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LOCAL DIRECTIVITY OF STRONG GROUND MOTION

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ABSTRACT

A definition of the directivity of three-dimensional ground motion during strong earthquakes, in terms of the invariants of the intensity tensor and the intensity deviation tensor, is presented and discussed. A specialized two-dimensional version of the general definition is applied to the horizontal components of 23 records of ground acceleration of the March 3, 1985 Central Chile earthquake, and to the components of ground velocity obtained thereof by integration. The analysis of the results reveals that the directivities of horizontal acceleration and horizontal velocity are strongly correlated for stations located not farther than 100 Km from the hypocenter, as well as for stations at which the invariant $(I_h)_a$ exceeds 3 m/s.

KEYWORDS

Directivity; intensity tensor; intensity deviation tensor; 1985 Central Chile earthquake.

THE EARTHQUAKE INTENSITY TENSOR

In the late 1960's, the author proposed a definition of earthquake intensity based on instrumental records of strong ground motion (Lange, 1968; Arnold, 1969; Arias et al., 1969; Arias, 1970). Briefly, the main underlying assumption at the basis of this definition was the idea that earthquake intensity, conceived as the capacity of ground motion to produce damage, can be represented by the amount of energy dissipated in the production of permanent effects on the structures belonging to some representative set. The final analytical expression of this definition depends on the properties attributed to the structures, the distribution of those properties over the representative set, and the physical entity taken as the point of comparison to express dissipated energy in relative terms. We shall omit the details which are not relevant for the purpose of the present paper, and go directly to the final results.

According to the original definition, earthquake intensity at a point 0 is a tensor quantity represented by the matrix:

$$I = [I_{rs}] \quad r,s = x\ y,\ z \tag{1}$$

whose elements are given by

$$I_{rs} = \frac{\pi}{2g} \int_0^{t_0} a_r(t)\, a_s(t)\, dt, \quad r,s = x,y,z \tag{2}$$

Here a_r (t), for r = x, y, z, are the components of acceleration recorded at 0 in the direction of the axes of a rectangular cartesian system of coordinates 0xyz, t is time, and t_0 is the total duration of the record.

The diagonal terms I_{xx}, I_{yy}, I_{zz} are the intensities at 0 in the direction of the x-, y-, and z-axis, respectively. In general, the intensity at 0 in the direction of the unit vector $\underline{e}$ is given by the product $\underline{e}^T\ \underline{I}\ \underline{e}$, where the superscript T denotes transpose. Therefore, the intensity tensor at 0 contains all the information neccesary to determine the intensity in any direction through 0.

It is always possible to diagonalize the intensity matrix by a suitable rotation of the coordinate axes, leaving 0 fixed. This transformation is essentially unique. Let $0x_1x_2x_3$ be the new axes, after rotation. The matrix that represents the same intensity tensor referred to the new axes will be of the form diag (I_1, I_2, I_3). The directions of the new axes are the principal directions of ground acceleration at 0, and the elements I_1, I_2, I_3 of the diagonalized matrix are the principal intensities at the point. The labels 1, 2, and 3 will be chosen so that the following inequalities hold

$$I_1 \geq I_2 \geq I_3 \tag{3}$$

Then the axes $0x_1$, $0x_2$, $0x_3$ will be respectively, the major, intermediate and minor axis of the intensity tensor. The intensity in any direction whatsoever through 0 lies necessarily between I_3 and I_1.

Any rotation of the coordinate axes around 0 will preserve the trace of the intensity matrix; i. e.

$$tr\ \underline{I} = I_{xx} + I_{yy} + I_{zz} = I_1 + I_2 + I_3 \tag{4}$$

is an invariant which may be called the scalar intensity at 0. We shall denote it by I_o. Obviously

$$I_o \equiv tr\ \underline{I} = \frac{\pi}{2g}\int_0^{t_0} \vec{a}^2(t)\, dt \tag{5}$$

where $\vec{a}(t)$ is the acceleration vector at 0.

J. Penzien and M. Watabe (Penzien and Watabe, 1975), led by statistical considerations, rediscovered the existence of principal directions of ground acceleration. In fact, if acceleration at a given point 0 is conceived as a three-component, zero-mean, ergodic stochastic process, then, except for the irrelevant factor $\pi/2g$, the matrix defined by eqs (1) and (2) is an estimate of the covariance matrix of acceleration based on just one realization of the process. These notions were extended to the frequency domain by Kubo and Penzien (1976, 1979).

When the motion is referred to the principal axes of ground acceleration, the variance terms (diagonal terms) have stationary values, whereas the cross-variance terms (off-diagonal terms) become equal to zero. This means that the components of acceleration along the principal axes are uncorrelated. If, besides the hypotheses enunciated in the preceding paragraph, the process is assumed to be gaussian, then the principal components of acceleration will be stochastically independent; a very convenient property for theoretical and practical purposes.

Alternative definitions of earthquake intensity as a tensor quantity

There are other possible definitions of tensorial quantities that reflect the directional properties of ground motion at a point. For example, in the definition of eqs. (1) and (2) ground velocity v(t) or the time derivative of ground acceleration $\dot{a}(t)$ can be used instead of acceleration. Each one of the entities so defined will enjoy the advantage of being a tensor quantity. Extensions to the frequency domain and the probabilistic interpretation of the intensity matrix as a single-realization-estimate of the covariance matrix of the underlying stochastic process are also possible for these alternative definitions. The simultaneous use of three intensity tensors, based respectively on ground velocity, ground acceleration and the time-derivative of ground acceleration was propounded by the author in his class-room lectures delivered at MIT during the spring term of the year 1969. Some results related with this proposition may be found in Sixsmith and Roesset (1970), and in Arias (1973). Those three tensors point to different regions of the frequency domain, and therefore give complementary information on the directional properties of ground motion in different ranges of the spectrum.

SOME EMPIRICAL FINDINGS ON PRINCIPAL DIRECTIONS

The first practical applications of the intensity tensor to the study of the spatial distribution of intensity at a point were made by G. Lange (Lange, 1968), who determined the principal directions for a small number of three-component strong-motion accelerograms: four of them recorded during the Parkfield earthquake of June 27, 1966 at the stations Temblor, N° 5, N° 8 and N° 12, and a fifth record obtained in Santiago, Chile, on March 28, 1965. The results revealed that in all cases one of the principal axes was nearly vertical, and, with only one exception, it was the minor axis the one that approached verticality. The exception corresponds to the Parkfield earthquake record with the lowest intensity.

In all the cases computed by Lange, it happened that the off-diagonal elements containing the index z (i.e, $I_{xz} = I_{zx}$ and $I_{yz} = I_{zy}$) had the smallest absolute values, as compared to the rest of the elements of the intensity matrix. In fact, those off-diagonal terms were always smaller than 4% of the trace.

R. Kubo and J. Penzien (Kubo and Penzien, 1976, 1979) examined a larger set of three-component accelerograms and came to a similar conclusion: in all cases, excepting records obtained in high-rise buildings, one of the principal axes was found to be nearly vertical. More specifically , using a moving window technique in the time domain, they found that during the early periods of low intensity motion at the beginning of the record, either the major or the intermediate principal axis is nearly vertical; later, except at stations near the epicenter, the near vertical position is taken by the minor principal axis. Similar results have been reported by Loh et al. (1982) from selected accelerograms recorded by the SMART 1 strong motion array located near Lotung, Taiwan, and also by Katayama et al. (1990), from records obtained in a three-dimensional seismometer array installed in the Chiba Experiment Station, University of Tokyo.

DIRECTIVITY

We are not concerned here with the description of the directional properties of ground motion as represented by the orientation of privileged or predominant directions at a point, because that can be done very nicely with the analytical instruments we already have at hand. For example, the principal directions of acceleration can be used for that purpose, as it has been done by several authors, starting with G.Lange (Lange, 1968); or else, we may employ the direction in which the vector $\vec{a}(t)$ points at the instant when $|\vec{a}(t)|$ attains its maximum value, as done by M. Ahumada (Ahumada, 1994), to cite only a recent example. What me have in mind is to define a single number, which we shall call directivity, to express in relative terms the lack of isotropy of the intensity tensor.

This number should not depend on the orientation of the principal axes, because it is not orientation what we want to describe; two motions having their major, intermediate and minor principal components of intensity, respectively equal to one another, should have the same directivity even if their principal directions do not coincide. Directivity should be a homogeneous function of degree zero of the components of the intensity tensor, so it does not depend on changes of scale; precisely this property is what we mean to say by the words "in relative terms"; if all the components of the intensity tensor are multiplied by a common arbitrary factor, directivity should remain the same. Directivity should be invariant for all rotations and changes of sense of the coordinate axes (orthogonal transformations). And, finally, it should be equal to zero when all three principal components of intensity are equal.

There are several possible options to define a number that satisfies all of the above conditions. The following appears to be satisfactory for three dimensional motion.

The directivity of ground acceleration intensity at a point 0, or simply the directivity of ground acceleration at 0, is herein defined, in terms of the three principal components of intensity, as the non-negative number δ given by

$$\delta^2 = \frac{1}{2I_0^2}\left[(I_1-I_2)^2+(I_2-I_3)^2+(I_3-I_1)^2\right] \tag{6}$$

It can be shown that $0 \leq \delta \leq 1$. For $\delta = 0$ it is necessary that $I_1 = I_2 = I_3$; i.e, the intensity tensor should be isotropic. To have $\delta=1$, the intermediate and the minor principal components should vanish.

Directivity and the Intensity Deviation Tensor

The intensity tensor at 0 can be written as the sum of a spherical plus a deviatoric part. Referred to the principal directions at 0, this decomposition takes the following form

$$I = diag(I_1,I_2,I_3) = \frac{1}{3}I_0[\delta_{ij}] + diag(I_1',I_2',I_3') \tag{7}$$

where $[\delta_{ij}]$ is the 3x3 identity matrix and

$$I_i' = I_i - \frac{1}{3}I_0 \qquad (i=1,2,3) \tag{8}$$

The first term on the right-hand side of eq. (7) is the spherical or isotropic part of I. The second term is the intensity deviation tensor

$$I' = diag(I_1', I_2', I_3') \tag{9}$$

that describes the deviation of I from isotropy. Observe that the principal direction of I and I' coincide.

The trace or first invariant J_1' of the deviation tensor vanishes identically. The second invariant J_2' is a homogeneous quadratic expression in the components of I'; it is by definition the negative of the sum of the three principal minors of I'. When referred to the principal directions

$$J_2' = -(I_1'I_2' + I_2'I_3' + I_3'I_1') = -\delta_{ij}I_1'I_j' \tag{10}$$

It is easy to show that

$$6J_2' = (I_1-I_2)^2 + (I_2-I_3)^2 + (I_3-I_1)^2 \tag{11}$$

and, therefore,

$$\delta^2 = \frac{3J_2'}{I_o^2} \tag{12}$$

Directivity in the Horizontal Plane

In practice, three-component strong-motion accelerographs are always installed so that one of the accelerometers picks up the vertical component; the other two accelerometers will then be aligned in two mutually orthogonal horizontal directions. Let 0z be vertical, with 0x (respectively, 0y) oriented in the direction of one the horizontal accelerometers. The sum $I_{xx} + I_{yy}$ remains constant when the accelerograph is rotated leaving 0z fixed. The sum

$$I_h = I_{xx} + I_{yy} \tag{13}$$

deserves then the name of intensity on the horizontal plane at 0.

It is evident that I_h is an upper bound for the intensity in any arbitrary horizontal direction. This upper bound is attained only if the ground motion at 0 is polarized on a vertical plane.

Structures are, in general, more sensitive to horizontal than to vertical ground motion. Therefore, the value of I_h and the azimuthal distribution of intensity of the horizontal components of ground motion are specially interesting objects of study from the standpoint of earthquake resistant design. All the necessary information for that kind of undertaking is contained in the matrix

$$I = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix} \tag{14}$$

It seems appropriate to remark here that the principal directions of this 2 x 2 matrix will coincide with two of the principal directions of the 3 x 3 matrix defined by the eqs (1) and (2) if and only if one of the principal directions of the latter is vertical.

Instead of adapting the general definition of eq (6) to the case of plane motion, we shall define local directivity in the xy-plane at the point 0 directly by the equation

$$\delta = \frac{I_1 - I_2}{I_1 + I_2} = \frac{I_1 - I_2}{I_h}, \; (I_1 \geq I_2) \tag{15}$$

where I_1 and I_2 are the principal components of the 2x2 matrix of eq. (14); in terms of the elements of that matrix

$$\delta = (I_{xx}+I_{yy})^{-1} \, [(I_{xx}-I_{yy})^2 + 4I_{xy}]^{1/2} \tag{16}$$

ACCELERATION AND VELOCITY DIRECTIVITIES FOR THE MARCH 3, 1985 EARTHQUAKE (M_s = 7.8)

A total of 23 three-component accelerograms obtained at stations operated by the University of Chile (Department of Geophysics and Department of Civil Engineering) during the March 3, 1985 earthquake that hit Central Chile were processed to obtain the local directivities δ_a, for horizontal ground acceleration, and δ_v, for horizontal ground velocity. Results are summarized in Table 1. The last two columns of that Table give the intensity on the horizontal plane based on ground acceleration $(I_h)_a$, and the distance of each station to the instrumental hypocenter, respectively.

Table 1. Local directivities of horizontal ground motion during the March 3, 1985, Central Chile earthquake.

Station identification	δ_a	δ_v	$(I_h)_a$ (m/s)	R (km)
1. Illapel	0.124	0.147	0.54	179
2. Los Vilos	0.191	0.097	0.65	138
3. La Ligua	0.088	0.217	1.03	97
4. Zapallar	0.110	0.063	3.75	75
5. Ventana	0.289	0.502	6.30	52
6. San Felipe	0.053	0.091	6.44	116
7. Llay Llay	0.110	0.180	10.77	92
8. Viña del Mar	0.502	0.629	8.51	31
9. El Almendral (Valp)	0.214	0.233	5.59	28
10. UTFSM (Valp)	0.248	0.648	1.85	28
11. Las Tórtolas	0.151	0.141	1.93	115
12. Quintay	0.240	0.427	3.35	30
13. Llolleo	0.379	0.418	22.21	64
14. Melipilla	0.056	0.084	18.89	89
15. Rapel	0.370	0.446	2.13	91
16. Pichilemu	0.407	0.029	2.52	143
17. San Fernando	0.166	0.259	3.90	182
18. Iloca	0.233	0.268	4.40	177
19. Hualañé	0.284	0.075	1.60	203
20. Constitución	0.349	0.349	1.00	251
21. Talca	0.044	0.264	1.63	255
22. Cauquenes	0.326	0.296	0.96	319
23. Chillán Viejo	0.169	0.407	0.27	410

When all pairs of values (δ_a , δ_v) given in Table 1 are considered, the correlation between these two variates is found to be weak; the simple correlation coefficient is r = 0.517. However, if pairs belonging to stations where the instrumental intensity on the horizontal plane was larger than 3 m/s are analyzed by separate, a strong and highly significant correlation is found : r = 0.929 for 11 pairs of values. For stations where $(I_h)_a$ < 3 m/s, there is practically no correlation: r = 0.086, for 12 pairs of values.

Similar results are obtained if the data are separated according to hypocentral distance. There are 11 stations less than 100 km away from the hyppocentre; for them, r = 0.816. For the remaining 12 stations there is no correlation at all (r = 0.028).

The same conclusions are reached using Spearman's test for rank correlation between two variates.

Regression equations for δ_v versus δ_a were established with the following results

$$\delta_v = 1.31\,\delta_a \quad for \quad (I_h)_a > 3\,m/s \tag{17}$$

$$\delta_v = 1.41\,\delta_a \quad for \quad R<100\ km \tag{18}$$

Thus, we may conclude that at distances from the hypocenter smaller than 100 km, as well as for stations where the instrumental intensity on the horizontal plane (based on ground acceleration) was larger than 3 m/s, there exists a strong correlation between the acceleration and the velocity directivities, δ_a and δ_v. Furthermore, under the same conditions, velocity exhibits a higher directivity than acceleration.

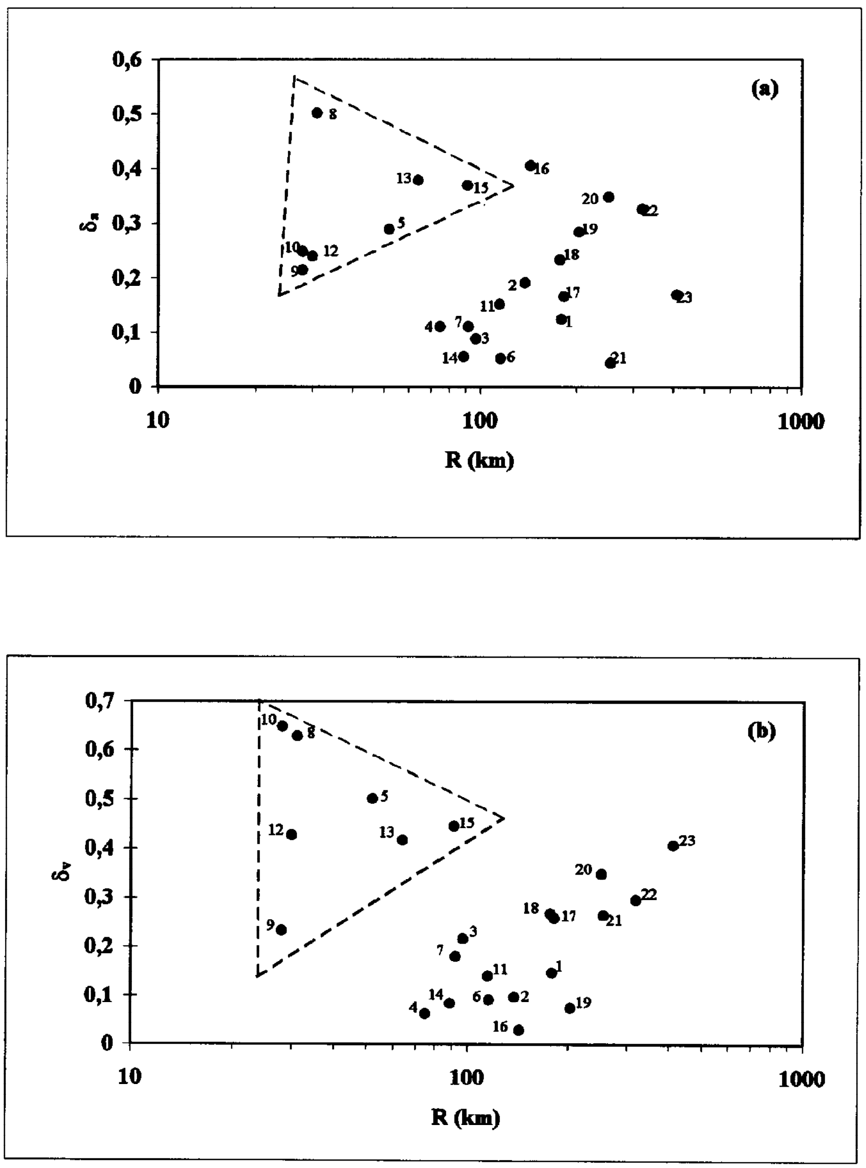

Fig. 1 Directivity of horizontal ground motion in the March 3, 1985 Central Chile earthquake versus hypocentral distance. (Dashed lines enclose all coastal stations in the macro-seismic area). (a) Directivity for ground acceleration; (b) Directivity for ground velocity.

If δ_a and δ_v are plotted as a function of hypocentral distance, then in both cases, the set of representative points clearly separates in two subsets: (a) a coastal subset, formed by stations Nos 5, 8, 9, 10, 12, 13, 15, and (b) a hinterland subset in which the rest of the stations is included. This separation may be inferred also from iso - δ_a and iso - δ_v maps (not reproduced here because of lack of space)

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