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Paper No. 1247. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

AMPLIFICATION FACTORS ESTIMATED FROM SPECTRAL RATIO BETWEEN HORIZONTAL AND VERTICAL COMPONENTS OF MICROTREMOR

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ABSTRACT

When properly smoothed, peak values in spectral ratios between horizontal and vertical components of Rayleigh waves are well correlated with amplification factors of vertically incident S waves in the surface ground. This is demonstrated by a series of numerical simulations for realistic ground models. For this reason, a smoothing function is newly proposed and applied to estimation of the amplification factors from microtremor data observed in Tokyo area. An amplification-factor map resulting from the estimation shows a good resemblance with an existing map to a satisfactory level of accuracy.

KEYWORDS

microtremor; fundamental mode Rayleigh wave; S wave; amplification factor; predominant period; spectral ratio; seismic microzonation

INTRODUCTION

A simple method was proposed to estimate dynamic characteristics of the ground. The method which was originally proposed by Nakamura (1989) is to calculate a spectral ratio between horizontal and vertical components (H/V ratio) of microtremors. In the method, a predominant period is given by a peak period in the H/V ratio, while an amplification factor at the periods is given by the ratio itself. Several recent applications of this method have been successful in estimating the predominant periods (e.g., Field and Jacob, 1993; Ohmachi *et al.*, 1994) and the relative amplification factors as well (e.g., Lermo and Chavez-Gracia, 1994; Konno and Ohmachi, 1995). However, a rigorous theoretical background for the method has been lacking.

Microtremors are considered to consist of body and surface waves. However H/V ratios of microtremors and those of fundamental mode Rayleigh waves estimated from geological date are very resemble at many sites (e.g.,Tokimatsu and Miyadera, 1992; Ohmachi *et al.*,1994). Lachet and Bard(1994) indicated that H/V ratios derived from noise simulation were governed by fundamental mode Rayleigh waves. Based on these results, we assume that H/V ratios of microtremors could be roughly interpreted with those of fundamental mode Rayleigh waves.

The H/V ratio of fundamental mode Rayleigh waves for soft site is infinite at the peak period. Therefore, we

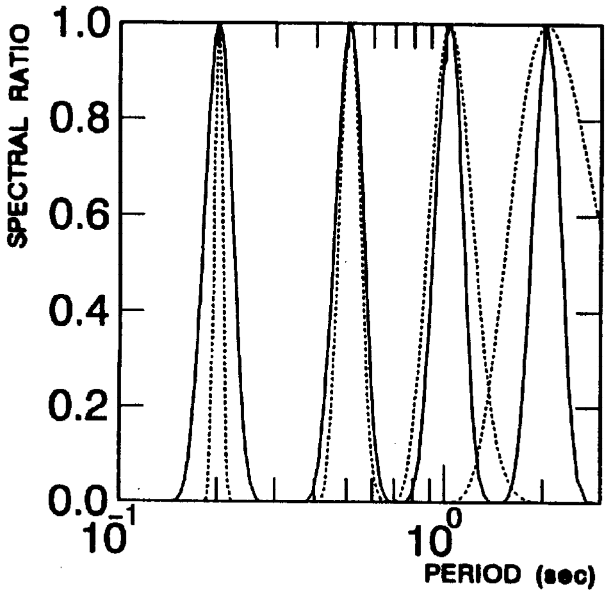


Fig. 1. Shapes of two types of smoothing function, when center frequencies are 0.5, 1.0, 2.0 and 5.0Hz. Dotted lines show Parzen window with band width of 0.5Hz and solid lines show smoothing functions W_B with $b = 20$.

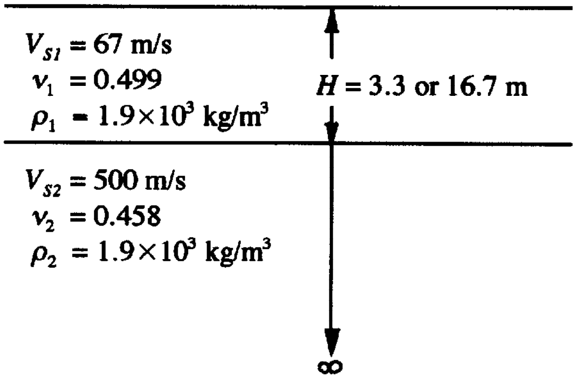


Fig. 2. Model of an elastic layer over a half-space. H is thickness, V_S is S wave velocity, ν is Poisson's ratio, and ρ is density.

cannot investigate a relation between peak values in H/V ratios of fundamental mode Rayleigh waves and amplification factors. For this reason, we have smoothed horizontal and vertical amplitudes of the fundamental mode Rayleigh waves individually before taking the ratio so as to avoid the infinite, and then investigate a relation between the smoothed peak values and the amplification factors.

PEAK VALUES IN H/V RATIOS OF FUNDAMENTAL MODE RAYLEIGH WAVES

Horizontal and vertical amplitudes of the surface for j th mode Rayleigh waves due to a vertical point force $L(\omega)$ at the surface are obtained as the following (Harkrider, 1964):

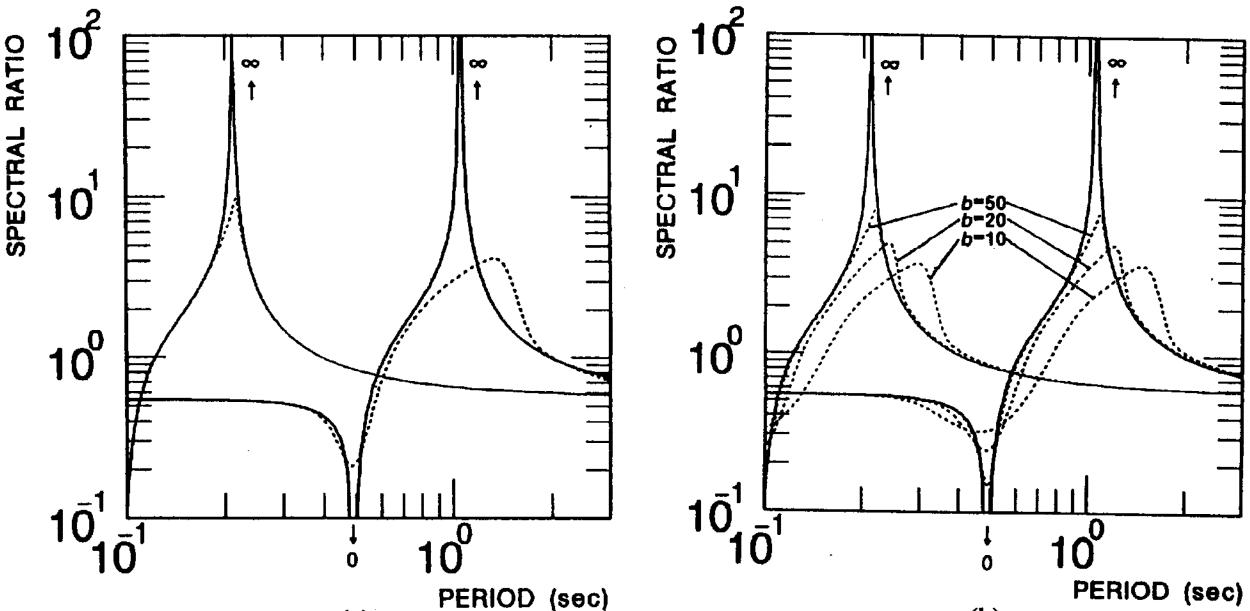
$$\dot{u}_j(\omega,r) = (L(\omega)/2)\,[\dot{u}_0{}^*/\dot{w}_0]_{H_j}\,A_{R_j}\;\mathrm{H}_1^{(2)}(k_j r)\cdot\omega \tag{1a}$$
$$\dot{w}_j(\omega,r) = (L(\omega)/2)\,A_{R_j}\;\mathrm{H}_0^{(2)}(k_j r)\cdot\omega \tag{1b}$$

where $\dot{u}_j(\omega,r)$ and $\dot{w}_j(\omega,r)$ are horizontal and vertical velocity amplitudes at the surface, respectively, ω is frequency, r is distance between the point force and observation station, $[\dot{u}_0{}^*/\dot{w}_0]_{H_j}$ is H/V ratio at large distance r defined by Haskell (1953), A_{R_j} is medium response and k_j is wave-number.

For simplicity, we considered only fundamental mode Rayleigh waves, that is $j = 0$ in equations (1). As for $L(\omega)$, we assumed $L(\omega)$ is proportional to ω - square as follows (Konno, 1995):

$$L(\omega) \propto \omega^{-2}. \tag{2}$$

We used two kinds of smoothing function against $\dot{u}_0(\omega,r)$ and $\dot{w}_0(\omega,r)$ at large distance r. One is a Parzen window with band width of 0.5 Hz, another is a newly proposed smoothing function $W_B(f,f_C)$ given by equation (3):



(a)

(b)

Fig. 3. Smoothed and not smoothed H/V ratios of fundamental mode Rayleigh waves for two cases of ground model shown in Fig. 2. The thickness is determined to give a predominant period of 0.2 and 1.0 s. Thick solid and thick dotted lines showing peaks at about 1.0 s represent H/V ratios for the model having a predominant period of 1.0 s. Thin solid and thin dotted lines showing peaks at about 0.2 s represent H/V ratios for the model having a predominant period of 0.2 s. Solid lines represent $[\dot{u}_0{}^*/\dot{w}_0]_{H_0}$. The dotted lines represent H/V ratios using (a) Parzen window with band width of 0.5 Hz and (b) smoothing function W_B for $b = 10$, 20 and 50.

$$W_B(f,f_C) = \left[\frac{\sin(b\log_{10}(f/f_C))}{b\log_{10}(f/f_C)}\right]^4 \tag{3}$$

where b is a coefficient which corresponds to band width, f is frequency, f_C is a center frequency at which the window is symmetry on a logarithmic scale. Figure 1 shows Parzen windows (dotted lines) with band width of 0.5 Hz and smoothing functions $W_B(f,f_C)$ (solid lines) with $b = 20$, when $f_C = 0.5$, 1.0, 2.0 and 5.0 Hz. The horizontal axis shows period.

Let us consider the fundamental mode Rayleigh waves in the simple ground model (Fig. 2). Figure 3 (a) shows H/V ratios of fundamental mode Rayleigh waves for two cases of ground model shown in Fig. 2. The thickness is determined to give a predominant period of 0.2 and 1.0 s. Thick solid and thick dotted lines showing peaks at about 1.0 s represent H/V ratios for the model having a predominant period of 1.0 s. Thin solid and thin dotted lines showing peaks at about 0.2 s represent H/V ratios for the model having a predominant period of 0.2 s. Solid lines represent $[\dot{u}_0{}^*/\dot{w}_0]_{H_0}$. The dotted lines represent H/V ratios using Parzen window with band width of 0.5 Hz. As a result, the smoothed H/V peak value depends on its peak period. Figure 3 (b) shows the same as Fig. 3 (a) except for smoothing function W_B for $b = 10$, 20 and 50 used instead of Parzen window. The H/V peak value smoothed by the function W_B does not depend on its peak periods. Amplification factors should not depend on the predominant periods, but it given by $2\rho_2 V_{S2}/(\rho_1 V_{S1})$, if attenuation of soil is neglected. For that reason, we will use smoothing function W_B to calculate H/V ratios of fundamental mode Rayleigh waves, and use $b = 20$ according to the result by Konno and Ohmachi (1995), hereafter.

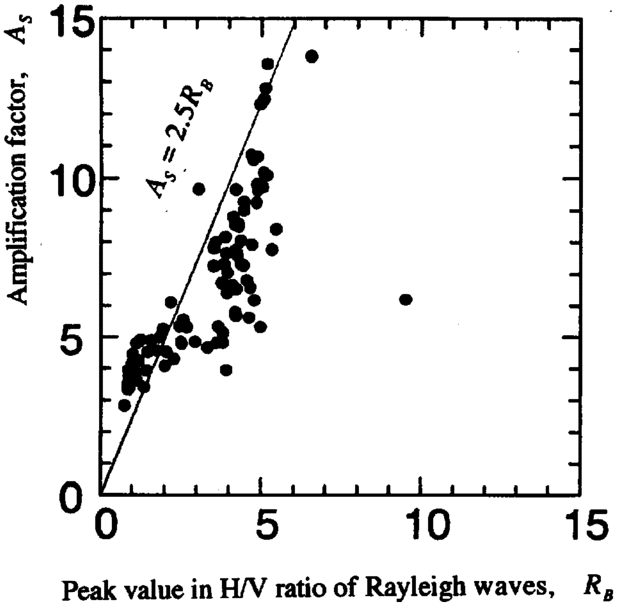


Fig. 4. Relation between peak values R_B in H/V ratios of fundamental mode Rayleigh waves and amplification factors A_S derived from transfer function of vertically incident S waves for 85 models. Solid line shows $A_S = 2.5R_B$.

Let us consider realistic ground models obtained from the down-hole measurements at 85 sites in and around Tokyo. Figure 4 shows a relation between peak values in H/V ratios of fundamental mode Rayleigh waves and amplification factors derived from transfer function of vertically incident S waves for the 85 models. There is a fairly good correlation between peak values in H/V ratios and amplification factors. Solid line shows

$$A_S = 2.5R_B \tag{4}$$

in which A_S is amplification factor and R_B is peak value in H/V ratio of fundamental mode Rayleigh waves.

ESTIMATION PROCEDURE FOR AMPLIFICATION FACTOR USING MICROTREMOR MEASUREMENTS

If Rayleigh waves components can be extracted from microtremors, we may evaluate amplification factors using the relation between A_S and R_B shown by equation (4). Let us consider H/V ratio of microtremors as follows:

$$R_M(T) = \left[H_L(T) + H_R(T)\right] / V_R(T) \tag{5}$$

where T is period, $H_L(T)$ is horizontal component of Love waves, $H_R(T)$ and $V_R(T)$ are horizontal and vertical components of fundamental mode Rayleigh waves, respectively, and $R_M(T)$ is simplified H/V ratio of microtremors. Next, let us define $\beta(T)$ as the proportion of horizontal component of Rayleigh waves to that of microtremors as follows:

$$\beta(T) = H_R(T) / \left[H_L(T) + H_R(T)\right]. \tag{6}$$

Combining equations (5) and (6) yields

$$R_M(T) = H_R(T) / \left[\beta(T) \cdot V_R(T)\right] \tag{7}$$

in which $H_R(T)/V_R(T)$ is H/V ratio of fundamental mode Rayleigh waves. We can obtain H/V ratio of microtremors by using smoothing function W_B as follows:

$$R_{MB}(T) = R_B(T)/\beta(T) \qquad (8)$$

in which $R_{MB}(T)$ and $R_B(T)$ are smoothed H/V ratios of microtremors and fundamental mode Rayleigh waves, respectively. Substituting equation (4) into equation (8), we can obtain equation (9):

$$A_S = 2.5\beta R_{MB} \qquad (9)$$

in which β and R_{MB} are $\beta(T)$ and $R_{MB}(T)$ at the peak periods of H/V ratios $R_{MB}(T)$, respectively. If we can evaluate β at each site, we can estimate amplification factors by using equation (9). However, there are few study for $\beta(T)$. In this paper, therefore, we use $\beta = 0.4$ according to the result by Miyadera and Tokimatsu (1992). Therefore we can express peak value R_{MB} by amplification factor A_S as follows:

$$A_S = R_{MB}. \qquad (10)$$

Verification

Figure 5 shows a comparison of H/V ratios of microtremors (thick solid line), transfer functions for vertically incident S waves (thin solid line) and H/V ratios of fundamental mode Rayleigh waves (dotted line). H/V ratios of fundamental mode Rayleigh waves and transfer functions are calculated for the ground models derived from the down-hole measurements at 14 sites in and around Tokyo.

Figure 6 shows a comparison of peak values R_{MB} in H/V ratios of microtremors and amplification factors A_S. We define R_{MB} as maximum value in the H/V ratio of microtremor between 0.05 and 3.0 s, and A_S is amplification factor at the fundamental predominant period. Each number in this figure corresponds to site number shown in Fig. 5. Solid line shows $R_{MB} = A_S$. There is a fairly good correlation between R_{MB} and A_S.

APPLICATION TO SEISMIC MICROZONATION

Amplification factors were estimated form microtremor measurements at 546 junior high schools in 23 wards in Tokyo. Figure 7 (a) and (b) show maps of amplification factors estimated from H/V ratios of microtremor using the smoothing function W_B defined by equation (3) and from surface geology (Shima, 1987). It seems that the amplification factors estimated from microtremors are large at clay sites and small at loam sites, and that here two maps are well correlated with one another.

Conclusively, as H/V ratios of microtremors are easily obtained, we can estimate amplification factors from the above-mentioned procedure to a satisfactory level of accuracy.

CONCLUSIONS

In this study, we have assumed that microtremors mainly consist of surface waves, and attempted to explain its H/V ratio in correlation with that of fundamental mode Rayleigh waves.

First, amplification factors were tried to correlate with H/V ratios of fundamental mode Rayleigh waves. Since the peak value is infinite at the peak period in the H/V ratios, we smoothed theoretical amplitudes of horizontal and vertical components of fundamental mode Rayleigh waves before taking spectral amplitude

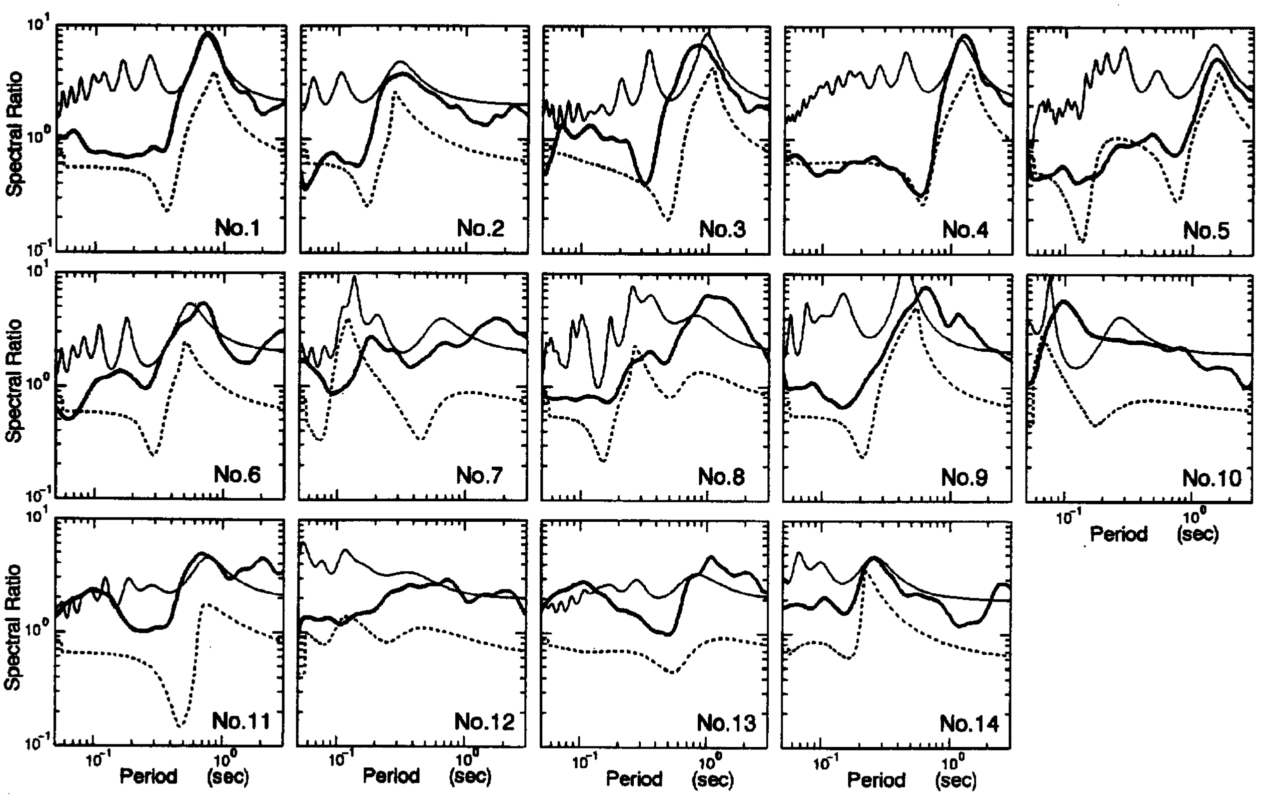


Fig. 5. Comparison of H/V ratios of microtremors (thick solid line), transfer functions for vertically incident S waves (thin solid line) and H/V ratios of fundamental mode Rayleigh waves (dotted line). H/V ratios of fundamental mode Rayleigh waves and transfer functions were calculated for the ground models derived from the down-hole measurements at 14 sites.

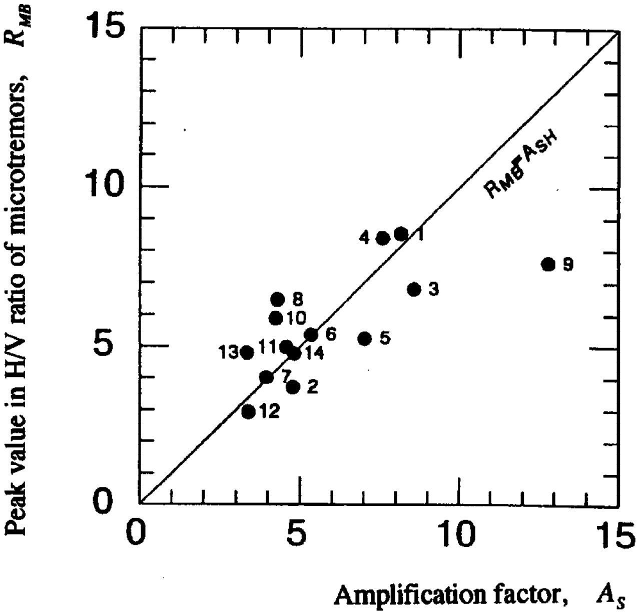
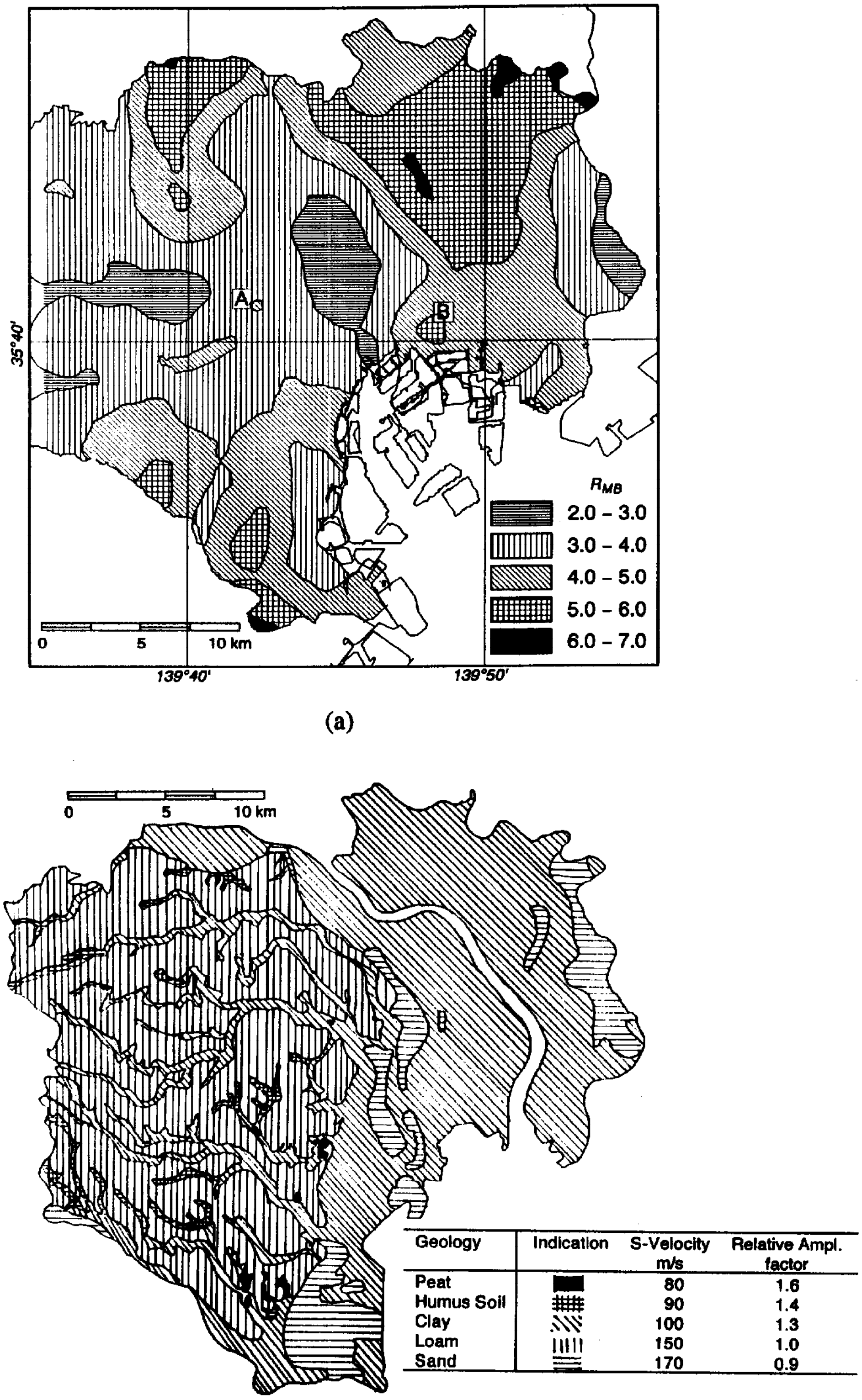


Fig. 6. Comparison of peak value R_{MB} in H/V ratios of microtremors and amplitude factors A_S derived from transfer function of vertically incident S waves. Solid line shows $R_{MB} = A_S$.



(a)

Geology	Indication	S-Velocity m/s	Relative Ampl. factor
Peat	■	80	1.6
Humus Soil	▦	90	1.4
Clay	▧	100	1.3
Loam	▥	150	1.0
Sand	▤	170	0.9

(b)

Fig. 7. Maps of amplification factors estimated from (a) microtremors and (b) surface geology (after Shima, 1987).

ratio. When Parzen window was used for a smoothing function, the peak value in H/V ratio was found to change depending on their peak periods. Hence, a smoothing function W_B was newly proposed, by which the peak value did not change with a peak period. When we used the smoothing function W_B, there was a fairly good correlation between the peak values in H/V ratios and amplification factors.

Next, we assumed that the proportion of horizontal component of Rayleigh waves to that of microtremor motion was 0.4, the peak value in H/V ratios of microtremors was found to be nearly equal to the amplification factor. We confirmed this hypothesis at 14 sites where V_S profiles were available.

Finally, we applied our procedure to seismic microzonation for 23 wards of Tokyo. After drawing an amplification factor map, we compared it with an existing map, with a satisfactory result.

Through this study, in the authors belief, the method to estimate amplification factors using H/V ratio of microtremors has been reinforced theoretically, and improved for practical use.

ACKNOWLEDGMENTS

The author would like to thank Prof. Tatsuo Ohmachi for his encouragement and constant guidance during the course of this study. He would also like to thank Prof. Saburoh Midorikawa, Prof. Takumi Toshinawa and Dr. Yutaka Nakamura for fruitful discussions and advice, and thank Tatsuya Endoh and Yohichi Nakamura for their cooperation in the field work.

REFERENCES

Field, E. and K. Jacob (1993). The theoretical response of sedimentary layers to ambient seismic noise. *Geophys. Res. Lett.*, **20**, No. 24. 2925-2928.

Harkrider, D. G. (1964). Surface waves in multilayered elastic media. *Bull. Seism. Soc. Am.*, **54**, No. 2, 627-679.

Haskell, N. A. (1953). The dispersion of surface waves on multilayered media. *Bull. Seism. Soc. Am.*, **43**, No. 1, 17-34.

Konno, K. (1995). A study on ground motion characteristics estimated from spectral ratio between horizontal and vertical components of microtremor. *Dr Thesis* (in Japanese), Tokyo Institute of Technology, Tokyo.

Konno, K. and T. Ohmachi (1995). A smoothing function suitable for estimation of amplification factor of the surface ground from microtremor and its application. *J. JSCE*, **525**, I-33, 247-259 (in Japanese).

Lachet, C. and P. Y. Bard (1994). Numerical and theoretical investigations on the possibilities and limitations of Nakamura's technique, *J. Phys. Earth*, **42**, 377-397.

Lermo, J. and F. J. Chavez-Gracia (1994). Are microtremors useful in site response evaluation? *Bull. Seism. Soc. Am.* **84**, No. 5, 1350-1364.

Miyadera, Y. and K. Tokimatsu (1992). Sampling of surface waves in short-period microtremors. *Proc.*, Annual Meeting of JSSMFE, 965-966 (in Japanese).

Nakamura, Y. (1989). A method for dynamic characteristics estimation of subsurface using microtremor on the ground surface. *QR of RTRI*, **30**, No. 1, 25-33.

Ohmachi, T., K. Konno, T. Endoh and T. Toshinawa (1994). Refinement and application of an estimation procedure for site natural periods using microtremor. *J. JSCE*, **489**, I-27, 251-260 (in Japanese).

Shima, E. (1987). Seismic microzoning map of Tokyo. *Proc. of the Second Intern. Conf. on Microzonation*, **1**, 433-443.

Tokimatsu, K. and Y. Miyadera (1992). Characteristics of Rayleigh waves in microtremors and their relation to underground structures. *J. Struct. Constr. Engng*, AIJ, **439**, 81-87 (in Japanese).