(\omega) = -\sum_{k=1}^{n}\Gamma_{ks}H^{(0)}_{ks}(\omega)\phi_{ks}(i) \qquad (i=1, 2, \ldots, n) \tag{17}$$

Mean-Square Responses of Structure Under White Noise Input

When the building structure is subjected to a White Noise input with power spectrum $S_{\ddot{x}_g}(\omega) = S_0$ (constant), the mean-square absolute accelerations at building floors can be respectively expressed as

$$<(\ddot{x}_{is} + \ddot{x}_g)^2> = S_0\int_{-\infty}^{+\infty}\left|1 - \omega^2X^*_{is}(\omega)\right|^2 d\omega \tag{18}$$

for the MSTMD application.

$$<(\ddot{x}_{is} + \ddot{x}_g)^2> = S_0\int_{-\infty}^{+\infty}\left|1 - \omega^2X^{(0)}_{is}(\omega)\right|^2 d\omega \tag{19}$$

for the structure without presence of dampers.

Equations (9) and (10) can be rewritten in the form of power balance (Chen and Soong, 1991)

$$\frac{2\xi_{ks}\omega_{ks}}{1+\mu} < \dot{q}^2_{ks} > + m < \ddot{q}_{kd}\dot{q}_{ks} > = -\Gamma_{ks} < \ddot{x}_g(t)\dot{q}_{ks} > \tag{20}$$

$$2\xi_d\omega_d < \dot{q}^2_{kd} > + m < \ddot{q}_{ks}\dot{q}_{kd} > = -m\Gamma_{ks} < \ddot{x}_g(t)\dot{q}_{kd} > \tag{21}$$

The right-hand terms in Eqs. (20) and (21) are respectively defined as power input to the structure and to the damper from ground motion. They are denoted here as $P_s^{(k)}$ and $P_d^{(k)}$. The $m < \ddot{q}_{kd}\dot{q}_{ks} >$ is the power flow transmitting from the structure to the damper, denoted as $P_{sd}^{(k)}$ and $m < \ddot{q}_{ks}\dot{q}_{kd} >$ can be expressed by $-P_{sd}^{(k)}$ due to the stationarity of the random input.

Equations (20) and (21) indicate complete uncoupling among vibration modes of the structure due to presence of dampers at each floor. The power input to the damper $P_d^{(k)}$ is zero for every mode as shown in the report by Chen and Soong (1993). The power thus flows unidirectional from the structure to the dampers for all vibration modes as shown in Eq. (21), which is equal to the dissipated power in the dampers. Consequently, the MSTMD suppresses seismic responses of its higher modes as well as fundamental mode.

ILLUSTRATIVE EXAMPLE

A six-story building as shown in Fig. 1 is considered as an example to illustrate the difference between the MSTMD and the conventional TMD and to demonstrate the seismic performance of the proposed MSTMD system. The floor mass throughout the building is equal to 1.0 kip sec^2/ft while the interval stiffness is assumed to be 5000 kip/ft. The natural frequencies of the building structure are 17.0, 50.1, 80.4 105.9, 125.2, and 137.4 rad/sec, respectively.

The modal transfer functions defined in Eqs. (11), (15) and (16) are presented in Fig. 2 as a function of the damper frequency. The damper-to-floor mass ratios are respectively taken as 0.065 and 0.24 for the MSTMD (six dampers) and for the TMD (one damper at the 5th floor). It is observed that the first mode transfer functions of the structure with the MSTMD or with the TMD can not be distinguished though the damper-to-floor mass ratio of the MSTMD is only about one-quarter of the one for the TMD. They are both reducing the modal transfer function of the structure alone (no damper). The MSTMD has insignificant influence on the second and third modal transfer functions and further reduces their magnitudes around the fundamental frequency because of the complete uncoupling among vibration modes of the structure.

The six-story building is subjected to a white noise random ground motion. This case is important not only because of ease for the mathematical manipulation but also because of the practical implication. The wide band ground input is one of the worst scenario for the MSTMD to be effective for seismic response reduction. The modal power flows transmitting from the structure to the damper are shown in Fig. 3 as a function of the frequency ratio between the damper and the fundamental mode of the structure and in Fig. 4 as a function of the damper-to-floor mass ratio with the damper tuned to the fundamental frequency. It is seen that the power flows are always positive, indicating the unidirectional transmission of energy from the structure to the damper or the response reduction for all modes of the structure. Figure 3 also implies that the maximum power flow is reached when the damper frequency is tuned to the structural frequency. Figure 4 indicates the increasing power flow as the mass ratio increases. But, the rate of the increasing power flow becomes small when the mass ratio is greater than 0.075, implying the practical range for the selection of the mass ratio.

Figure 5 shows the reduction of the mean-square modal accelerations as the frequency ratio changes. It confirms the conclusion drawn from Fig. 3. It is noted that the mean-square modal accelerations are reduced to about 35% by adding the MSTMD. The mean-square floor accelerations are presented in Fig. 6 as the damper frequency varies. About 50% reduction can be observed for the mean-square accelerations.

PRACTICAL CONSIDERATIONS

The proposed MSTMD system is more robust than the conventional TMD. When one of the dampers is out of order during strong earthquake, the remaining dampers will take over and can still effectively mitigate dynamic responses of the structure. This redundancy of damping effect leads to a stable dampers performance even if the dampers parameters slightly deviate from their optimal values due to reasons such as manufacture tolerance.

The masses of the dampers in the MSTMD system are substantially lighter than one big mass in the conventional TMD. The in-phase motion between the dampers and the structure is significantly smaller in magnitude and shorter in period before the dampers are set in motion out of phase with the structure movement. This effect becomes more significant when the dampers are not perfectly synchronized in their initial movement during earthquake. Under a severe earthquake event, overstroke impact will not generate large inertia forces on the building structure, which may otherwise cause severe damage to the structure.

The number of dampers in the MSTMD system can be practically smaller than the number of building story. It is prudent to limit the number of dampers in such a way that every activated mode of the structure by an earthquake with wide frequency range is added with one damper. This topic is beyond the scope of this paper and will be discussed in a subsequent paper.
The total cost of the MSTMD system can be roughly grouped into cost for engineering design, material, manufacture, maintenance and occupied space. It is expected comparable with that of the conventional TMD base on the following considerations:

1. The engineering design is less complicated due to less stringent design requirement for dampers.
2. Demand for more dampers can be compensated for with the material saving from a lighter damper.
3. Less amount of labors is needed for making one damper due to less sophisticated manufacture process.
4. Same design and working environment for the dampers only warrants need for inspecting one or two dampers and then replacing parts without discrimination.
5. The MSTMD system can make full use of some spare spaces at different floors of the building instead of a big area for a conventional TMD and its accessories.

CONCLUDING REMARKS

The presented MSTMD system consists of many oscillators that are all tuned to the fundamental frequency of the building structure. The multiple tuning completely decouples all modes of the building structure and conveys no power transmission among the modes. The MSTMD consequently suppresses seismic responses of the higher modes as well as the fundamental mode of the structure. The seismic performance can be measured with the power flow transmitting from the structure to the damper or equivalent damping coefficient defined in Eq. (5).

Analyses on the six-story building showed about 50% reduction of the mean-square absolute accelerations by adding a MSTMD system when the building is subjected to a white noise base input.

In addition to the performance improvement, the presented system is robust. It will not generate great inertia forces on the building structure in the event of an unexpected earthquake. The light dampers are also easier to set in motion out of phase with the structure.

Although the MSTMD seems a promising system for earthquake application, many technical considerations need to be addressed before practical applications. Future research is necessary in the following areas: 1).shake table tests for confirmation of seismic performance; 2).development of design criteria for the determination of the number of dampers and their distribution in building; 3).cost evaluation.

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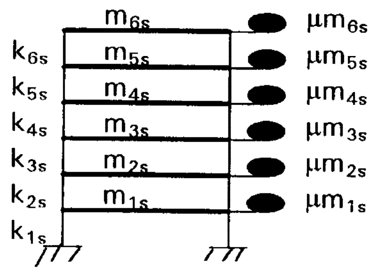

Fig.1 Example building structure:

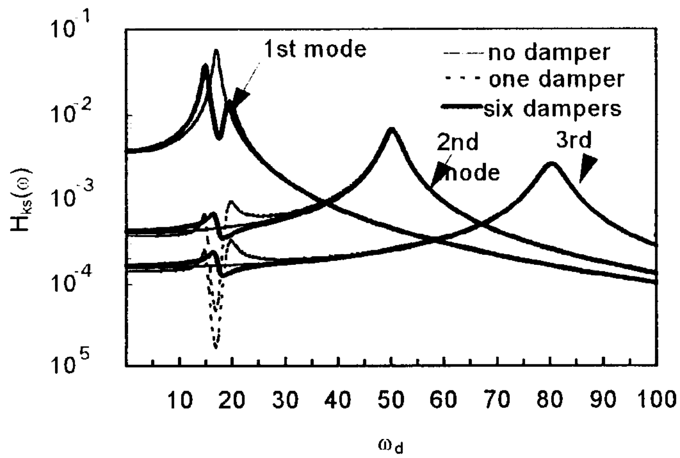

Fig.2 Comparison of modal transfer functions
$\xi_s = 0.03, \xi_d = 0.05$

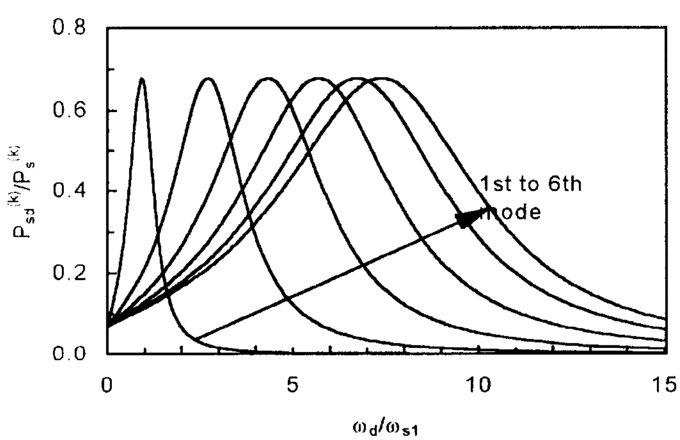

Fig.3 Modal power flow transmitting from structure to damper vs. damper-to-structure frequency ratio
$\xi_s = 0.03, \xi_d = 0.15, \mu = 0.065$

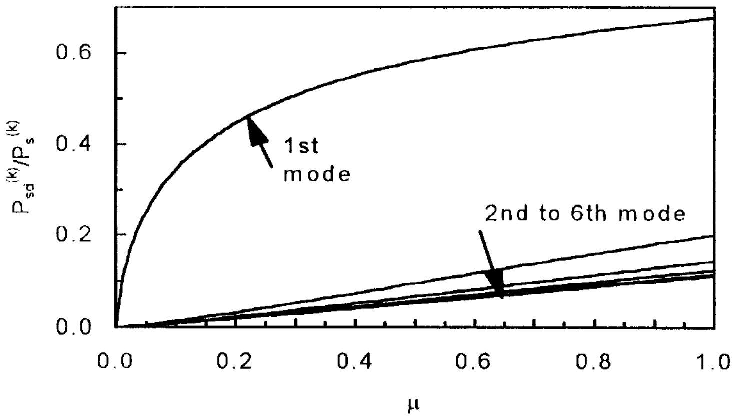

Fig.4 Modal power flow transmitting from structure to damper vs. mass ratio: $\xi_s = 0.03$, $\xi_d = \sqrt{3\mu/8(1+\mu)}$, $\omega_d / \omega_{s1} = 1/(1+\mu)$

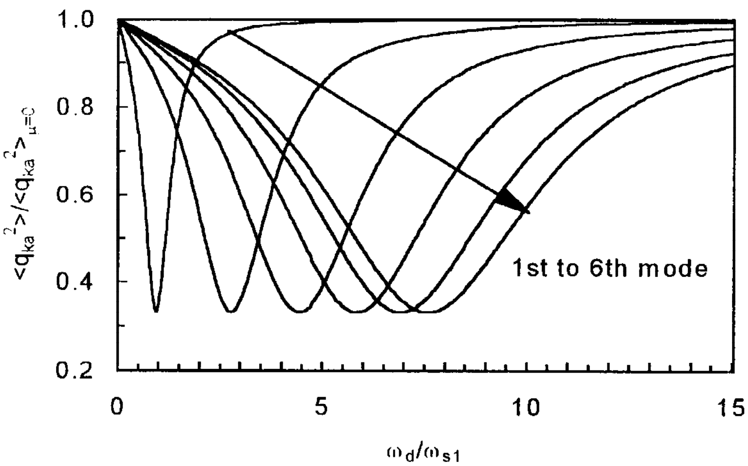

Fig.5 Mean-square modal acceleration vs. damper-structure frequency ratio: $\xi_s = 0.03, \xi_d = 0.15, \mu = 0.065$

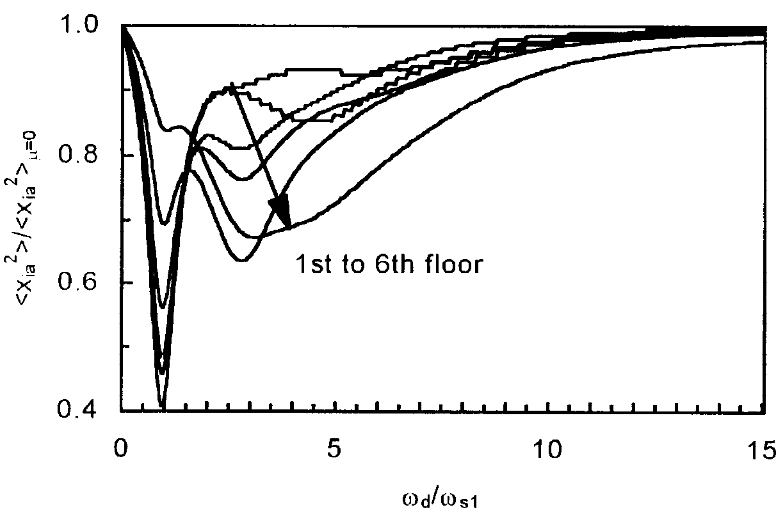

Fig.6 Mean-square floor acceleration vs. damper-structure frequency ratio: $\xi_s = 0.03, \xi_d = 0.15, \mu = 0.065$