Paper No. 1285. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

A PROBABILISTIC APPROACH FOR COMPUTER AIDED RESPONSE SPECTRA

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ABSTRACT

The present study aims at developing computer aided probabilistic response spectra for viscously damped single degree of freedom systems which can be directly used for aseismic design. The spectra are generated by using actual probability distributions of maximum elastic response, strength factors and reduction factors. The spectra obtained are compared with the three extreme value distributions for the two ensembles of artificial earthquakes. It is concluded that extreme type 1 and type 2 distributions are within 15 % of the actual probability distribution upto a confidence level of 85 % beyond which they differ considerably. A relationship between elastic and inelastic systems was derived in the form of reduction factors. It is concluded that the design forces in an inelastic system can be reduced by a factor of 1.1 to 9 depending upon the acceptable inelastic damage and the confidence level for survival of the structure. Finally, the probabilistic response spectra are developed which can be directly used for elastic and inelastic design for the desired confidence levels and ductility ratios

KEYWORDS

Amplification Factor; Confidence Level; Ductility Ratio; Reduction Factor; Strength Factor; Probabilistic Spectra; Extreme Value Distribution; Trapezoidal Spectra.

INTRODUCTION

The uncertainties involved in earthquake motions, structural systems and the resulting structural response suggest that the seismic response of structures can be best represented in probabilistic terms. Murakami and Penzien (1975, 1977) obtained probabilistic response spectra assuming that the probability distribution function of extreme values of structural response for a single class of earthquake follows closely Gumbel Type 1 Distribution (Benjamin and Cornell, 1970). Newmark and Riddell (1979,1980) studied the statistical response of single degree of freedom systems subjected to ten real earthquakes and proposed amplification factors to obtain trapezoidal spectra. Pal *et. al.*(1987) carried out statistical analysis for ten artificial earthquakes and presented constant strength, constant ductility, reduction factor and inelastic spectra. These analyses were based on the assumption that the inelastic response follows the Gaussian probability distribution.

This paper aims at the generation of probabilistic response spectra for elastic and inelastic systems using the analytical probability distribution (Pal, 1989; Siddal, 1983) for viscously damped single degree of freedom systems. Two ensembles of 50 records each were generated using the non stationary shot noise modelling (Murakami and Penzien, 1975). Earthquake E1 was of 5 sec duration and its peak ground acceleration varied between 0.15 g and 0.3 g .It simulated a shallow ground motion of magnitude 4.5 to 5.5. Earthquake E2 was of 30 sec duration and peak ground acceleration varied between 0.25 g and 0.4 g. It simulates a motion of magnitude 7 close to a fault.

A computer code IRS has been developed which generates the elastic response spectra, constant strength spectra, constant ductility spectra and reduction factor spectra for the given excitations. Another computer program NONDET has been written which directly reads the output created by the program IRS; carries out the probabilistic analysis of the elastic response, amplification factors, strength factors and reduction factors; and finally computes the probabilistic elastic and inelastic response spectra for different confidence levels and ductility ratios.

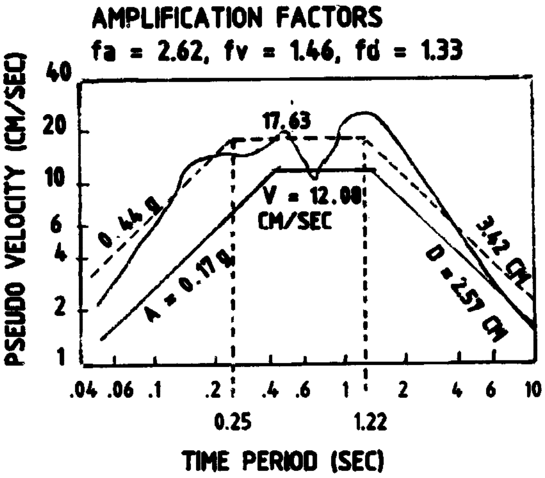
The equation of motion has been solved by numerical integration by using a variable time increment procedure so as not to miss any peak or trough in the earthquake excitation. Two types of hysteresis models i.e. elasto plastic and stiffness degrading models were used for inelastic analysis. The spectra were generated using 27 values of strength factors ranging from 0.001 to 4 and ductility ratios of 1, 1.5, 2, 2.5, 3, 4, 5, 6, 7 and 8. The response was computed for fifty six values of time periods ranging from 0.05 to 10 sec. A small interval of time period was selected in the shorter time period range since the response spectra is very sensitive to the system characteristics in this range.

ELASTIC SPECTRA

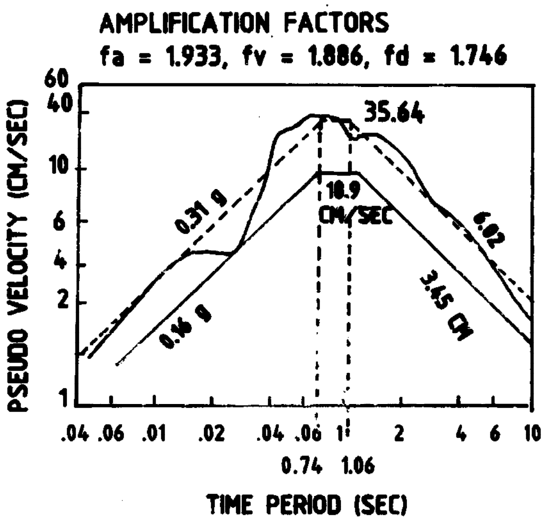
The mean, standard deviation, coefficient of variation and probability density functions of maximum elastic displacement were computed for statistical and probabilistic analysis of elastic response. The response in terms of maximum elastic displacement, pseudo velocity and pseudo acceleration for desired confidence levels were computed from the probability density functions and the plots were obtained in the form of a tripartite response spectra (Pal, 1989,1995).

Table 1. Comparison of 84.1% Confidence Level Spectra with (Mean + σ) Spectra - Earthquake E1

Time Period (sec)	Maximum Elastic Displacement (cm)		Percentage Difference (%)
	84.1% level	mean + σ	
0.05	0.024	0.024	0
0.5	2.655	2.604	1.9
1.0	4.781	4.913	2.8
2.0	6.262	6.150	4.3
4.0	4.833	4.702	2.7
6.0	4.329	4.166	3.8
8.0	3.704	3.717	0.4
10.0	3.533	3.440	2.6



(a) RECORD NO 9



(b) RECORD NO 35

FIG. 1 VARIATION IN KNEE PERIODS OF ELASIC SPECTRA - EARTHQUAKE E 1

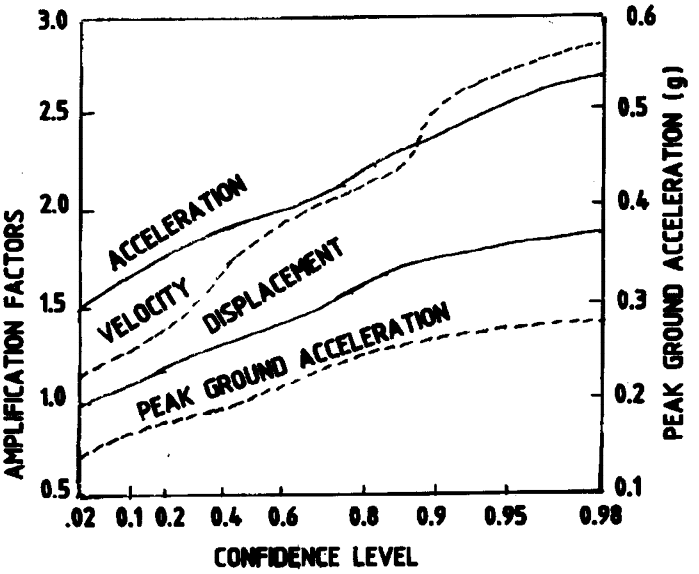


FIG. 2 DISTRIBUTION OF AMPLIFICATION FACTORS
EARTHQUAKE E 1

To verify the significance of assuming normal distribution for maximum elastic response a comparison of mean, (mean + σ) and (mean + 2σ) spectra with 50, 84.1 and 97.7% confidence level spectra respectively was made. It was observed that the average percentage difference in 84.1% confidence level spectra and (mean +σ) spectra is of the order of 5 percent only (as shown in Table 1), whereas the comparison of mean spectra and (mean + 2σ) spectra with their respective confidence levels showed an average difference of about 12%.

Amplification Factors

With a view to determine the design spectra from the estimates of peak ground motion parameters, the trapezoidal lines were fitted to the elastic spectra of each earthquake record in the velocity, displacement and acceleration regions of the tripartite plot by adopting the procedure suggested by Riddell and Newmark (1979). It was observed that there is a considerable dispersion in the knee period value of individual earthquake record. For earthquake E1, The knee period T_{av} varied from 0.2 sec to 1 sec, and T_{vd} varied from 0.4 to 1.7 sec. For E2 earthquake the values were 0.35 to 0.9 sec and 1.8 to 9 sec, respectively. The trapezoidal lines fitted to mean spectra of E1 earthquake gave the knee periods as 0.31 sec and 0.95 sec. Fig. 1 shows the typical variation in the knee periods for record numbers 9 and 35 for earthquake E1. Fig. 2 shows the distribution of amplification factors for different confidence levels.

INELASTIC SPECTRA

For inelastic systems, the strength factors and reduction factors were computed for desired ductility ratios and their statistical and probabilistic analysis was carried out. Actual probability distribution of strength factors and reduction factors also differed considerably with the three extreme value distributions. The inelastic spectra have been obtained in the form of constant strength spectra, constant ductility spectra, reduction factor spectra, and tripartite inelastic response spectra for different confidence levels and ductility ratios . The detailed results are available elsewhere (Pal, 1989, 1995).

Probability Distributions

Four probabilistic models were employed in the study to examine their suitability in representing the random behaviour of inelastic response, viz. : Extreme value distribution type 1, type 2, type 3 or Weibull distribution, and the analytical probability distribution model. The probability density function of the analytical model is computed using the theory of curve fitting and interpolation on the available data. More details can be seen in Pal (1989, 1985) and Siddal (1983).

The various probability distributions of the strength factors are shown in Fig. 3. It can be seen that the type 1 and type 2 distributions are fairly close to the analytical distribution till about 85% confidence level beyond which the difference is quite large. Similarly, the various probability distributions for reduction factors are shown in Fig. 4. The type 1 and type 2 distributions are very close to the analytical distribution except in lower confidence level range (less than about 20%) which is not of much practical relevance. The probability distributions of reduction factors for earthquake E1 for time period of 1.0 sec for all ductility ratios are shown in Fig. 5. The type 1 distribution compares well with the analytical probability distribution.

The confidence level spectra of strength factors for ductility of 4 for different time periods is shown in Fig. 6. It also shows statistical curve of mean + standard deviation for strength factors which corresponds to 84.1% confidence level assuming Gaussian distribution. It can be seen that the statistical curve is close to the probabilistic curve within about 10%. Fig. 7 shows the comparison of reduction factor spectra for elasto plastic and stiffness degrading models for 85% confidence level. It is observed that the reduction factors generally vary from about 1.1 to 9 depending upon the ductility ratio.

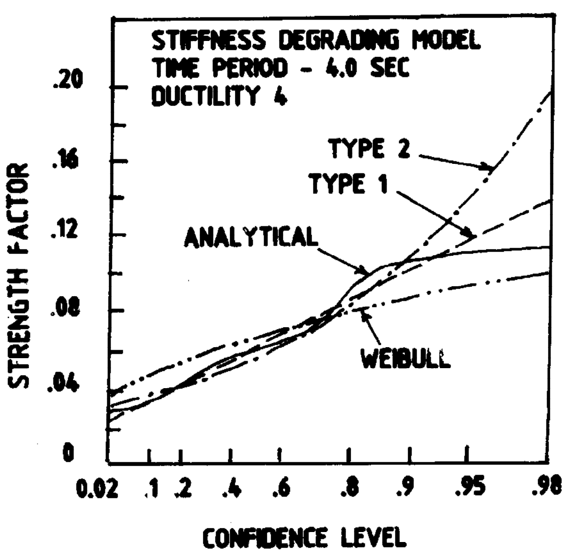


FIG. 3 COMPARISON OF PROBABILITY DISTRIBUTION OF STRENGTH FACTORS - EARTHQUAKE E 2

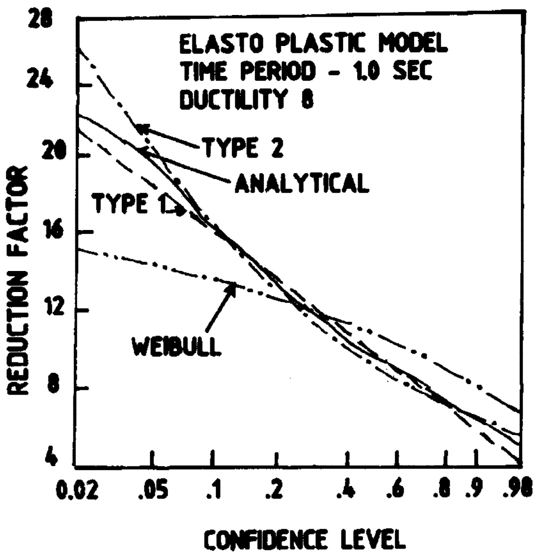


FIG. 4 COMPARISON OF PROBABILITY DISTRIBUTION OF REDUCTION FACTORS - EARTHQUAKE E 1

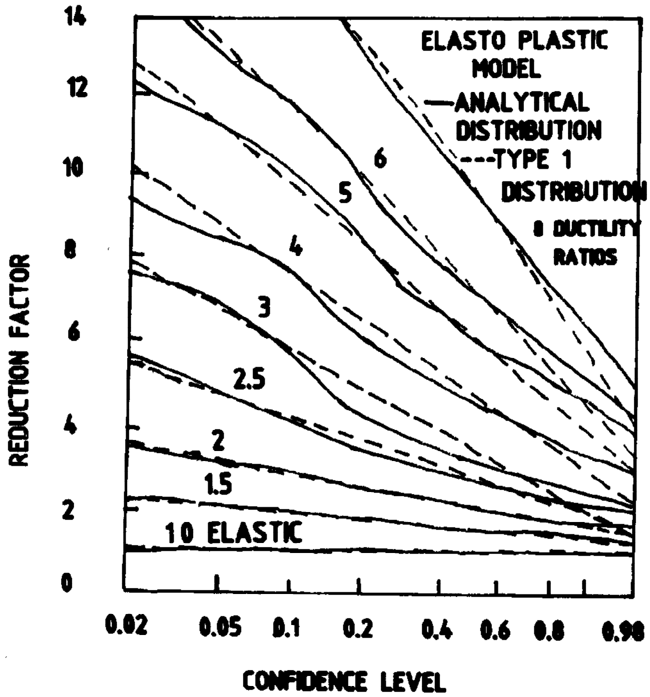


FIG. 5 PROBABILITY DISTRIBUTION OF REDUCTION FACTORS - EARTHQUAKE E 1 (TIME PERIOD = 1.0 SEC)

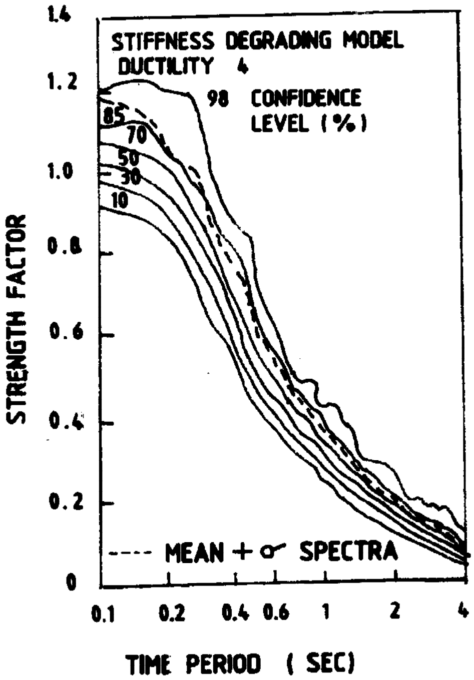


FIG. 6. CONFIDENCE LEVEL SECTRA OF STRENGTH FACTORS-EARTHQUAKE E 2

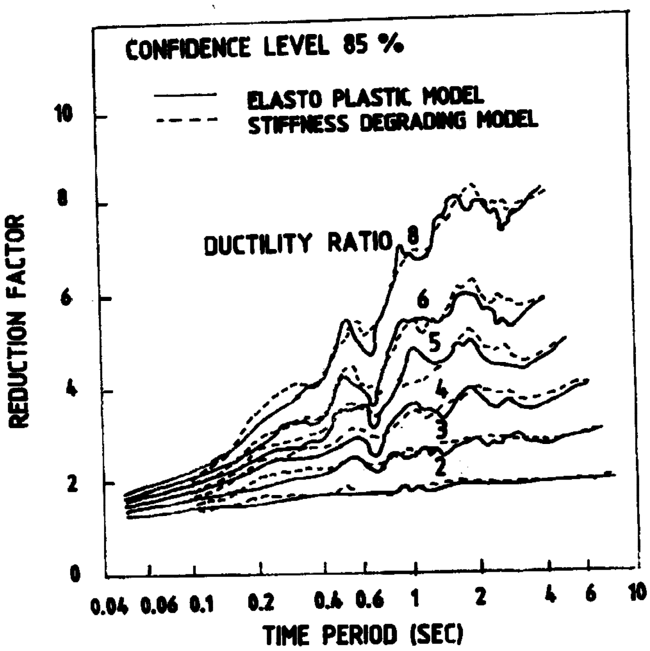


FIG. 7. REDUCTION FACTOR SPECTRA FOR 8 5 % CONFIDENCE LEVEL - EARTHQUAKE E 1

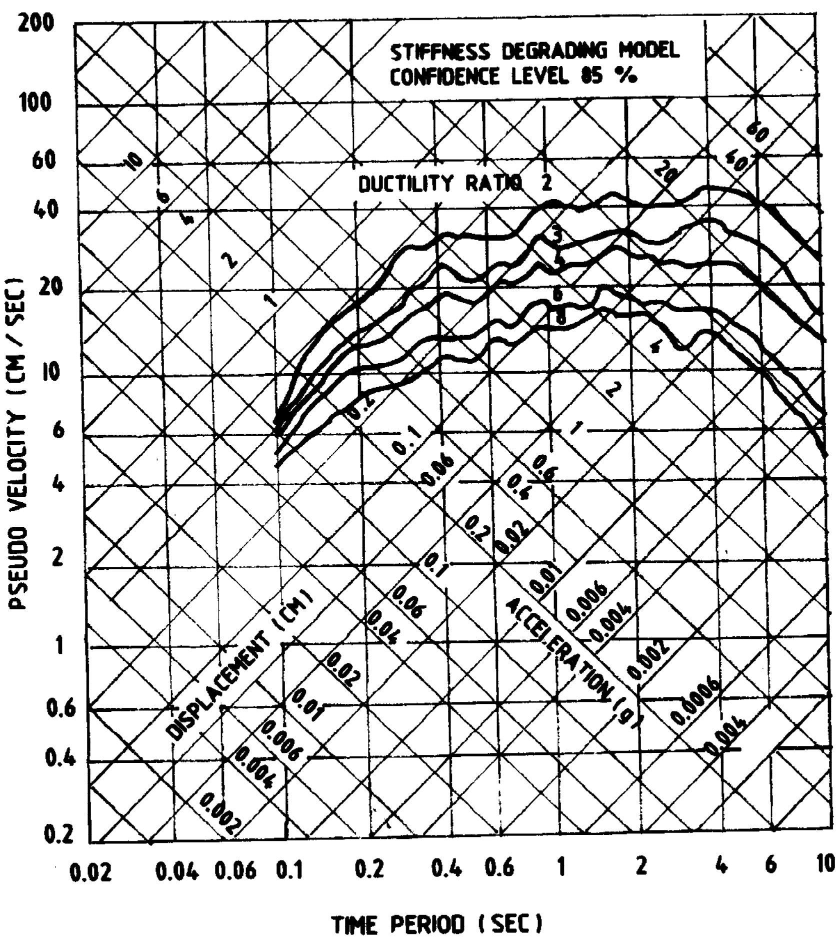


FIG. 8 INELASTIC RESPONSE SPECTRA FOR 85 % CONFIDENCE LEVEL - EARTHQUAKE E 2

Inelastic Design Spectra

Elastic spectra for different confidence levels were toned down by applying the reduction factors corresponding to the desired confidence levels to the maximum elastic displacement to obtain the inelastic response spectra. By reducing the maximum elastic displacement by reduction factors, will result in the yield displacement. The yield displacements so computed are plotted on the displacement axis of a tripartite logarithmic paper to obtain the tripartite inelastic response spectra. Such spectra for 85% confidence level and for different ductility ratios is shown in Fig. 8 . From these spectra one can directly read the yield displacement x_y , pseudo velocity ωx_y and pseudo acceleration $\omega^2 x_y$ on the respective axes of tripartite plot. To get the yield force or maximum force in the elasto plastic systems, $\omega^2 x_y$ is multiplied by the mass of the system. For stiffness degrading model, the maximum force can be computed by multiplying the mass with $\omega^2 x_y\ [1 + s(\mu-1)]$ where μ is the desired ductility, and s is the strain hardening slope expressed as a fraction of initial elastic stiffness of the system . Such spectra can, therefore, be directly used for inelastic design and are termed as *Inelastic Design Spectra.*

The procedure for obtaining the inelastic response spectra from the estimates of peak ground motion parameters, is explained in Fig. 9. First the elastic spectra is obtained by applying the amplification factors obtained from elastic analysis for desired confidence levels. Then the reduction factors are applied to the elastic spectra to obtain Inelastic Design Spectra for desired ductility ratios and confidence levels. In Fig. 9 , the trapezoidal spectra due to Riddell and Newmark (1980) by using their amplification and de amplification factors, has also been shown in dotted lines. The elastic spectra due to two studies are quite close to each other. However, the trapezoidal inelastic spectra gives the lower acceleration values than the spectra obtained by present approach. This is due to the difference in approaches (statistical and probabilistic) adopted in the two studies and due to the reason that the trapezoidal spectra is greatly simplified.

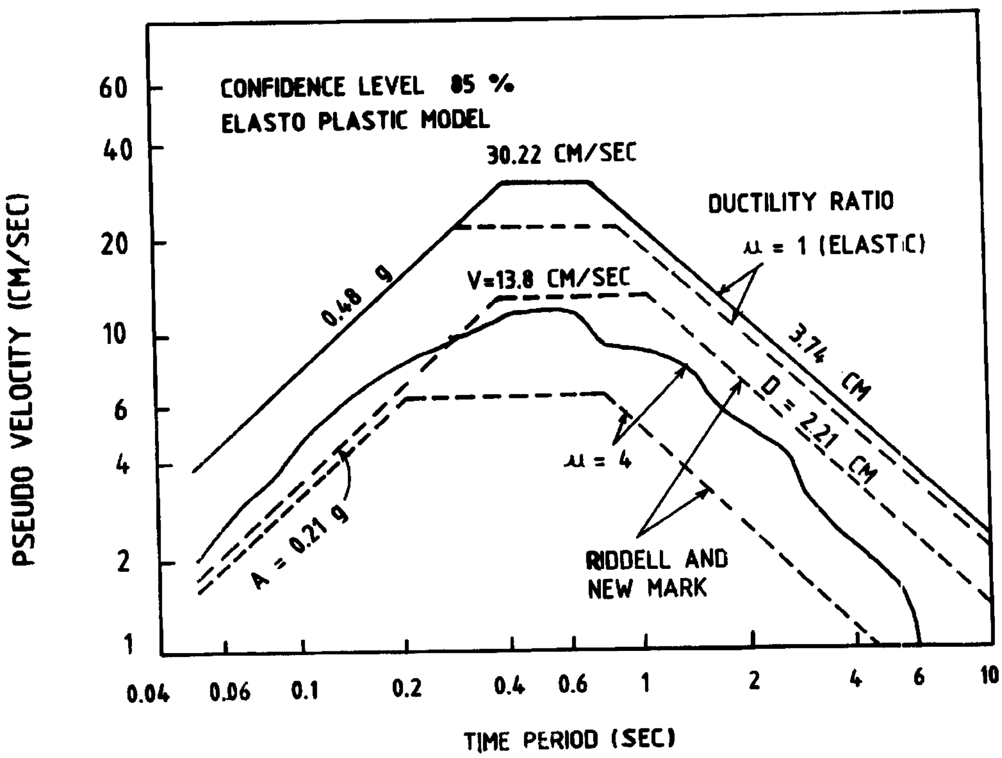


FIG. 9 DESIGN SPECTRA FROM PEAK GROUND MOTION PARAMETERS : EARTHQUAKE - E1

CONCLUSIONS

Based on the results presented in this paper, the following significant conclusions are drawn :

(1) For elastic spectra, the average percentage difference in 84.1% confidence level spectra and (mean + σ) spectra is of the order of 5 percent only, whereas the difference of mean spectra and (mean + 2σ) spectra with their respective confidence levels on an average is about 12%.

(2) A considerable difference is observed in the knee periods in the spectra of individual earthquake record. They also differ considerably from the knee periods of the mean spectra. This results in the considerable difference in the amplification factors in the velocity region.

(3) For the strength and reduction factors, the extreme value type 1 and type 2 distributions are within 10% of the analytical distribution upto a confidence level of 85% beyond which the difference is about 15%. The Weibull distribution does not give satisfactory results.

(4) The inelastic spectra obtained by using the simpler elasto plastic model are comparable within 8% accuracy with those obtained by using the more complicated stiffness degrading hysteresis model.

(5) The reduction factors generally vary from 1.1 to 9 depending upon the desired ductility ratio and confidence level.

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