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EARTHQUAKE RESPONSE OF MULTI-SPAN CONTINUOUS BRIDGES

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ABSTRACT

The performance of multi-span continuous bridges under seismic excitation and the influence of spatial variability of seimic waves on the dynamic response of bridge structures are investigated. A seismic wave which is spatially variable may be evaluated with a simplified cross-spectral matrix of the modified Kanai-Tajimi type. This seismic wave is assumed to arrive at each of the pier footing with a phase lag and with a variation in dynamic characteristics. The equations motion for the structure including the soil-foundation system are obtained by the substructure method. Since these equations contain nonproportional damping matrices, complex eigenvalue analysis is performed. The results of random vibration analysis are expressed using *rms* displacements and *rms* sectional forces at typical nodal points of the model. It is shown that the nonuniform excitation of supports of mutli-span continuous bridges due to spatial variation of seismic wave has particularly predominant effect for out-of-plane vibrations.

KEY WORDS

Multi-span continuous bridges, caisson-soil foundation, seismic wave, spatial variability, random response

INTRODUCTION

The earthquake response analysis of civil engineering structures is often performed applying identical excitation at all support points. While this approach may be satisfactory for ordinary building structures, long-span structures such as industrial buildings, bridges and dams experience different support movements due to the spatial variability of input seismic wave motion. Several researchers in the past (for example, Somaini (1987), Zerva *et al* (1988), Kawano *et al* (1991), Berrah and Kausel (1992), Kiureghian and Neuenhofer (1992), Nazmy and Abdel-Ghaffar (1992)) have carried out studies related to this problem and have identified some of the major effects of nonuniform support excitations on the response evaluations. The emphasis in the present study is to examine the influence of spatial variation of seismic motion on the in-plane and out-of-plane vibrations of bridge structures. The dynamic soil-structure interaction between the superstructure and the soil-foundation system, which is influenced by the proximity of their natural frequencies and the stiffness and damping of the soil-foundation, also plays important roles on the dynamic response and is considered. Figure 1 shows the model of a multi-span continuous bridge including foundation considered for study. The ground motion is assumed along the bridge axis for in-plane vibrations and perpendicular to the bridge axis for out-of-plane vibrations.

FORMULATION

The governing equation of motion

The multi-span continuous bridge is discretized by finite element method. The equation of motion is obtained by the substructure method in which the total system is hypothetically divided into the superstructure and the caisson-soil foundation system. The displacement of the superstructure can be expressed as the sum of the dynamic displacement of the structure on a fixed base, and the quasi-static displacement due to the interactions with the foundation. The dynamic displacements of the fixed-base structure are treated as the linear combination of the dominating lower vibrational modes. The dynamic response due to seismic motions are mainly dependent upon these modes. The equation of motion of the superstructure is expressed in terms of generalized coordinates which correspond to the lower vibration modes obtained by eigenvalue analysis.

The caisson-soil foundation system, at each support of bridge pier, is modelled as a rigid body which has frequency-independent impedance functions, with two degree-of-freedom of horizontal translation and rocking about its center of gravity. This kind of modelling of impedance functions corresponds to the soil condition in which an uniform layer overlies on a firm half-space substratum (Beredugo and Novak (1972), Yamada *et al* (1979)). Figure 2 shows the caisson-soil foundation model used in the study.

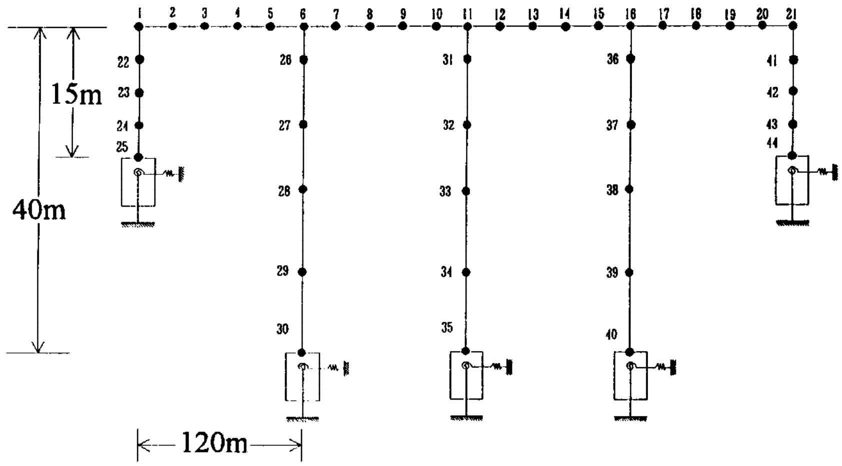

Fig.1 Analytical model of multi-span continuous bridge
including footing-soil foundation system

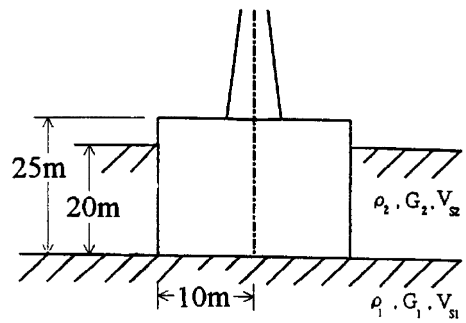

Fig.2 Section of a main caisson footing

The governing equation of motion for the total system is obtained by combining the equation of motion for the superstructure and the equation of motion for the caisson-soil foundation system, using the compatibility conditions of displacements and the equilibrium conditions of forces at the base nodal points, as

$$[M_r]\{\ddot{r}\}+[C_r]\{\dot{r}\}+[K_r]\{r\}=[F_r]\{\ddot{u}_g\} \tag{1}$$

where

$$[M_r]=\begin{bmatrix}[\ddots I \ddots] & [\tilde{M}_{sf}] \\ [\tilde{M}_{sf}]^T & [\tilde{M}_f]\end{bmatrix},\quad [C_r]=\begin{bmatrix}[\ddots 2\beta_{sj}\omega_{sj} \ddots] & [0] \\ [0] & [\tilde{C}_f]\end{bmatrix},\quad [K_r]=\begin{bmatrix}[\ddots \omega_{sj}^2 \ddots] & [0] \\ [0] & [\tilde{K}_f]\end{bmatrix},\quad [F_r]=\begin{bmatrix}[\tilde{M}_{sf}] \\ [\tilde{M}_f]\end{bmatrix},$$

$$\{r\}=\begin{Bmatrix}\{q\} \\ \{u_f\}\end{Bmatrix},\quad \{u_a^c\}=[\Phi]\{q\}$$

in which $[\ddots I \ddots]$ is the unit matrix, subscript s denotes the superstructure, subscript f denotes the caisson-soil foundation, $[\Phi]$ is the modal matrix of the undamped superstructure with fixed-base, ω_{sj} is the natural frequency of the superstructure with fixed-base for jth vibration mode, β_{sj} is the corresponding damping ratio, $\ddot{u}_g$ is the ground acceleration, u_f is the displacement of the gravity center of the foundation (which consists of translational and rotational components) , u_a^c is the dynamic displacement of the unrestrained nodal points of the superstructure with fixed-base and q is the modal displacement of the superstructure.

The equation of motion (Eq.(1)) contains nonproportional damping matrix. Therefore, the conventional modal superposition tecnnique based on classical damping, in which the damping matrix is orthogonal with respect to its undamped real-valued mode shapes and the equations of motion can be decoupled, can not be applied (Villaverde (1988)). One of the approaches for dealing with such problems is the application of complex eigenvalue analysis. For this purpose, firstly Eq.(1) is transformed into a first order differential equation as

$$\{\dot{s}\} + [D]\{s\} = [Q]\{\ddot{u}_g\} \tag{2}$$

where

$$[D] = \begin{bmatrix} [M_r]^{-1}[C_r] & [M_r]^{-1}[K_r] \\ -[\cdot.I\cdot.] & [0] \end{bmatrix}, \quad [Q] = \begin{bmatrix} [M_r]^{-1}[F_r] \\ [0] \end{bmatrix}, \quad \{s\} = \begin{Bmatrix} \{\dot{r}\} \\ \{r\} \end{Bmatrix}$$

The homogeneous solution of Eq.(2) consists of complex-valued eigenvalues λ_j and complex-valued modal vectors Ψ. If ω_j is the natural frequency of the structure including the caisson-soil foundation system and β_j is the damping ratio which includes the structural damping and the radiation damping through the caisson-soil foundation system, the complex eigenvalues are given as

$$\lambda_j = -\beta_j\omega_j + \mathrm{i}\omega_j\sqrt{\left(1-\beta_j^2\right)} \tag{3}$$

where i denotes the unit imaginary number. Now using the complex eigenvalues and complex modal vectors, Eq.(2) can be rewritten as

$$\{\dot{y}\} + [\cdot.\lambda_j\cdot.]\{y\} = [W]\{\ddot{u}_g\} \tag{4}$$

where

$$[\Psi]^{-1}[D][\Psi] = [\cdot.\lambda_j\cdot.], \quad [\Psi]^{-1}[Q] = [W], \quad \{s\} = [\Psi]\{y\}$$

Random response analysis

The dynamic analysis for spatially varying seismic inputs can be performed in time domain or in frequency domain. For the time-domain approach, input acceleration at each support point consists of a complete time-history and often the results vary significantly when different acceleration records are chosen as inputs. On the other hand, the frequency-domain approach uses a statistical value of ground acceleration at the support points, expressed using power spectral density functions which give the amplitudes and frequency components of ground motions for a given intensity. The main advantage of this method is that the response values can be taken as representative from a statistical viewpoint and this approach is used in this paper.

Now applying the Fourier Transform to both sides of Eq.(4) and rearranging, the generalized response in the frequency-domain is obtained as

$$\{\bar{y}(\omega)\} = [G(\omega)][W]\{\bar{\ddot{u}}_g(\omega)\} \tag{5}$$

where

$$\{\bar{y}(\omega)\} = \int_{-\infty}^{\infty}\{y\exp(-\mathrm{i}\omega t)\,\mathrm{d}t\}, \quad \{\bar{\ddot{u}}_g(\omega)\} = \int_{-\infty}^{\infty}\{\ddot{u}_g\exp(-\mathrm{i}\omega t)\,\mathrm{d}t\}, \quad [G(\omega)] = [\cdot.i\omega+\lambda_j\cdot.]^{-1}$$

Now if a statistical value of seismic motion such as the power spectral density function (PSDF) of ground acceleration $\left[S_{\bar{\ddot{u}}_g\bar{\ddot{u}}_g}(\omega)\right]$ at each support point is given, the PSDF of response $\left[S_{\bar{y}\bar{y}}(\omega)\right]$ can be obtained as

$$\left[S_{\ddot{y}\ddot{y}}(\omega)\right]=\left[G(\omega)\right]\left[W\right]\left[S_{\ddot{u}_g\ddot{u}_g}(\omega)\right]\left[W^*\right]\left[G(\omega)^*\right] \tag{6}$$

in which * denotes the complex conjugate matrix. Now using the Wiener-Khintchine relationship between the autocorrelation function and PSDF, the covariance matrix of the response can be obtained as

$$\mathrm{E}\left[\{y\}\{y\}^{\mathrm{T}}\right]=\left[R_{yy}(0)\right]=\int_{-\infty}^{\infty}\left[S_{\bar{y}\bar{y}}(\omega)\right]\mathrm{d}\omega \tag{7}$$

From these results, the *rms* response dispacements of superstructure are determined as

$$\mathrm{E}\left[\{u_a^c\}\{u_a^c\}^{\mathrm{T}}\right]=[\Phi]\,\mathrm{E}\left[\{y\}\{y\}^{\mathrm{T}}\right][\Phi]^{\mathrm{T}} \tag{8}$$

and the *rms* response member forces of each element can be obtained using the element stiffness matrix $\left[K_e\right]$and the corresponding PSDF of displacement response $\left[S_{ee}(\omega)\right]$as follows

$$\mathrm{E}\left[\{F_e\}\{F_e\}^{\mathrm{T}}\right]=\int_{-\infty}^{\infty}\left[K_e\right]\left[S_{ee}(\omega)\right]\left[K_e\right]^{\mathrm{T}}\mathrm{d}\omega \tag{9}$$

<u>The power spectral density function (PSDF) of ground motion including spatial variability effects</u>

The spatial variation of ground motion at support points of multi-span structures is caused by various factors. A detailed discussion on this topic is presented by Kiureghian and Neuenhofer (1992). The main causes are: a) wave passage effect: due to the difference in arrival times of the seismic waves at different support points, b) incoherence effect: due to the numerous reflections and refractions of waves in the soil medium and the signals from the sourse superimposing differently at each support, and c) local effect: due to the local site conditions which influence the wave propogation in the bedrock to the foundation. The PSDF of ground accelerations including the spatial variability of ground motions can be expressed as

$$\left[S_{\ddot{u}_g\ddot{u}_g}(\omega)\right]=\left[S_{x_m x_n}(\omega)\right]\left[S_1(\omega)\right]\left[S_2(\omega)\right]S_0 \tag{10}$$

where

$$\left[S_{x_m x_n}(\omega)\right]=\exp\left[-\left(\omega h_{ef}+\mathrm{i}\omega\right)\frac{|x_m-x_n|}{C_s}\right], \qquad S_0=\frac{\sigma_{\ddot{u}_g}^2}{\pi\left(1+4h_g^2\right)\omega_g/\left(2h_g\right)}$$

$$\left[S_1(\omega)\right]=\frac{\left(\omega/\omega_f\right)^4}{\left(1-\left(\omega/\omega_f\right)^2\right)^2+4h_f^2\left(\omega/\omega_f\right)^2}, \qquad \left[S_2(\omega)\right]=\frac{1+4h_g^2\left(\omega/\omega_g\right)^2}{\left(1-\left(\omega/\omega_g\right)^2\right)^2+4h_g^2\left(\omega/\omega_g\right)^2}$$

x_m, x_n are coordinates of the reference points, C_s is the phase velocity of seismic motion, ω_g, h_g are the filter parameters (characteristic ground frequency and characteristic ground damping ratio respectively) of the well-known Kanai-Tajimi type, $\sigma_{\ddot{u}_g}$ is the *rms* value of ground acceleration, S_0 is the intensity of white noise at a support, ω_f, h_f are the filter parameters (frequency parameter and damping parameter respectively) of a second filter, introduced to overcome the limitations of Kanai-Tajimi type filter occurring in the region of low-frequencies. On substituting Eq.(10) into Eq.(6) and carrying out random response analysis as explained before, the *rms* displacements and section forces of the structure can be determined and the effects of spatial variation of ground motion can be quantitatively expressed.

NUMERICAL RESULTS AND DISCUSSIONS

A numerical computation is carried out for the high-elevated four-span continuous bridge structure shown in Fig.1. The structure is idealized by finite element method into 44 nodes and 43 elements. The total length of the bridge is 480m with each span being 120m. The height of abutment piers is 15m where as the main piers are 40m high. Out of three translational degrees of freedom and three rotational degrees of freedom at each nodal point, two translational (one vertical and one horizontal) and one rotational degrees of freedom are considered for in-plane motion whereas one translational (one horizontal in the transverse direction) and two rotational degrees of freedom are considered for out-of-plane motion. The equation of motion for the superstructure (deck and piers) is formulated with predominant vibration modes which correspond to natural frequencies of less than 30 rad/s. The structural damping ratio of structure subsystem is assumed to be 2%. The foundation is made of caisson footing, embedded 20m (18m for abutment piers) in the surface layer of stratum on a firm substratum as shown in Fig.2. The equation of motion for the footing-soil foundation is obtained using frequency-independent impedance functions corresponding to the soil condition in which an uniform layer overlies on a firm half-space substratum. The equation of motion for the total system is obtained by the substructure method. The earthquake response of this structure is then determined using the frequency-domain random-vibration approach and complex eigenvalue analysis.

The shear wave velocity in the firm substratum V_{s1} is taken as 300m/s and the ratio of shear wave velocity in the overlying layer to that in the firm substratum is assumed as, V_{s2} / V_{s1}=0.3 for the analysis since significant soil-structure interaction effects are found to occur at these values. Tables 1 and 2 show the values of natural frequencies and damping ratios of the total system for upto 9th vibration mode obtained by complex eigenvalue analysis for free-vibrations of in-plane and out-of-plane cases respectively. As the natural frequencies of these two cases are well-separated, they can be considered independently for the analysis.

The PSDF of Eq.(10) is used as the seismic wave input incorporating the spatial variability effects. The assumed parameters of this equation are: $\omega_g = 15$ rad/s, $\omega_f = 1.6$ rad/s, $h_g = h_f = 0.6$, indicating relatively firm soil condition and *rms* ground acceleration $\sigma_{\ddot{u}_g} = 100$ gal corresponding to a severe earthquake situation.

Table 1. Natural frequencies & damping ratios (in-plane vibration)

Vibration mode	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Natural frequency (rad/s)	3.281	3.817	4.914	5.055	6.479	13.06	13.19	13.69	25.34
Damping ratio (%)	2.01	2.00	9.18	2.17	2.00	29.21	29.39	28.03	28.63

Table 2. Natural frequencies & damping ratios (out-of-plane vibration)

Vibration mode	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Natural frequency (rad/s)	4.980	6.490	9.383	10.86	10.92	11.05	17.05	20.60	20.75
Damping ratio (%)	3.67	3.19	2.87	38.60	38.32	37.78	2.38	3.12	46.06

rms displacement responses:

In-plane vibration: Figures 3 and 4 show the *rms* displacements of some of the typical nodes of the structure in the vertical and horizontal (longitudinal) directions against phase velocity of a seismic wave, with the seismic-induced motion of each support being along the bridge axis. The variations in the vertical displacements are generally smaller whereas the horizontal displaments increase with the increase in the phase velocity of seismic wave and approach a constant value. This is due to the fact that when the phase velocity is higher, all the supports are subjected to mostly uniform motion because the seismic wave may be considered to arrive at all support points simultaneously. Also the vertical displacements are larger than horizontal displacements for deck nodes and viceversa for pier nodes (results for pier nodes are not shown) as expected.

Out-of-plane vibration: Figures 5 and 6 show the *rms* displacements of typical nodes of deck, and pier in the horizontal (transverse) direction, plotted against the phase velocity of the seismic wave, with the seismic-induced motion of each support being in a direction transverse to the bridge axis. The response values increase with the increase in phase velocity and approach a steady value. The responses for both deck and pier nodes are several times larger than their equivalent values for in-plane vibration indicating that spatial variation of seismic wave is particularly important for out-of-plane vibrations.

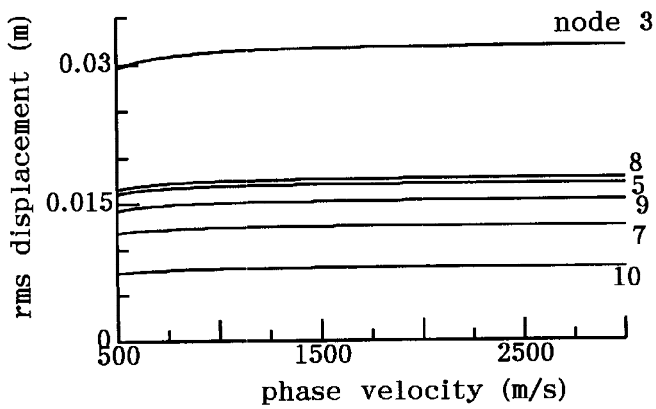

Fig.3 Vertical displacements of deck (in-plane)

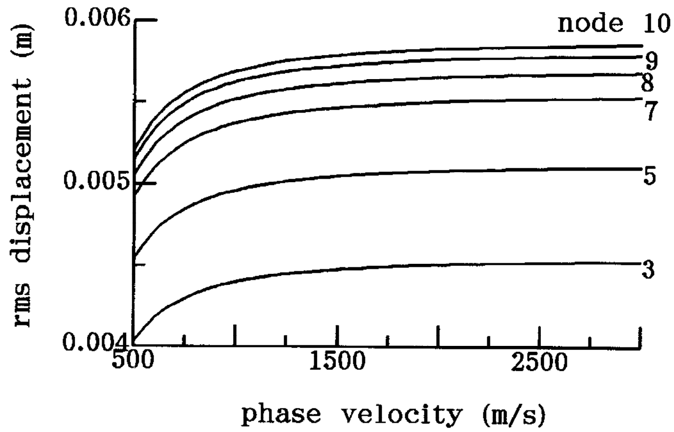

Fig.4 Horizontal displacements of deck (in-plane)

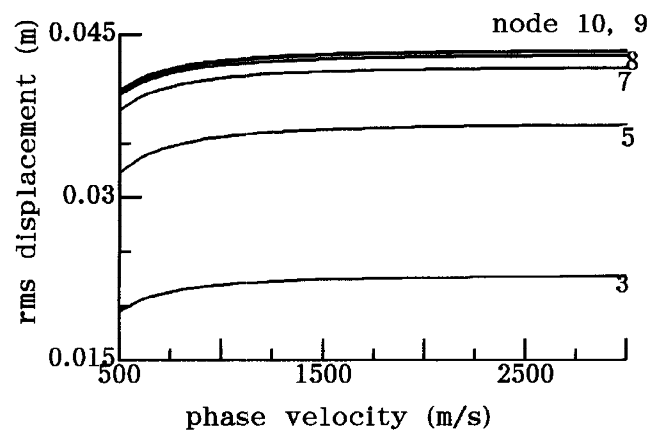

Fig.5 Transverse displacements of deck (out-of-plane)

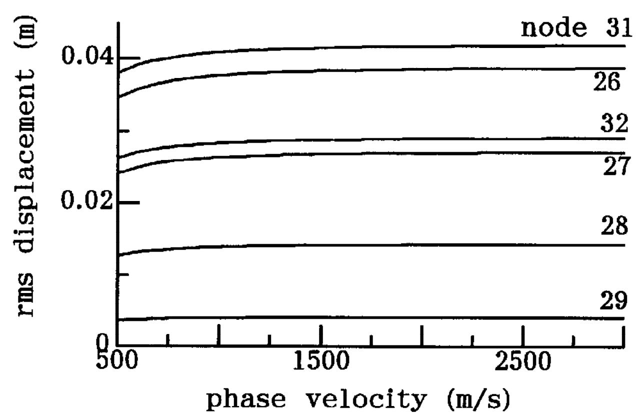

Fig.6 Transverse displacements of pier (out-of-plane)

Section force responses:

In-plane vibration: Figures 7 and 8 show the shear force and bending moment respectively at typical nodes of deck plotted against the phase velocity of seismic wave. Figures 9 and 10 show the correponding section forces at typical nodes of piers. The shear forces in the deck are generally higher (except at node 10) whereas the shear forces at nodes 26 and 27 in the piers are higher for lower values of phase velocity and are reduced with increasing phase velocity. When the phase velocity is lower, the support are subjected to nonuniform motion thereby causing larger shearing forces in the deck and in those nodes in the pier which are located near the center of the structure. For both deck and piers, the bending moments at different nodes show different variation patterns with increase in phase velocity and approach a constant value. This kind of complicated response characteristics indicate the presence of strong influence from second and higher vibration modes of the system.

Out-of-plane vibration: Figures 11 and 12 show the shear force and bending moment respectively at typical nodes of the deck plotted against the phase velocity of seismic wave. Figures 13 and 14 show the correponding section forces at typical nodes of piers. The shear forces at the nodes for deck as well as for piers decrease with increasing phase velocity and approach a steady value for the same reason mentioned above. Bending moments show different patterns for different nodes indicating the presence of higher order mode effects. Also, compared to the results of in-plane vibration, the section forces are several times higher.

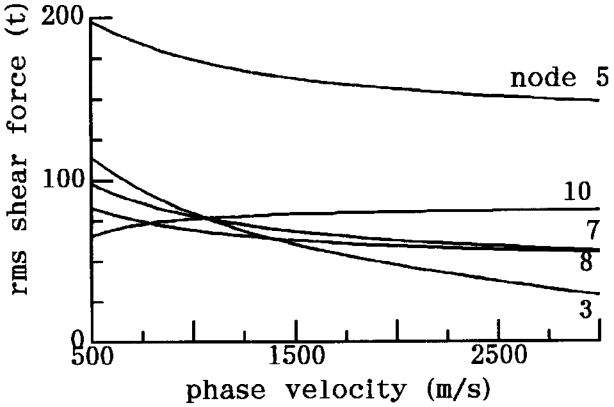

Fig.7 Shear force of deck (in-plane)

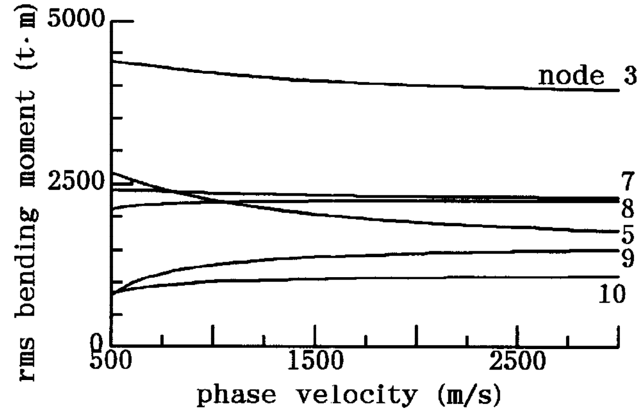

Fig.8 Bending moment of deck (in-plane)

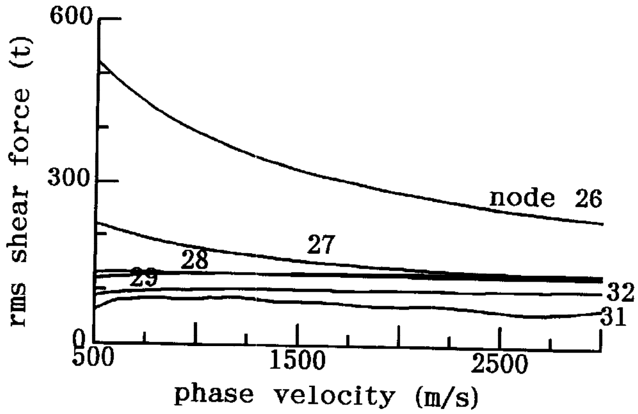

Fig.9 Shear force of pier (in-plane)

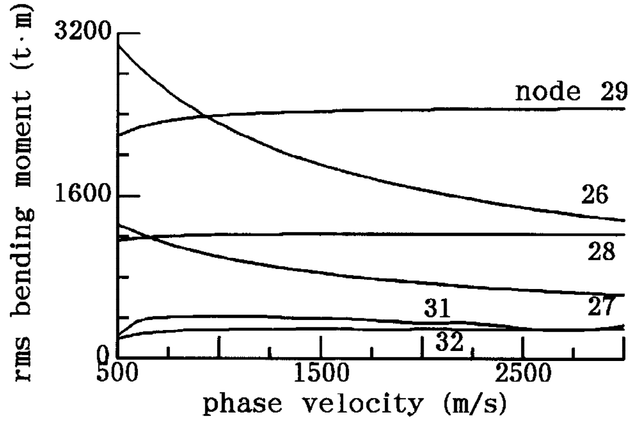

Fig.10 Bending moment of pier (in-plane)

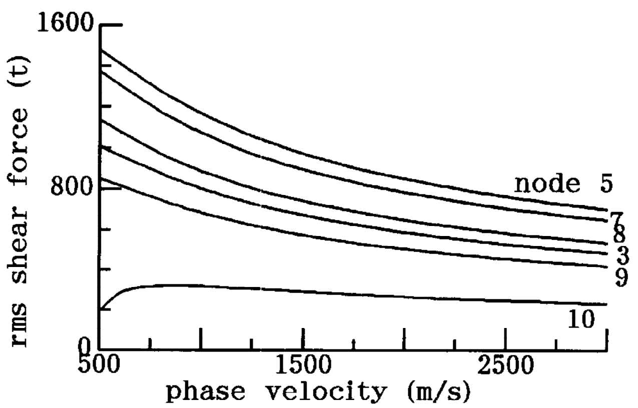

Fig.11 Shear force of deck (out-of-plane)

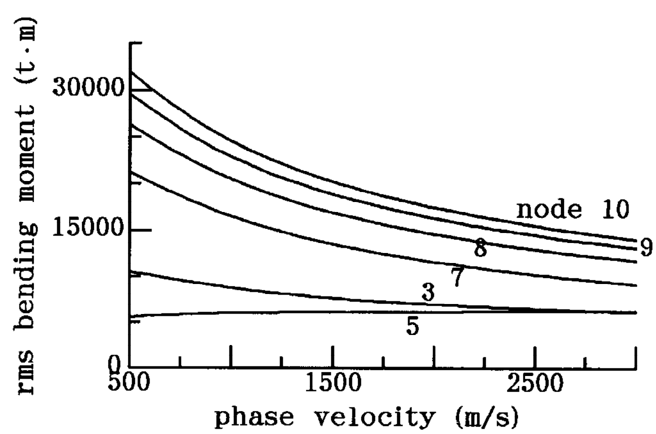

Fig.12 Bending moment of deck (out-of-plane)

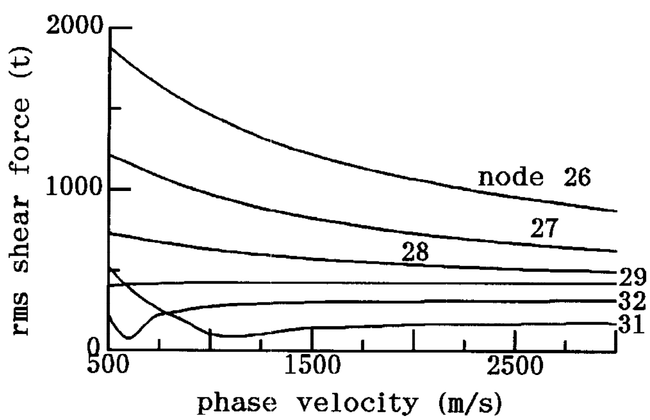

Fig.13 Shear force of pier (out-of-plane)

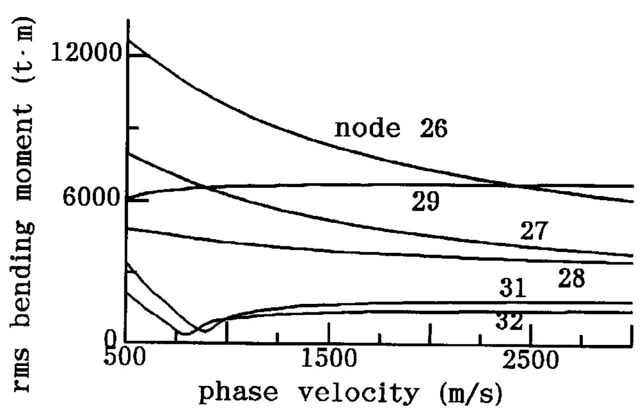

Fig.14 Bending moment of pier (out-of-plane)

CONCLUSIONS

The dynamic response analysis of a multi-span high-elevated continuous bridge including footing-soil foundation system, subjected to a spatially varying ground motion, is carried out using frequency-domain random vibration aproach and complex eigenvalue analysis. The main results are summerized as follows:

1. As the multi-span bridge including foundation system has longer natural periods, several vibration modes, starting from the first and lowest, contribute significantly to the dynamic response in terms of displacements and section forces of the structure. It is extremely important to determine accurately these vibration mode shapes and their natural frequencies.
2. The *rms* displacements of deck as well as pier are slightly influenced by the spatial variation of seismic wave at lower phase velocities. The response values generally increase with increasing phase velocity and approach a steady value when the phase velocity reaches around 3000m/s. When the phase velocity is equal or higher than this value, the supports are subjected to uniform motions.
3. The effects of nonuniform vibration of supports, due to a phase difference of input seismic wave, are especially important for sectional forces (namely, axial forces, shear forces, bending moments and torsional moments) of structural nodes, for both in-plane and out-of-plane vibrations. The results show that the sectional forces at certain nodes of the bridge, considering the phase lag of seismic wave, may be upto 2 to 3 times larger than their corresponding values when the seismic wave arrives at all supports at the same instant. Therefore careful considerations must be given not only for the structural characteristics but also the phase lag of the seismic wave.
4. The *rms* displacements as well as section forces for the case of out-of-plane vibrations are several times higher relative to their corresponding values for the case of in-plane vibrations. Therefore this research shows that the spatial variability of seismic wave is particularly important for out-of-plane vibrations of multi-span bridge structures.

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